The Egg Roulette Game

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Overview of Lesson

This lesson uses a probability game and computer simulations to explore the law of large numbers, conditional events, sampling distributions, and the central limit theorem. Video clips from The Late Night show are shown to students where Jimmy Fallon plays the egg roulette game with celebrities. Students play the game and are asked several questions regarding the probabilities of winning. The class data is used to estimate an unknown probability. Part I of the lesson utilizes a preconstructed Fathom simulation to collect more data, estimate an unknown probability, and demonstrate the law of large numbers. A discussion of how to calculate the probability follows. In Part II of the lesson a free online computer applet is used to display the sampling distribution of a sample proportion by using the assumed probability obtained in Part I. Students explore variability, mean, and properties of an empirical sampling distribution. Applications of the central limit theorem are discussed using extensions of the egg roulette game.

The main goals of this lesson are: (1) to recognize conditional probabilities, (2) understand that an unknown probability can be estimated using the law of large numbers, (3) construct and explore the variability of sampling distributions and central limit theorem using randomization-based simulations.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are: formulate a question, design and implement a plan to collect data, analyze the data, and interpret results in the context of the original question. This is a **GAISE Level C** activity.

Common Core State Standards for Mathematical Practice

1. Make inferences and justify conclusions from samples.
2. Understand independence and conditional probability and use them to interpret data.
3. Use probability to evaluate outcomes of decisions.

Learning Objectives Alignment with Common Core and NCTM PSSM (Grades 9 – 12)

Learning Objectives	**Common Core State Standards**	**NCTM Principles and Standards for School Mathematics**
Use data from a sample to estimate a population proportion; use simulations to explore the variability of sample statistics from a known population and construct	HSS.IC.B4	Grades 9 – 12 D3.f

sampling distributions.		
Recognize and explain the concepts of conditional probability and independent events	HSS.CP.A5	Grades 9 – 12 D4.j
Analyze decisions and strategies using probability concepts. Use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty	HSS.MD.B7	Grades 9 – 12 D.g

Prerequisites
Students should know the advantages of having a large sample vs. a small sample, the basic concept of a probability, and be able to determine if events are independent or conditional.

Time Required
70 – 90 minutes or (1 - 2 class periods)

Materials and Preparation Required

- Brown paper bags or envelopes (1 for each pair of students)
- White beads and red beads (8 white beads and 4 red beads for each bag)
- 12 plastic eggs and confetti can be used instead of beads to have one pair of students act out the game before the entire class plays the game in pairs
- Fathom software. Available at http://fathom.concord.org. A free 30 day trial is available.
- Fathom file Simulating the Egg Roulette Game
- Instructional lesson plan
- Student activity sheet
- Computer internet access is necessary for Part II. The applet works well on most browsers, Google, Firefox, or Internet Explorer. The simulation can be found under Categorical Response/One Proportion. Note: ‘probability of heads’ will change to ‘probability of success’ when you change the probability from 0.5 to 0.55 and hit the enter key. http://www.rossmanchance.com/ISIapplets.html

The Egg Roulette Game
Teacher's Lesson Plan

Part I

Describe the Context and Formulate a Question

Begin by showing the two video clips of the egg roulette game recorded on The Late Night show found at the following links

https://www.youtube.com/watch?v=y6jb0cOYqAI,
https://www.youtube.com/watch?v=ZVUfnJipFh0.

After watching the videos, ask a student to review the rules of the game: one dozen eggs are presented; 8 hard boiled and 4 raw. There is no visible difference in the eggs. Players take turns selecting an egg and cracking that egg on their own head. The first person to crack two raw eggs on his head loses. Guest of the show always goes first. Two student volunteers can act out playing the game with plastic eggs and confetti. Four plastic eggs contain confetti to represent raw eggs, while empty eggs represent hardboiled eggs. Using colored eggs may prevent students from seeing which eggs are empty, or a towel can be placed over the eggs so students cannot see.

Ask students: Do you think each player has the same probability of winning the game when guest of the show goes first? Is the probability of winning similar to flipping a coin? Are the events of selecting eggs conditional or independent events? Encourage students to generate their own investigative questions by having students write down their answers to the previous questions and a question they could investigate concerning the probabilities.

