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How long are the words in the Gettysburg Address?

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Overview of Lesson
In this lesson, each student tries two methods for selecting a sample from the population of words in the Gettysburg Address: self-selection and simple random sampling. Then, as a class, students construct dotplots and calculate numerical summaries to show how sample means vary from sample to sample. Using these plots and numerical summaries, students see that self-selected samples tend to over-represent the longer words in the Gettysburg Address and produce sample means that are larger than the actual population mean (*biased* sampling method). On the other hand, simple random samples tend to be representative of the population and produce sample means balanced on both sides of the population mean (*unbiased* sampling method). Finally, students explore the relationship between sample size and sample-to-sample variability. Students discover that larger random samples produce sample means with less variability.

Type of Data
- Choose one: One quantitative variable
- Choose one: Static dataset provided by lesson plan authors

Learning Objectives
- **CCSSM.7.SP.A.1**: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- **CCSSM.7.SP.A.2**: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Audience
- The authors have tested this lesson with middle and high school students, pre-service and in-service teachers, and mathematics teacher educators.
- This lesson is also appropriate for post-secondary students.
- *Prerequisites:* Displaying numerical data in plots on a number line (e.g., dotplots), calculating quantitative measures of center and spread (e.g., mean and range), and some experience with data collection.

Time Required
- Two 50-75 minute periods

Technology and Other Materials

- *Technology:* Random digits generator (random.org, calculator, or spreadsheet)
- Student handout (includes sequentially numbered list of words in the Gettysburg Address)
- Three Sheets from a Large Gridded Pad scaled from 1 to 11 (provide 5 spaces between each tick). Label these as Dotplots 1.2, 2.2, and 3.1

Lesson Plan

The following investigation is adapted from NCTM's *Developing Understanding of Statistics for Teaching Mathematics in Grades 6-8* (2013).

Authorship of literary works is often a topic for debate. One method researchers use to decide who was the author is to look at word patterns from known writing of the author and compare these findings to an unknown work. To help us understand this process we will analyze the length of the words in the Gettysburg Address, authored by Abraham Lincoln.

The Gettysburg Address shown below was delivered by Abraham Lincoln on November 19, 1863 on the battlefield near Gettysburg, Pennsylvania. It is one of the most famous speeches ever given by an American President.

> Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal.
>
> Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battlefield of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.
>
> But, in a larger sense, we cannot dedicate -- we cannot consecrate -- we cannot hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us -- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.

In this lesson, students collect their own samples from the Gettysburg address. Using the mean word length from those samples, students estimate the population mean word length.

The lesson plan consists of three parts. In each part, students collect their own samples of words then work together as a class to describe sample-to-sample variability. In Part I, each student uses non-random sampling method (self-selection) to choose ten words from the Gettysburg Address. In Part II, each student uses a random number generator to select a simple random sample of size ten. In Part III, each student selects another simple random sample, this time of size five.

Part I – Self-Selected Sampling

Formulate a Statistical Question

Begin by pointing out to students that the *length* of a word (number of characters) in the Gettysburg Address varies from word to word. For example, the first word "Four" is of length 4 (4 characters), while the second word, "score", has 5 characters, so the length is 5. Note that length of a word is a discrete quantitative variable. A statistical question we might ask is:

How do the lengths of words in the Gettysburg Address vary?

1. a. Have students identify the population of interest (all words of the Gettysburg Address) and the variable of interest (length of a word).

1. b. Describe how would you select a sample of words to produce a sample that is similar to (representative of) the population?

Note: The numbering system in this lesson plan corresponds to the numbered questions on the attached student handouts.

Point out that the word lengths vary and that the population of all word lengths has a distribution, which can be summarized by describing its shape, center, and the variability. To address the question, *"How do the lengths of words in the Gettysburg Address vary*?", we could examine the distribution of word length for all words in the Gettysburg Address. Instead, we will select a subset (sample) of words with the goal of producing a sample of words that has a distribution that is similar to the population distribution of word length. That is, the shape, center and variability of the sample data distribution are similar to that of the population distribution. In this investigation, we will explore two methods for selecting a sample of words.

