

Happy Birthday to . . . Two?

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Overview of Lesson

This activity is related to the Birthday Problem, originally posed by Richard von Mises (1939, reprinted in English in 1964), which can be stated as “How many people must be in a room so that there is a greater than 50% chance that at least two of them share a birthday?” Using real data, students take repeated samples of various sizes to determine answers to this question.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level A activity.

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
5. Use appropriate tools strategically.

Common Core State Standards Grade Level Content (Grades 6 and 7)

6-SP. 5. Summarize numerical data sets in relation to their context, such as by:

- a. Reporting the number of observations.
- b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

7-SP. 1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7-SP. 7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

NCTM Principles and Standards for School Mathematics
Data Analysis and Probability Standards for Grades 6-8

Develop and evaluate inferences and predictions that are based on data:

- use conjectures to formulate new questions and plan new studies to answer them.

Understand and apply basic concepts of probability:

- use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.

Prerequisites

Prior to this lesson, students should be able to organize numerical and categorical data, compare and order rational numbers, and understand that probability is a numerical measure of the likelihood that an event will occur. Students will also need an understanding of the phrases “equally likely” and “more likely than not.” Students should also have some experience working with a spreadsheet, as this will facilitate the sorting and organization of a large set of data.

It will also be helpful to introduce students to the term *Julian date*, which is a count of the number of days that have passed from January 1 to a given date. The data set for this lesson lists date of birth as the Julian date. For example, $DOB = 17$ for a person born on January 17. For a person born on February 5, $DOB = 36$ (31 days in January + 5 days in February).

This activity will be most effective after students have had exposure using data to compute experimental probabilities.

Learning Targets

1. Students will be able to collect and organize numerical data (related to birthdates and frequencies of repetitions) and categorical data.
2. Students will be able to calculate proportions and interpret them as experimental probabilities of an event.
3. Students will be able to compare experimental probabilities and make inferences about the context.
4. Students will understand that experimental probabilities are affected by the sample size, or number of observations, that are used to determine the probability.

Time Required

This lesson may be completed in 90-120 minutes.

Materials Required

Students need an Activity Sheet (page 9) and students need access to the data set with 736 cases. Because of the large number of cases, it would be ideal for students to use software to work with the data (such as a spreadsheet or statistics software).

Instructional Lesson Plan

The GAISE Statistical Problem-Solving Procedure

I. Formulate Question(s)

As this is a GAISE Level A activity, the teacher poses the question. Begin by asking for everyone's birthday to see if anyone in the classroom shares a birthday. Next, ask students if they would be surprised if two people in the classroom next door shared a birthday. Would they be surprised if two people in their grade level shared a birthday? What about if two people in the school shared a birthday?

Ask students, "How many people would need to be in a room so that it was more likely than not that at least two of them shared a birthday?" Richard von Mises originally posed this problem in 1939, and many people have explored it and its multiple variations.

Ask, "What does it mean that something is 'more likely than not' to occur? What are some examples of things that are 'more likely than not' to occur? What fraction or percent of the time would something need to occur that would lead us to say that it was 'more likely than not'?" The key idea is that we are investigating situations where the probability of an outcome is more than 0.5, which means the event would occur with more than 50% of the time.

Solicit students' hypotheses about the number of people that would need to be in a room so that there was a greater than 50% chance that at least two of the people share a birthday. Certainly, if the number of people in the room is greater than the number of days in a year, at least two of them would have to share a birthday. Challenge students to come up with a number of people for which the probability that at least two people share a birthday is close to 50%. It may help to give some concrete examples, such as the number of students in the school, in the grade, in the classrooms in a particular hallway, or in a single classroom. (Common guesses include 180 [approximately half of the number of days in a year], 100, and 50.)

II. Design and Implement a Plan to Collect the Data

For this lesson, we will examine a data set of 736 birthdays of athletes that participated in the 2010 World Cup, an international soccer tournament. We assume that the birthdates are independent of each other and the distribution of the sample resembles the distribution of birthdates of all people in the world. Have students examine the data set (either a hard copy or an electronic copy), and tell them what variables are shown and what they may represent. For clarification, a portion of the data table is shown and the variables are explained below.