Some common student responses and questions:

- The events are conditional events because whatever egg is selected there is one less of that type of egg to choose from, which changes the probability.
- Does the guest of the show have an advantage since he goes first and there are more hard boiled eggs to select from at the start?
- Does the guest have a disadvantage if there are only raw eggs left to choose from at the end that player must select the raw egg?
- Maybe it depends on what happens in the first few turns because that will determine how many raw eggs are left to choose from at the end for each player?

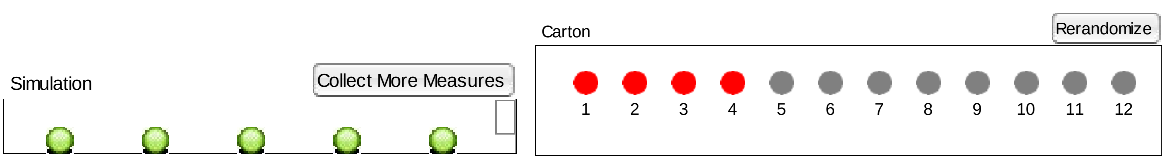
Collect Data

Pair students and pass out the student activity worksheet and presorted bags of beads. There are 8 white beads and 4 red beads in each bag; white beads represent hard boiled eggs and red beads represent raw eggs. Emphasize that in order to have an unbiased, random sample students must not look in the bag when selecting a bead. Beads are selected without replacement. Name one student Player 1 (Guest) and the other student Player 2 (Jimmy Fallon); instruct students to play the game 5 times recording the number of wins for each player. <u>It is imperative that Player 1 (Guest) takes the first turn in all 5 trials</u>. Have each pair of students record the number of wins for each player, in their 5 trials. Then the teacher will total the results of the class and record the total number of wins for each player as a running tally on the board. Using the class totals, the teacher can calculate the proportion of wins for Player 2. This proportion is an empirical probability that Player 2 (Fallon) wins the game, when guest of the show selects the first egg.

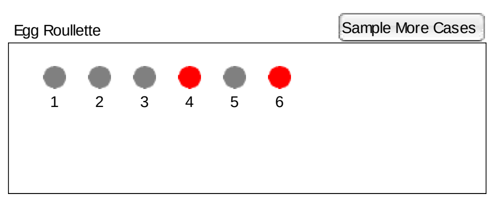
Analyze Data
Ask students: Does this empirical probability give Fallon an advantage or disadvantage? What did you notice while you were playing the game? Based on your experience, does either player have an advantage? It is important to note that we are discussing the possible advantage of a player when guest of the show takes the first turn. Is the sample size large enough to use the class results as an empirical probability estimate of the actual probability? Could we also use a theoretical approach to compute the probability, such as a tree diagram to list all possible outcomes? Time permitting, the teacher can have students attempt to build a tree diagram. The theoretical calculation of the probability is challenging using a tree diagram and can demonstrate the benefits of using a computer simulation to estimate this unknown probability.

After playing the game, most students will report that Player 2 (Fallon) has an advantage. Some will say Player 1 has an advantage if that player won more games in their 5 trials. Students have enough knowledge of probability and sampling to say we need a larger number of trials, i.e., to play the game a large number of times and look at the proportion of wins for Player 2 to estimate the probability. This discussion transitions to the Fathom simulation and discovery of the law of large numbers.

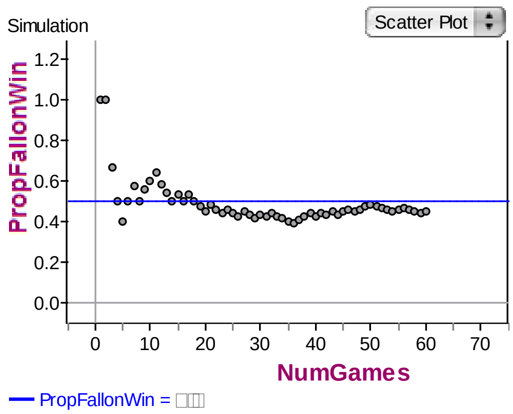
Collect More Data
There are two options for collecting more data using a computer simulation. 1) Use the fathom file to simulate games played and discover the probability. 2) Use the free online applet to generate results of games played. The second option does not allow for discovery of the probability, but can be used to construct and explore the variability of sampling distributions and the central limit theorem. This will be discussed in Part II of the lesson. Open the Fathom file, Simulating the Egg Roulette Game, and explain that the computer will play the game 5 games at a time by clicking on 'Collect More Measures'. The carton image displays raw eggs as red circles and hardboiled eggs as gray circles.