Collect Appropriate Data

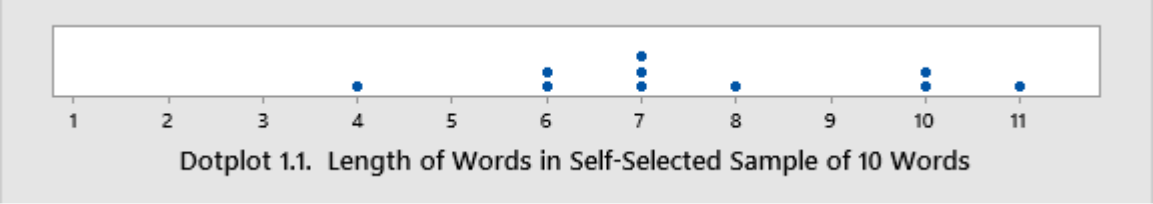
Have each student circle 10 words they think are "representative" of the varying word lengths in the Gettysburg Address. The resulting 10 words constitute a self-selected sample from the population of 268 words in the entire Gettysburg Address. Have students record their words and word lengths in Table 1.1 (provided in student materials), as illustrated below.

Table 1.1. – Sample of 10 Words/Sample 1

Word Number	Selected Word	Length of Word
1	created	7
2	testing	7
3	battlefield	11
4	larger	6
5	consecrate	10
6	remember	8
7	advanced	7
8	rather	6
9	unfinished	10
10	here	4
	Total	*76*

Analyze Data and Interpret Results

2. Using the plot provided in the student materials (Dotplot 1.1), have each student create a dotplot for their word-length data. The dotplot for the data displayed in Table 1.1 is shown below. Have students compare their distribution with other students. Are they the same or different? Results will vary from student to student.



Dotplot 1.1. Length of Words in Self-Selected Sample of 10 Words

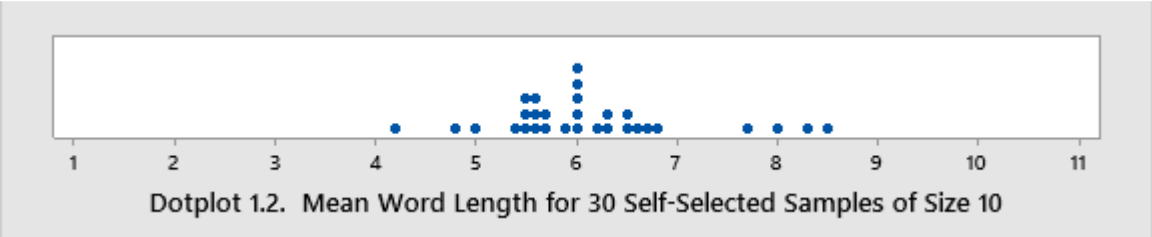
Based on their dotplot, have each student summarize the data in their sample. For example, based on the above plot, the 10 words have lengths between 4 and 11 characters, and the center appears to be between around 7 characters. The range is 7 characters. The shape is not very clear, which may be the result of the small sample size.

3. Ask for volunteers to share their dotplot and to describe their strategy (if any) for the sample they selected. Answers will vary.

4. What is a typical length for the words in the Gettysburg Address? One quantity that is used to measure "typical" is the mean. Have each student compute the mean for their sample. (The mean for the data in Table 1.1 is 76/10 = 7.6 characters). Ask the class "Do all the samples produce the same mean?" Have some students report their mean and point out that the sample means vary from one sample to another.

5. Using a sheet from a Large Gridded Pad have each student place a "sticky dot" on the plot as close as possible over their sample mean to create a dotplot displaying the

sample means for the entire class. Students whose data have the same mean should stack one dot above another. Results will vary from class to class. A hypothetical dotplot for a class of 30 students is shown below:



Dotplot 1.2. Mean Word Length for 30 Self-Selected Samples of Size 10

Remind students that this plot is displaying **sample means** and illustrates the sample-to-sample variability in these 30 means.