Country	Group	Num	Position	DOB
France	A	2	Defender	45
Greece	B	12	Goalkeeper	224
Chile	H	7	Forward	353
Honduras	H	17	Midfielder	18

- **Country** The athlete's home country and team name. There are 32 different countries in this data set.
- **Group** The 32 teams are divided into eight groups, with four countries in each group. Groups are named by the letters A, B, C, D, E, F, G, & H.
- **Num** The player's jersey number. There are 23 athletes on each team, and the numbers are the whole numbers ranging from 1 to 23.
- **Position** The athlete's position, which is either Forward, Midfielder, Defender, or Goalkeeper. Each team has exactly 3 Goalkeepers.
- **DOB** The Julian date of the athlete's date of birth. The Julian date is a count of the number of days that have passed from January 1 to a given date. For example, DOB = 17 for an athlete born on January 17. For an athlete born on February 5, DOB = 36 (31 days in January + 5 days in February).
Note: This particular data set contains no birthdates on February 29, so leap day has been ignored. Therefore, a birthdate of March 1 has DOB = 60 (31 days in January + 28 days in February + 1 day in March) even if the birthday occurred in a leap year. DOB = 365 corresponds to an athlete born on December 31.

Ask students to discuss ideas for how these data can help investigate the question, "How many people would need to be in a room so that it was more likely than not that at least two of them shared a birthday?" One possible idea that students may offer is to start at the beginning of (or some other location on) the list and count entries until there is a repeated value in the DOB column (indicating that two people in the set share a birthday). Another possibility would be to randomly select entries from the table one at a time, and keep a count until there is a repeated value for DOB. Another possibility is described in below.

In this lesson, students will divide the 736 athletes into subsets of equal size. These subsets are determined using the values of the other variables (i.e. Group, Country, or Position) in the data set. We will then determine whether or not each subset contains at least two athletes who share a birthday. After this, the proportion of subsets that contain shared birthdays will be calculated, and interpreted as an experimental probability. This process will then be repeated five or six times, using a different number of athletes per subset in each repetition.

III. Analyze the Data

Begin by modeling this problem with the whole class. We can obtain subsets of size 184 using the following partition of athletes: Groups A & B; Groups C & D; Groups E & F; Groups G & H. Have students examine the birthdays within each of the subsets to see if at least two athletes share a birthday. (Note that once a shared birthday is found in a subset, we don't need to search the subset any further.) Students should find that shared birthdays are located in each of the four subsets. Therefore, using a subset of size 184, the proportion of these subsets with shared birthdays is 4/4. This may be interpreted as the experimental probability is 100% that at least two people will share a birthday if 184 people are in the room.

Note that it may be possible to get a sample of 184 where there are no shared birthdays. Ask students to give an example of how this might happen, and whether or not the occurrence of a sample of this type would be likely.

Having worked through this example, ask students whether we should examine subsets with fewer or more athletes in order to find a proportion closer to 0.5. Students should notice that a smaller subset is needed.

Choose a smaller number of athletes per subset, perhaps 92, which can be found by letting each subset consist of the athletes in the eight Groups (A through H). Here, it will probably be helpful to divide the students into groups, with each group examining a single subset. Students then share their results (shared birthday or no shared birthday), and calculate the proportion of subsets containing shared birthdays. Based on their results, they will likely wish to examine subsets with fewer athletes.

Continue this cycle five or six times, being sure to examine subsets of size 23 (which is the answer to the question using theoretical probability), as well as some subsets with fewer than 23 athletes. The table below contains the proportions for subsets that students have used in the past.