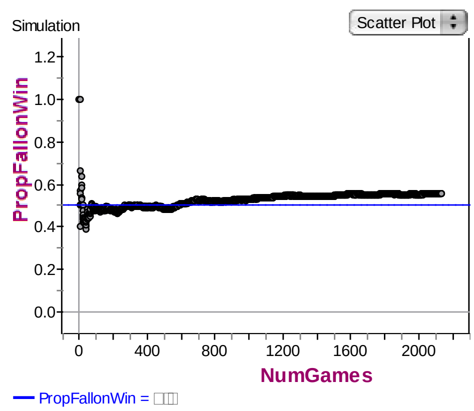


The most recent game results are displayed here:



In the picture above, Fallon lost this game; he selects on the even numbered turns as Guest takes the first turn. Both scatterplots display the proportion of wins for Fallon; the plot on the left after 60 trials and the right scatterplot after more than 2000 trials. Demonstrated in the table are the simulation results: the number of trials, the number of wins for Fallon, and the proportion of wins for Fallon. Continue to collect more measures until the proportion of wins for Fallon seems to stabilize. Since the simulation is random, this may occur after 1000, 2000, or upwards of 10, 000 trials. To increase the number of games played at one time, double click inside the simulation box in the blank space above the green dots. The 'Inspect Simulation' box will open. Select the far right heading, 'Collect More Measures', (turn the animation off to avoid lag time), and enter a numeric value in the text box ‘Measures’ for the number of games played at once. Entering 100 measures will collect the results of 100 games instead of the standard setting of 5 games at a time. Note: The fathom file starts with 60 games played. To start from 0 games played, select ‘Replace existing cases’. The proportion should begin to hover around $0.\overline{5}$. Build a discussion around the law of large numbers using the data to estimate the probability Fallon has of winning the game.





Simulation

	FallonRaw	GuestRaw	FallonWin	NumGames	NumFallonWin	PropFallonWin	<new>
2127	0	2	1	2127	1181	0.555242	
2128	1	2	1	2128	1182	0.555451	
2129	0	2	1	2129	1183	0.55566	
2130	1	2	1	2130	1184	0.555869	
2131	1	2	1	2131	1185	0.556077	
2132	2	0	0	2132	1185	0.555816	
2133	1	2	1	2133	1186	0.556024	
2134	2	1	0	2134	1186	0.555764	
2135	2	0	0	2135	1186	0.555504	

Interpret Results

Have students use the cumulative proportion of time Fallon wins to consider the original questions:

Do you think each player has the same probability of winning the game when the guest of the show goes first? Is the probability of either player winning similar to flipping a coin?

Using the law of large numbers and the data in the simulated example shown above, we could reasonably estimate the probability of Fallon winning the game as approximately 0.55, meaning that Fallon has a slight advantage.

If this wasn’t done earlier, the teacher can ask students how they might begin to calculate the probability. Consider building a tree diagram to explore all possible outcomes for Fallon winning the game, and conditional probabilities. This will be time consuming and should possibly be considered as a second extension lesson. Using a tree diagram can show how challenging and time consuming it would be to calculate the probability using traditional methods and the power of using a computer simulation. Another option is to explore a negative, hypergeometric distribution to calculate the probability Fallon wins to 5/9. However, this model is most likely beyond the scope of an introductory statistics course.

Part II

After students have engaged in playing the game and discovered the probability, students can explore how likely their empirical results, (the class proportion of wins for Fallon) is to occur. Assuming Fallon has a 0.55 probability of winning the game, we can use this probability as the assumed population proportion of wins Fallon may have over his entire career. Students will explore the shape and variability of a sampling distribution, and what constitutes an unlikely event to answer the following questions.

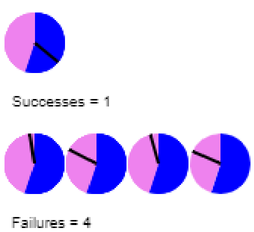
Formulate a Question

Let's assume that Fallon has a 55% chance of winning. If he played the game 24 times per year (about twice per month) with different celebrities on the Tonight Show, what proportion of wins or losses would surprise you? In other words, what percent of games would Fallon have to lose in a year for you to consider this to be highly unusual?