Based on Dotplot 1.2 for their class, have students summarize the results of their means. For example, based on the above plot, the means have between 4.2 and 8.7 characters, and the center is around 6 characters. The range is 4.5 characters, and the display is skewed right. If we consider the two values above 8 as unusually large, a range of typical values for a sample mean using self-selected sampling might be from 4.2 to 8.0 characters.

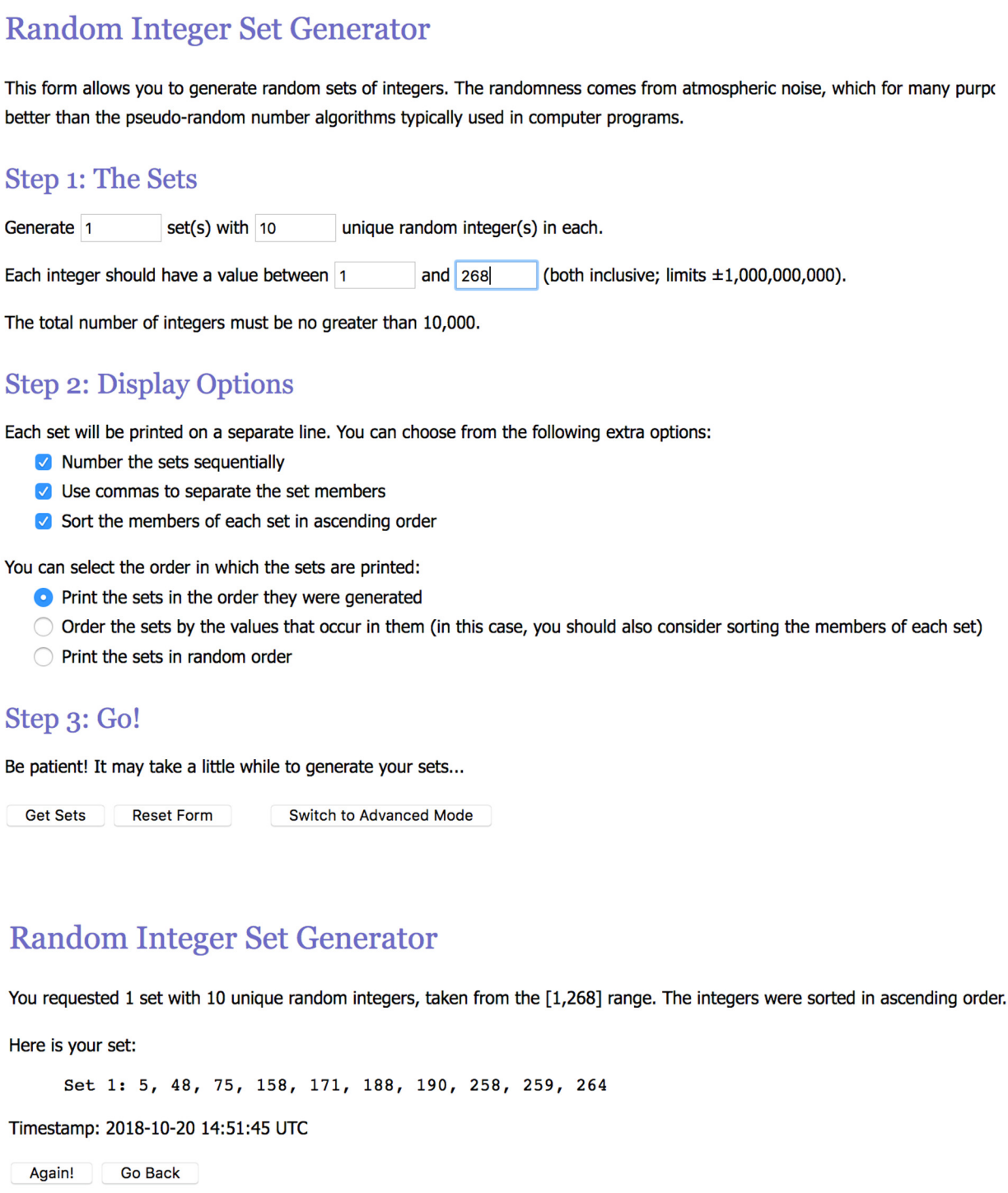
Part II – Simple Random Sampling

Collect Appropriate Data

Another way to select a sample from a population is to use simple random sampling. How do we select a simple random sample of 10 words from the Gettysburg Address?

On the last pages of the attached student handout file are copies of a list of the words in the Gettysburg Address. Note that there are 268 words, and each word is assigned a number from 1 (001) to 268. To randomly select 10 words from the Gettysburg Address we will generate 10 distinct random integers between 1 and 268 and select the corresponding word from the Gettysburg Address. Many calculators or computer apps are useful for generating random integers. One such computer application can be found and used free of charge at www.random.org

At this web-site, have each student select "Numbers" and "Integer Set Generator." Complete Step 1 as indicated below and click on "Get Sets" in Step 3. Sample output from this app are shown below.



Random Integer Set Generator

This form allows you to generate random sets of integers. The randomness comes from atmospheric noise, which for many purpc better than the pseudo-random number algorithms typically used in computer programs.

Step 1: The Sets

Generate 1 set(s) with 10 unique random integer(s) in each.