Description of subset	Number of subsets	Number of athletes in each subset	Experimental probability that at least two athletes in a subset shared a birthday
Pairs of Groups (listed in order from A to H)	4	184	4/4 (100%)
Group	8	92	8/8 (100%)
Non-Goalkeepers in each group	8	80	8/8 (100%)
Pairs of countries (listed alphabetically)	16	46	15/16 (93.75%)
Country	32	23	15/32 (46.875%)
Non-Goalkeepers in each country	32	20	13/32 (40.625%)
Goalkeepers in each group	8	12	4/8 (50%)
Athletes numbered 12 to 23 in each country	32	12	5/32 (15.625%)
Goalkeepers in each country	32	3	2/32 (6.25%)

IV. Interpret the Results

Have students share their results with each other. Using the shared results that students have generated, ask students to create a table (similar to the one shown above) and organize the information from largest subset size to smallest subset size.

Ask, “What patterns do you notice in the column of experimental probabilities?” Students should note that as the number of athletes in a subset decreases, the experimental probability that at least two people share a birthday also decreases. They should also see that this pattern is not strictly decreasing. For example, note that the row of “Goalkeepers in each group” in the table above has only 12 athletes in a subset, but a larger experimental probability than the row above, which had 20 athletes in a subset. One explanation for this phenomenon is that there were only 8 subsets of “Goalkeepers in each group,” but 32 subsets for the previous row.

Ask students to indicate which subset sizes yielded experimental probabilities near 0.5, and how they might use this information to answer the original question.

In the table shown above, two different types of subsets had size 12, and the resulting experimental probabilities are very different. Have students explain this discrepancy. One possible explanation is that while there were the same number of athletes in each subset, there were fewer subsets considered in the case of Goalkeepers in each group (8) compared to Athletes numbered 12 to 23 in each country (32). Ask students which of these two probabilities is closest to the theoretical value of 0.5. Students may indicate that by using a greater number of subsets, the experimental results will more accurately reflect the theoretical results. This is a good illustration of the Law of Large Numbers.

You may wish to let students know the answer to the problem, using theoretical probability techniques and assuming each birthday is equally likely, is 23. If this information is shared, it should only be done after students feel fairly confident that the number of people should be near that value, based on their interpretation of the experimental probabilities.

Here is a derivation of the theoretical probability to answer the question “How many people must be in a room so that there is a greater than 50% chance that at least two of them share a birthday?” The key lies in finding the probability of the complement of the event in question. If the event is “at least two people in the room share a birthday,” then the complement is “no one in the room shares a birthday.”

If there are n people in a room, then the probability that no one shares a birthday is

$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots \frac{365 - n + 1}{365} = \frac{1}{365^n} \cdot \frac{365!}{(365 - n)!} = \frac{1}{365^n} \cdot n! \binom{365}{n}$$

Therefore, the probability that at least two people in the room of n people share a birthday is

$$1 - \frac{1}{365^n} \cdot n! \binom{365}{n}$$

When $n = 22$, this probability is approximately 0.476. The probability becomes larger than 0.5 for all $n > 23$. When $n = 23$, the probability is approximately 0.507.

Possible answers to formative assessment questions:

1. The results would likely be similar for cases (a), (b), (c), and (e). In the case of (d), it would depend on whether the data were gathered over the course of an entire year or more, or during a shorter span (such as the past week or month).
2. Subsets of different sizes would need to be examined to answer this question. Based on the results contained in the table above, the number is somewhere between 23 and 46. Calculations with theoretical probability indicate that about 34 people are needed.
3. There would need to be more people in the room in order to have at least three people sharing a birthday. This question would be a good way to launch an extension to the activity.

Summative assessment questions

1. Use a number line (with marks indicating 0 and 1) to indicate your estimate for the probability that at least two people will share a birthday if there are:
 - (a) 4 people in the room

 - (b) 17 people in the room

 - (c) 31 people in the room

 - (d) 205 people in the room

2. Robbie, Vonda, and Missy were discussing the question, “How many people need to be in a room so that it is more likely than not that at least two people were born in the same month?” They all had different ideas. Robbie said that 6 people would be needed. Vonda thought that the number would need to be less than 6. Missy thought that the number would be greater than 6. Who do you agree with? Please explain your reasoning.