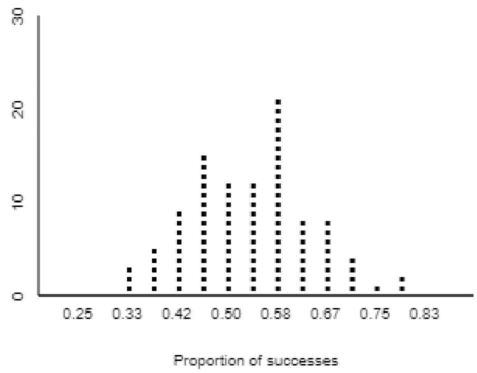
How could we determine if our empirical results, (the class generated data) is unusual? In a class of 30 students, 15 pairs of students play 5 games for a total of 75 games, suppose the class results are 50 wins for Fallon resulting in a sample proportion of approximately 0.67. Would these results seem unusual? How many of the 75 games would you expect for Fallon to win?

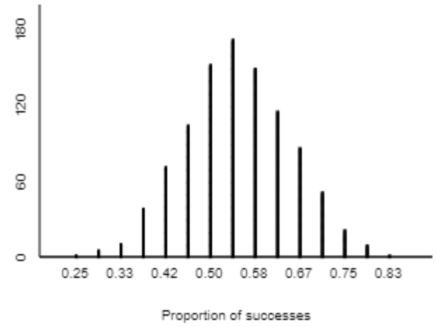
Collect Data

Open the computer applet located at http://www.rossmanchance.com/ISIapplets.html. Enter 0.55 for the 'probability of heads'. (This will change to 'probability of success' after the probability changes to 0.55.) Change 'number of successes' to 'proportion of successes'. Begin with a small sample; say $n = 5$ to explain the concept of the spinner. The computer will play the game 5 times. Each spinner represents the results of one game. Assuming Fallon has a 55% chance of winning, if the spinner lands on blue this means that Fallon won that game. The blue region represents 55% of the area of the spinner. Assuming there is a 45% chance of Fallon losing the game, if the spinner lands in the pink region this means Fallon lost that game. The following results are 1 win for Fallon and 4 games lost.

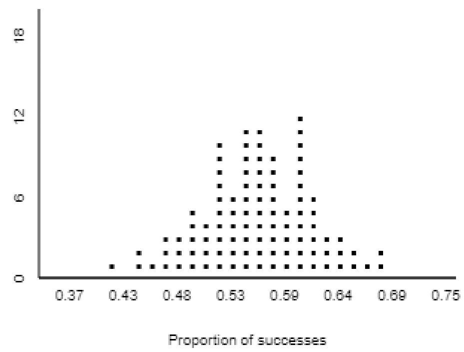


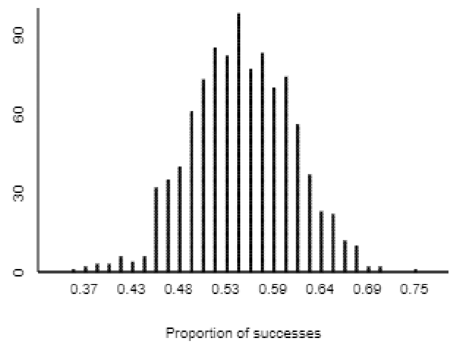
Increase the sample size (n) from 5 to 24 to represent the number of games Fallon will play in one year. Increasing the number of samples to 100 allows us to see what would occur in alternate timelines of those 24 guests. Continue to resample 1000 samples. Dots plots for 100 and 1000 samples are given below, left and right, respectively.





Increase the sample size to 75, (use the total number of games played by the class), and increase the number of samples to 100. Continue to resample 1000 samples of size 75. Dots plots for 100 and 1000 samples are given below, left and right, respectively, each of sample size 75.





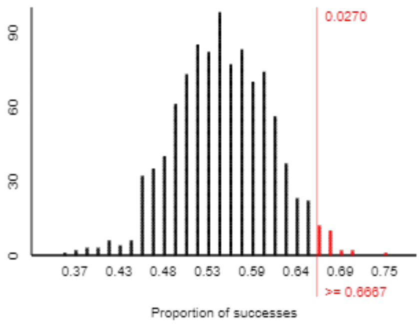
Analyze Data

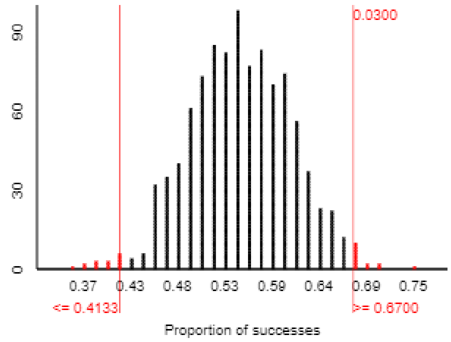
Have students explore the shape, center, and variability of the sampling distribution using the sample sizes of 24 and then 75. What happens to the shape of the distribution as the number of samples increases? What change occurs in the distribution as the sample size increases from 24 to 75? Where is the center of the distribution? What proportion of wins appear to happen less often?