Each integer should have a value between 1 and 268 (both inclusive; limits ±1,000,000,000).

The total number of integers must be no greater than 10,000.

Step 2: Display Options

Each set will be printed on a separate line. You can choose from the following extra options:

- [x] Number the sets sequentially
- [x] Use commas to separate the set members
- [x] Sort the members of each set in ascending order

You can select the order in which the sets are printed:

- (•) Print the sets in the order they were generated
- ( ) Order the sets by the values that occur in them (in this case, you should also consider sorting the members of each set)
- ( ) Print the sets in random order

Step 3: Go!

Be patient! It may take a little while to generate your sets...

Get Sets | Reset Form | Switch to Advanced Mode

Random Integer Set Generator

You requested 1 set with 10 unique random integers, taken from the [1,268] range. The integers were sorted in ascending order.

Here is your set:

```
Set 1: 5, 48, 75, 158, 171, 188, 190, 258, 259, 264
```

Timestamp: 2018-10-20 14:51:45 UTC

Again! | Go Back

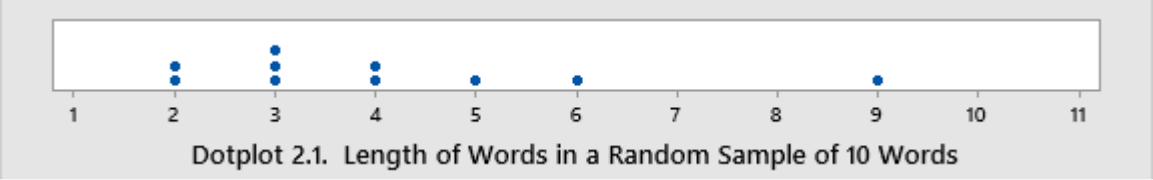
Have students enter their 10 random integers in Table 2.1. Next, provide them with a copy of the list of the words so they can identify the corresponding words from the Gettysburg Address and the length of each word as illustrated below:

Table 2.1. Random Sample of 10 Words/Sample 2

Random Integer	Selected Word	Length of Word
5	years	5
48	conceived	9
75	as	2
158	they	4
171	here	4
188	is	2
190	for	3
258	the	3
259	people	6
264	not	3
	Total	*41*

Analyze Data and Interpret Results

6. Using the plot provided in the student materials (Dotplot 2.1), have each student create a dotplot for their word-length data. Results will vary from student to student. The dotplot for the data displayed in Table 2.1 is shown below:

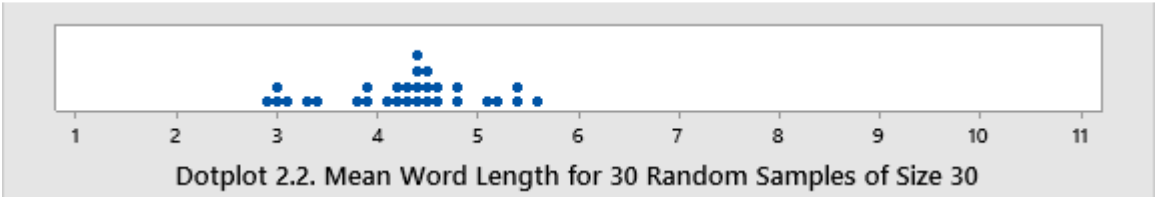


Dotplot 2.1. Length of Words in a Random Sample of 10 Words

Based on their dotplot, have students summarize the data in their random sample. For example, based on the above plot, the 10 words have lengths between 2 and 9 characters, the center appears to be between 3 and 4 characters. The range is 7 characters, and the shape is skewed right.

7. Have each student compute the mean for their random sample. (The mean for the data in Table 2.1 is 41/10 = 4.1 characters.) Do all samples produce the same mean? Ask some students to report their mean and point out that the sample means vary from one sample to another.

8. Using a sheet from a Large Gridded Pad, have each student place a "sticky dot" on the plot as close as possible over their sample mean to create a dotplot displaying the sample means for the entire class. Students having the same mean should stack one dot above another. Results will vary from class to class. A hypothetical dotplot for a class of 30 students is shown below:

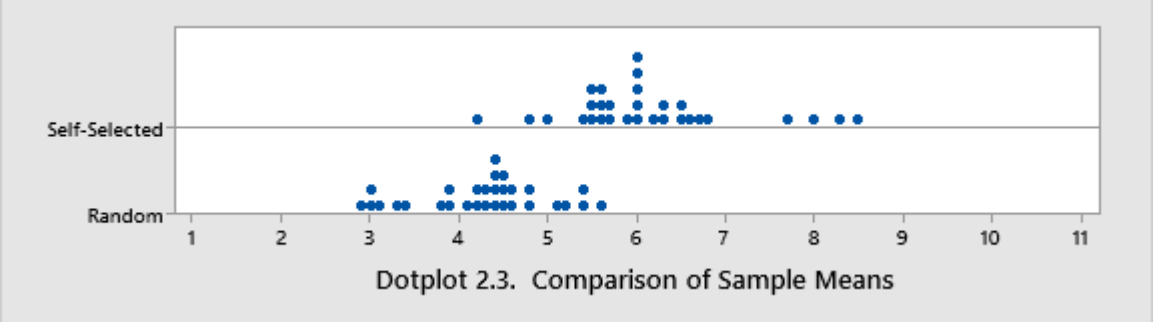


Dotplot 2.2. Mean Word Length for 30 Random Samples of Size 30

Remind students that this plot is displaying **sample means** and illustrates the sample-to-sample variability in these 30 means.

Based on Dotplot 2.2 for the class data, have students summarize their results. For example, based on the above plot, the means have between 2.9 and 5.6 characters, and the center is around 4.4 characters. The range is 2.7 characters, and the display is reasonably symmetric. A range of typical values for a sample mean using simple random sampling might be from 2.9 to 5.6 characters.

9. Have students compare Dotplots 1.2 (30 Means from Self-Selected Samples) and Dotplot 2.2 (30 Means from Simple Random Samples). Specifically, have them compare their centers and variability.