3. Describe a way to investigate and answer the question, “How many people need to be in a room so that it is more likely than not that at least two people have the same first initial?”

Possible answers to summative assessment questions:

1. (a) The mark for A should be close to 0.
(b) The mark for B should be between 0 and 0.5 (the theoretical value is near $1/3$).
(c) The mark for C should be between 0.5 and 1 (the theoretical value is near $3/4$).
(d) The mark for D should be close to (or equal to) 1.
2. Vonda is correct – fewer than 6 people would be needed. The number 6 is half of the number of options, but given the nature of the activity, students should recognize that using 6 will have a probability greater than 0.5.
3. Using a large list of people's names, divide the list into subsets of equal size and determine the proportion of subsets that have at least two people with the same first initial. Based on those results, adjust the size of the subset and repeat the process. Do this with several different sizes of subset to get a good approximation of the number of people needed.

Note: In some ways, this is very different than the birthday problem, as it depends on languages and popularity of names. Also, while there may be some reason to expect that each birthday is equally likely, there is no reason to expect that each letter is equally likely to be a first initial. For this reason, this question is very difficult to answer using theoretical probability. On the other hand, it is a good extension of the above activity.

Possible Extensions

Below are some suggested extensions. More possible extensions are suggested in the assessment items above.

- Use random samples of various sizes, such as using a calculator to generate random integers between 1 and 736. Discuss why random samples may be selected, and how this may help eliminate some issues in the previous sampling method, such as unfriendly numbers in a subset, or possible bias for top athletes in a particular country be born at the same time of the year.
- (If students have an understanding of distributions.) Examine the distribution of birthdays in this set – describe it in terms of shape, center, and spread. Is it reasonable to assume that all birthdays are equally likely? That is to say, does the distribution of DOB appear to be uniform?
- (If students have an understanding of sampling distributions.) Take 100 random samples of size 23 and record the proportion of samples with a shared birthday. Repeat this 30 times to obtain 30 proportions, and then graph the sampling distribution of proportions for samples of size 23. Examine the relationship of the shape, center, and spread of the sampling distribution.
- (If students have an understanding of sampling distributions.) Repeat the previous activity with samples of different size – perhaps 12, 20, 22, 24, 30, 40, or 180. Compare the various sampling distributions, and describe what changes in the sampling distribution as the sample size increases.

References

1. Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K–12 Curriculum Framework*. Alexandria, Va.: American Statistical Association, 2007.
2. This lesson was based on the following article: Jones, D. L. (2012). The birthday problem, empirically. *Mathematics Teacher*, 105(6), 480.
3. Data from World Cup athletes was downloaded from <http://soccernet.espn.go.com/world-cup/?cc=5901&ver=us>
4. von Mises, Richard. “Über Aufteilungen – und Besetzungen – Wahrscheinlichkeiten.” *Revue de la Faculté des Sciences de l’Université d’Istanbul*, N.S. 4, 1939, 145-63. Reprinted in *Selected Papers of Richard von Mises*, vol. 2, 313-34, edited by Philipp Frank, Sydney Goldstein, Mark Kac, William Prager, Gábor Szegő, and Garrett Birkhoff. Providence, R.I.: American Mathematical Society, 1964.

Happy Birthday to ... Two? Activity Sheet – Part 1

In 1939, Richard von Mises posed the **Birthday Problem**: “How many people would need to be in a room so that it was more likely than not that at least two of them shared a birthday?”

1. What are some examples of things that are ‘more likely than not’ to occur?
2. What fraction or percent of the time would something need to occur that would lead us to say that it was ‘more likely than not’?”
3. What is your initial guess at an answer for the Birthday Problem?

You will examine a data set of 736 birthdays of athletes that participated in the 2010 World Cup, an international soccer tournament. Below is a portion of the data table and an explanation of the variables.

Country	Group	Num	Position	DOB
France	A	2	Defender	45
Greece	B	12	Goalkeeper	224
Chile	H	7	Forward	353
Honduras	H	17	Midfielder	18

Country The athlete’s home country and team name. There are 32 different countries in this data set.