Interpret Results

How does the center and spread relate to the probability that Fallon wins the game? Come back to the original question: What proportion of wins or losses for Fallon would surprise you? If we consider an unusual or surprising event to occur less than 5% of the time, what proportion of wins and losses might fall in this range?

Consider the second question: How can we determine if our class results are unusual? Enter the sample proportion of the class generated data in the 'count as extreme as box'. By using the empirical sampling distribution, we can estimate the likelihood of obtaining our class results or something more extreme.





The teacher can use this example to explain the Central Limit Theorem for a sample proportion. Use frequencies to show that the majority of sample proportions are close to the population proportion. Ask students if they can think of a larger context that the results of this simulation can be applied to?

Students commonly report that the distribution becomes more normally distributed as the number of samples increases. One student commented, “Isn’t this just like the law of large numbers, since the center is close to 0.55?” Another student, “Is this how political races are predicted; by looking at a large number of samples and seeing if the proportion of voters fall in a certain range?”

Possible Differentiation

This lesson could be adapted to a Gaise Level B activity. Follow the same introduction by showing and discussing the videos. Have students play the egg roulette game 9 times recording the number of wins for Player 2, (Fallon) on a dot plot displayed on a board in the front of the room. Students then play the game another 9 times and record their results on the dot plot. After the results are recorded, change the scale from number of wins to proportion of wins. Discuss the shape, spread, and center of the distribution. Ask students to conjecture if they were to play more games and record their results, can they predict what will happen to the distribution? Next, open the fathom file and have the computer play the game a large number of times. Ask students if they were correct in their predictions. Relative frequencies can be used to discuss the empirical probability that Fallon wins the game.

References

This lesson plan is adapted from an activity created by James Bush and Jennifer Bready, for the United States Conference on Teaching Statistics (2015).

Walker, A. N., & White, A. (2017, May) *Exploring the law of large numbers through the egg roulette game in fathom.* Poster presented at the United States Conference on Teaching Statistics, State College, PA.

Chance, B., Cobb, G., Rossman, A., Roy, S., Swanson, T., Tintle, N., VanderStoep, J. (2014). *Applets for Introduction to Statistical Investigation Process*. [Computer Software]. Retrieved from http://www.rossmanchance.com/ISIapplets.html.

Lock, R., Swanson, T., & Tintle, T. (2016, June). *Teaching the Statistical Investigation Process with Randomization-Based Inference.* Workshop presented at the Joint Mathematics Meetings in San Antonio, TX.

[The Tonight Show Starring Jimmy Fallon]. (2013, April 12). *Egg Roulette with Tom Cruise – Late Night with Jimmy Fallon*. [Video File]. Retrieved from https://www.youtube.com/watch?v=y6jb0cOYqAI.

[The Tonight Show Starring Jimmy Fallon]. (2014, October 17). *Egg Roulette with Bradley Cooper – Late Night with Jimmy Fallon*. [Video File]. Retrieved from https://www.youtube.com/watch?v=ZVUfnJipFh0.

White, A. (2015). Egg Roulette Fathom Simulation [Computer Software].

Further Reading About the Topic

Rossman, A. J. and Chance, B. L. (2014), Using simulation-based inference for learning introductory statistics. WIREs Comp Stat, 6: 211–221. doi: 10.1002/wics.1302

Retention of Statistical Concepts in a Preliminary Randomization-Based Introductory Statistics Curriculum. (2013). *College Mathematics Journal*, 44(3), 252-253.

Chen, F. (2017, March 29). Hey, Jimmy, egg Russian roulette is rigged…in your favor! *Huffington Post.* Retrieved from http://www.huffingtonpost.com/entry/hey-jimmy-egg-russian-roulette-is-riggedin-your_us_58dac868e4b0f087a3041d9c

The Egg Roulette Game
Student Handouts

Part I:
Rules of Egg Roulette:

Reflecting on the videos

1. Do you think each player has the same probability of winning the game when guest of the show goes first?

2. Is the probability of winning similar to flipping a coin?

3. Are the events of selecting eggs conditional or independent events?

4. What are some other questions you could investigate concerning the probabilities each player has of winning the game?

With your partner play the egg roulette game 5 times and tally the number of wins for each player in the table below. Guest of the show must take the first turn in all 5 games!