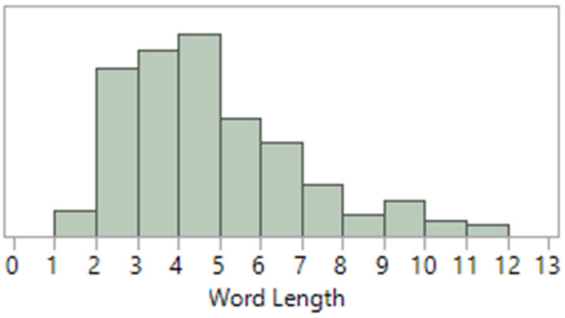


Dotplot 2.3. Comparison of Sample Means

Note that the center for the means of simple random samples (Sample 2) is around 4.4 versus around 6 for self-selected sample (Sample 1). Also, there appears to be less variability in the means for simple random samples. (Range for Simple Random is 2.7 versus 4.5 for Self-Selected). The range of typical values for Simple Random Sampling is 2.9 to 5.6, much narrower than a range of 4.2 to 8.0 for Self-Selected Sampling. Results will vary from class to class.

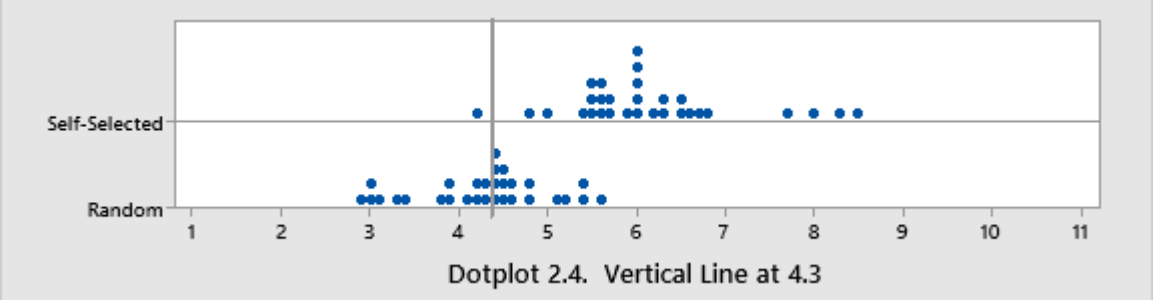
10. Ask students to think about which sampling method they think is more likely to produce a representative sample. Follow up with a class discussion.

11. Next, have students examine the histogram of word lengths for the entire population of all 268 words in the Gettysburg Address (provided in the student handout) and shown below.



Ask students to comment on the population distribution. Specifically, the word lengths are between 1 and 11 characters, the center is around 4 characters, and the range is 10 characters. Also, point out that most words in the Gettysburg Address are fairly short (2, 3, or 4 characters), and only a few words are long (8 or more characters).

12. Inform students that the mean of the population of 268 words is approximately 4.3 characters. Have students draw a vertical line at 4.3 in Dotplot 2.3 as shown below:



Dotplot 2.4. Vertical Line at 4.3

Have students compare the sample means from the two sampling methods with the actual population mean of 4.3.

For example, using the hypothetical results for 30 self-selected samples (Sample 1), only one sample mean is below the population mean of 4.3, none is equal to 4.3 and twenty-nine (29) are greater than 4.3. Thus, the samples resulting from self-selection tend to produce samples with means that are larger than the population mean. With self-selection longer words tend to be over-represented in a sample. This over-representation of the longer words is called *sampling bias*.

On the other hand, of the 30 means from simple random sampling (Sample 2) 12 are below 4.3, two are equal to 4.3 and 16 are above 4.3. Thus, simple random sampling tends to produce sample means on both sides of the population mean. When selecting samples of words from the Gettysburg Address simple random sampling tends to produce samples that are representative of the population.

Part III – Simple Random Sampling and Sample Size

Parts I and II of this investigation are designed to illustrate that simple random sampling is an unbiased sampling method. In Part III, students explore the relationship between sample size and how close a sample mean from a simple random sample is to the actual population mean.