Group The 32 teams are divided into eight groups, with four countries in each group. Groups are named by the letters A, B, C, D, E, F, G, & H.

Num The player’s jersey number. There are 23 athletes on each team, and the numbers are the whole numbers ranging from 1 to 23.

Position The athlete’s position, which is either Forward, Midfielder, Defender, or Goalkeeper. Each team has exactly 3 Goalkeepers.

DOB The Julian date of the athlete’s date of birth. The Julian date is a count of the number of days that have passed from January 1 to a given date. For example, DOB = 18 for an athlete born on January 18. For an athlete born on February 14, DOB = 45 (31 days in January + 14 days in February).

Note: This particular set contains no birthdates on February 29, so leap day has been ignored.

4. How may we use these data to investigate the Birthday Problem?

Happy Birthday to ... Two? Activity Sheet – Part 2

Investigate subsets of size 184 using the following partition of athletes: Groups A & B; Groups C & D; Groups E & F; Groups G & H. Your teacher will assign you to look at one of these subsets.

My subset is: _____ size = 184 Is there a shared birthday? _____

Examine your subset. If you have a spreadsheet, it may be helpful to sort the data by Group, and then by DOB. This will put the Groups in order, and then the birthdates. Once you find a shared birthday, you don't need to look any further.

Combine your results with the rest of the class to find the proportion of these subsets in which shared birthdays occur.

The proportion of subsets that contain shared birthdays can be thought of as the *experimental probability* that at least two people in a subset share a birthday.

5. Based on the results above, should we examine subsets with fewer or more athletes in order to find a proportion (or experimental probability) closer to 0.5?
6. Choose a different number of athletes per subset, perhaps 92, which can be found by letting each subset consist of the athletes in the eight Groups (A through H).

My subset is: _____ size = 92 Is there a shared birthday? _____

Combine your results with the rest of the class to find the proportion of these subsets in which shared birthdays occur. What is that proportion?

7. Continue this process, choosing a different number of athletes per subset, examining the data set for shared birthdays within each subset, and then computing the proportion of subsets in which shared birthdays occur. Record your results in the table on the next page.

Description of subset	Number of subsets	Number of athletes in each subset	Experimental probability that at least two athletes in a subset shared a birthday
Pairs of Groups (listed in order from A to H)	4	184	
Group	8	92	

With your class, organize a table similar to the one above, and order the list according to the number of athletes per subset.

8. What patterns do you notice in the column of experimental probabilities?

9. Which subset sizes yielded experimental probabilities near 0.5, and how might we use this information to answer the Birthday Problem?

Note

The data are included in a table that spans the next eight pages of this document.

The data below has been sorted first by Group, then Country, and finally by DOB.

When using a spreadsheet, it is possible to sort the data in various ways, which may be helpful for students as they examine various subsets. Typically, there is a “sort” button on the toolbar, which opens a dialog box to indicate how the data should be sorted. This capability is also present within statistical software packages.

Country	Group	Num	Position	DOB
France	A	2	Defender	45
France	A	6	Defender	66
France	A	21	Forward	73
France	A	16	Goalkeeper	82
France	A	7	Midfielder	97
France	A	17	Defender	101
France	A	19	Midfielder	131
France	A	11	Forward	132
France	A	13	Defender	135
France	A	15	Midfielder	164
France	A	8	Midfielder	192
France	A	18	Midfielder	198
France	A	22	Defender	207
France	A	10	Forward	208
France	A	9	Forward	224
France	A	12	Forward	229
France	A	5	Defender	229
France	A	14	Midfielder	253
France	A	3	Defender	254
France	A	20	Midfielder	271
France	A	4	Defender	314
France	A	1	Goalkeeper	360
France	A	23	Goalkeeper	364
Mexico	A	20	Defender	16
Mexico	A	15	Defender	17
Mexico	A	10	Forward	17
Mexico	A	1	Goalkeeper	32
Mexico	A	4	Defender	44
Mexico	A	16	Defender	53
Mexico	A	11	Forward	60
Mexico	A	12	Defender	65
Mexico	A	5	Defender	89
Mexico	A	3	Defender	92
Mexico	A	6	Midfielder	120
Mexico	A	17	Midfielder	131
Mexico	A	21	Forward	135
Mexico	A	22	Midfielder	149
Mexico	A	14	Forward	152
Mexico	A	7	Midfielder	172
Mexico	A	13	Goalkeeper	194
Mexico	A	23	Goalkeeper	202
Mexico	A	18	Midfielder	271
Mexico	A	2	Defender	293