	Player 1: (Guest)	Player 2: (Fallon)
Game 1		
Game 2		
Game 3		
Game 4		
Game 5		
Total # of Wins		

5. Using the class totals, what is the empirical probability Fallon wins the game? Does this give him an advantage or disadvantage, why?

6. Using the Fathom simulation, estimate the probability Fallon wins the game.

7. Explain the **Law of Large Numbers**

Part II:

8. Open the computer applet located at http://www.rossmanchance.com/ISIapplets.html. Select 'Categorical Response/One Proportion'. Enter 0.55 for 'probability of heads'. Note that this heading will change to 'probability of success' once the numeric value changes from 0.5 to 0.55. Enter 24 for 'sample size', and number of samples to 100. Describe the shape and variability of the sampling distribution of a sample proportion after 100 samples.

9. What changes do you notice in the graph as the number of samples increases to 1000?

10. What proportion of wins or losses for Fallon would surprise you? If we consider an unusual or surprising event to occur less than 5% of the time, what proportion of wins and losses fall in this range?

11. How can we determine if our class results are unusual? Enter the sample proportion of the class generated data in the 'count as extreme as box'. By using the empirical sampling distribution, we can estimate the likelihood of obtaining our class results or something more extreme. Interpret the results.

12. Explain the **Central Limit Theorem** and how this relates to the theoretical probability that Fallon has of winning the game.

13. Can you describe how the Law of Large Numbers or Central Limit Theorem might be applied to a larger context, or real world example?

Sample Solutions for Student Handouts

Part I:

Rules of Egg Roulette: one dozen eggs are presented; 8 hard boiled and 4 raw. There is no visible difference in the eggs. Players take turns selecting an egg and cracking that egg on their own head. The first person to crack two raw eggs on his head loses. Guest of the show always goes first.

Reflecting on the videos

1.

- Does the guest of the show have an advantage since he goes first and there are more hard boiled eggs to select from at the start?
- Does the guest have a disadvantage if there are only raw eggs left to choose from at the end that player must select the raw egg?
- Maybe it depends on what happens in the first few turns because that will determine how many raw eggs are left to choose from at the end for each player?

2. If both players have an equal chance of winning then the probability would be the same as flipping a coin. If one player has a higher chance of winning then the probabilities will not be 0.5.

3. The events are conditional events because whatever egg is selected there is one less of that type of egg to choose from, which changes the probability.

4. (DATA depends on experimental results obtained by students)

5. Computed probability depends on data.

6. After a large number of trials, we see the cumulative proportion of wins for Fallon stabilizes around 0.55. Therefore, we can assume the probability Fallon wins is

$$\frac{5}{9} = 0.\overline{5}$$

7. The empirical probability of an event occurring, or sample proportion, will approach the expected value, or unknown population proportion, if the number of samples is large enough.

Part II:

8. The distribution is roughly normal with a mean of 0.55-0.56. The proportions range from approximately 0.37 to 0.75. (These values are based on the distributions within the lesson, and may be slightly different when students use the computer simulation)

9. As the number of samples increases the distribution becomes more normally distributed. The center of the distribution is approximately 0.55, the assumed probability of Fallon winning.

10. The surprising results occur in the tails. If we select 100 samples, then the outer most 5 samples are surprising. When selecting 1000 samples, then the outer most 50 samples are surprising.

11. The results show the proportion of wins that are as or more extreme than the class results. We can interpret this as the likelihood of obtaining our class results as a random sample.

12. The CLT states that the sampling distribution of a sample proportion is approximately normally distributed with a center of p and standard deviation $\sqrt{np(1-p)}$. The mean of all of the sample proportions, $\mu_{\hat{p}} = p$. For the egg roulette game, this means that the probability that Fallon wins the game is the center (mean) of the sampling distribution of a large number of sample proportions, and is close to 0.55.

13. The majority of sample portions are centered around the population proportion we are trying to estimate. So a single sample, (so long as it is large enough and representative of the population), can be used to:

- estimate the proportion of voters for a certain candidate
- estimate the proportion of a population that has a certain disease, income level, or other characteristic.