Collect Appropriate Data

Begin by having students use a random number generator to generate 5 distinct random Integers between 1 and 268, and to use these to select a simple random sample of 5 words from the Gettysburg Address. Have students enter their results in Table 3.1 along with the corresponding words from the Gettysburg Address and the length of each word as illustrated below:

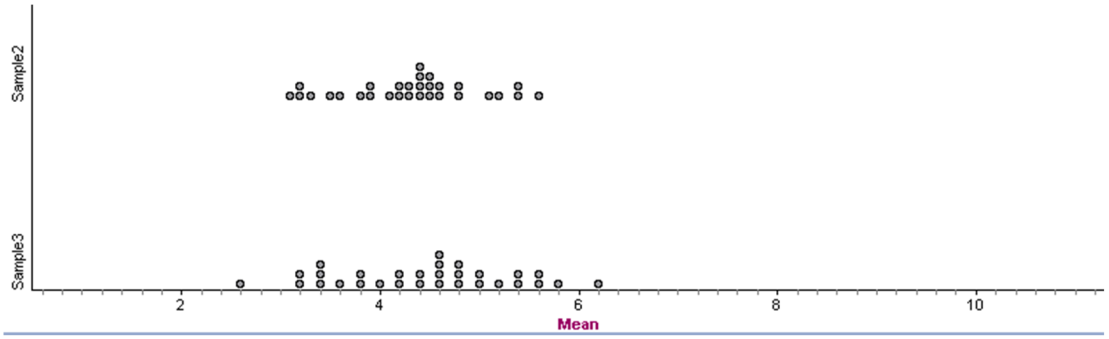
Table 3.1 – Random Sample of 5 Words/Sample 3

Random Integer	Selected Word	Length of Word
7	our	3
45	any	3
167	rather	6
201	before	6
254	of	2
	Total	*20*

Analyze Data and Interpret Results

13. Have each student compute the mean for their sample. (The mean for the data in Table 3.1 is 20/5 = 4.0 characters). Using a sheet from a Large Gridded Pad have each student place a "sticky dot" on the plot (Dotplot 3.1) as close as possible over their sample mean to create a dotplot displaying the sample means for the entire class. Students having the same mean should stack one dot above another.

    Have students compare the results displayed in Dotplot 2.2 (Means from samples of size 10) and Dotplot 3.1 (Means from samples of size 5). Results will vary class to class. Hypothetical dotplots for a class of 30 students are shown below:



Means for Random Samples of Size 10 (Sample 2/Dotplot 2.2) and
Means for Random Samples of Size 5 (Sample 3/Dotplot 3.1)

    How do these two distributions compare? For example, the displays above are both fairly symmetrical and centered around 4.3 (the population mean). The major difference between the two groups is that there is more variability in the sample means for samples of size 5 than there is in the variability in the sample means for samples of size of 10. Also, a range of typical values for a sample mean from samples of size 10 is 2.9 to 5.6 versus a range of 2.6 to 6.2 for a sample mean from samples of size 5. Thus, larger sample sizes result in a narrower range of typical values.

Part 3 of this lesson illustrates an important relationship between sample size and how close a sample mean is to the population mean – with simple random samples, the larger the sample size, the closer sample means tend to be to the population mean.

Summary

When sampling words from the Gettysburg Address, self-selected samples tend to over-represent longer words when compared to how often they occur in the population. This phenomenon occurred even though we were able to see the entire population, which is uncommon when sampling in statistics. Using simple random sampling tends to produce samples that are representative of the population. While the sample mean may not be the same as the population mean, we expect the sample mean to within a range of values centered around the population mean. Larger simple random samples tend to produce sample means closer to the population mean.

Attached Materials

- Student handouts
    - The last two pages of the document provide a list of the words in the Gettysburg Address. There are 268 words, and each word is assigned a number, 001 to 268.

References

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., and Scheaffer, R. (2005), *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework,* Alexandria, VA: American Statistical Association.

Kader, G. and Jacobbe, T. (2013), *Developing Essential Understanding of Statistics for Teaching Mathematics in Grades 6-8,* Reston, VA: National Council of Teachers of Mathematics.