Mexico	A	9	Forward	307
Mexico	A	19	Defender	325
Mexico	A	8	Midfielder	354
South Africa	A	20	Defender	6
South Africa	A	14	Defender	73
South Africa	A	17	Forward	75
South Africa	A	10	Midfielder	76
South Africa	A	19	Midfielder	79
South Africa	A	2	Defender	96
South Africa	A	7	Midfielder	101
South Africa	A	3	Defender	125
South Africa	A	1	Goalkeeper	139
South Africa	A	23	Midfielder	142
South Africa	A	12	Midfielder	160
South Africa	A	16	Goalkeeper	171
South Africa	A	8	Midfielder	268
South Africa	A	21	Defender	272
South Africa	A	15	Defender	292
South Africa	A	5	Defender	293
South Africa	A	13	Midfielder	328
South Africa	A	4	Defender	329
South Africa	A	6	Midfielder	329
South Africa	A	9	Forward	333
South Africa	A	18	Forward	336
South Africa	A	11	Midfielder	356
South Africa	A	22	Goalkeeper	360
Uruguay	A	8	Midfielder	8
Uruguay	A	9	Forward	24
Uruguay	A	7	Forward	45
Uruguay	A	3	Defender	47
Uruguay	A	14	Midfielder	80
Uruguay	A	23	Goalkeeper	84
Uruguay	A	22	Defender	97
Uruguay	A	12	Goalkeeper	107
Uruguay	A	16	Midfielder	128
Uruguay	A	18	Midfielder	134
Uruguay	A	15	Midfielder	138
Uruguay	A	10	Forward	139
Uruguay	A	21	Forward	143
Uruguay	A	1	Goalkeeper	167
Uruguay	A	5	Midfielder	208
Uruguay	A	17	Midfielder	270
Uruguay	A	6	Defender	284
Uruguay	A	20	Midfielder	284

Uruguay	A	13	Forward	290
Uruguay	A	2	Defender	306
Uruguay	A	4	Defender	323
Uruguay	A	11	Midfielder	332
Uruguay	A	19	Defender	348
Argentina	B	20	Midfielder	2
Argentina	B	11	Forward	36
Argentina	B	15	Defender	43
Argentina	B	7	Midfielder	45
Argentina	B	1	Goalkeeper	47
Argentina	B	5	Midfielder	48
Argentina	B	8	Midfielder	68
Argentina	B	6	Defender	78
Argentina	B	13	Defender	82
Argentina	B	4	Defender	102
Argentina	B	16	Forward	153
Argentina	B	14	Midfielder	159
Argentina	B	19	Forward	163
Argentina	B	23	Midfielder	171
Argentina	B	10	Forward	175
Argentina	B	17	Midfielder	186
Argentina	B	12	Defender	195
Argentina	B	22	Goalkeeper	203
Argentina	B	21	Goalkeeper	211
Argentina	B	3	Defender	212
Argentina	B	18	Forward	311
Argentina	B	9	Forward	344
Argentina	B	2	Defender	354
Greece	B	23	Midfielder	9
Greece	B	11	Defender	36
Greece	B	9	Forward	40
Greece	B	6	Midfielder	44
Greece	B	7	Forward	52
Greece	B	10	Midfielder	65
Greece	B	22	Defender	70
Greece	B	3	Midfielder	78
Greece	B	18	Midfielder	93
Greece	B	17	Forward	143
Greece	B	1	Goalkeeper	150
Greece	B	2	Defender	155
Greece	B	20	Forward	159
Greece	B	19	Defender	160
Greece	B	15	Defender	161
Greece	B	21	Midfielder	172

Greece	B	16	Defender	204
Greece	B	12	Goalkeeper	224
Greece	B	14	Forward	230
Greece	B	5	Defender	238
Greece	B	13	Goalkeeper	252
Greece	B	4	Defender	283
Greece	B	8	Defender	337
Nigeria	B	7	Midfielder	8
Nigeria	B	21	Defender	20
Nigeria	B	18	Forward	84
Nigeria	B	16	Goalkeeper	98
Nigeria	B	3	Defender	106
Nigeria	B	5	Defender	108
Nigeria	B	14	Midfielder	122
Nigeria	B	19	Forward	152
Nigeria	B	20	Midfielder	159
Nigeria	B	11	Forward	196
Nigeria	B	4	Forward	213
Nigeria	B	1	Goalkeeper	241
Nigeria	B	6	Defender	245
Nigeria	B	2	Defender	249
Nigeria	B	10	Forward	283
Nigeria	B	9	Forward	301
Nigeria	B	13	Midfielder	308
Nigeria	B	12	Midfielder	319
Nigeria	B	23	Goalkeeper	324
Nigeria	B	8	Forward	326
Nigeria	B	15	Midfielder	338
Nigeria	B	17	Defender	351
Nigeria	B	22	Defender	359
South Korea	B	18	Goalkeeper	4
South Korea	B	14	Defender	8
South Korea	B	16	Midfielder	24
South Korea	B	9	Forward	27
South Korea	B	15	Defender	29
South Korea	B	23	Defender	45
South Korea	B	7	Midfielder	56
South Korea	B	11	Forward	65
South Korea	B	5	Midfielder	73
South Korea	B	19	Midfielder	89
South Korea	B	12	Defender	113
South Korea	B	1	Goalkeeper	116
South Korea	B	3	Defender	117
South Korea	B	20	Forward	119

South Korea	B	8	Midfielder	129
South Korea	B	21	Goalkeeper	179
South Korea	B	17	Midfielder	183
South Korea	B	10	Forward	191
South Korea	B	22	Defender	206
South Korea	B	2	Defender	210
South Korea	B	13	Midfielder	276
South Korea	B	6	Midfielder	279
South Korea	B	4	Defender	307
Algeria	C	10	Forward	38
Algeria	C	22	Midfielder	45
Algeria	C	7	Midfielder	50
Algeria	C	6	Midfielder	56
Algeria	C	11	Forward	67
Algeria	C	4	Defender	80
Algeria	C	12	Defender	87
Algeria	C	23	Goalkeeper	115
Algeria	C	19	Midfielder	134
Algeria	C	18	Defender	135
Algeria	C	8	Midfielder	135
Algeria	C	3	Defender	169
Algeria	C	13	Midfielder	176
Algeria	C	14	Defender	210
Algeria	C	15	Midfielder	229
Algeria	C	5	Defender	245
Algeria	C	1	Goalkeeper	271
Algeria	C	2	Defender	280
Algeria	C	20	Midfielder	282
Algeria	C	17	Midfielder	316
Algeria	C	16	Goalkeeper	339
Algeria	C	21	Midfielder	339
Algeria	C	9	Forward	339
England	C	16	Midfielder	4
England	C	21	Forward	11
England	C	12	Goalkeeper	18
England	C	18	Defender	28
England	C	9	Forward	30
England	C	14	Midfielder	54
England	C	7	Midfielder	106
England	C	15	Defender	108
England	C	23	Goalkeeper	109
England	C	4	Midfielder	150
England	C	8	Midfielder	171
England	C	22	Midfielder	209

England	C	1	Goalkeeper	213
England	C	2	Defender	235
England	C	19	Forward	280
England	C	20	Defender	285
England	C	10	Forward	297
England	C	17	Midfielder	298
England	C	11	Midfielder	312
England	C	5	Defender	322
England	C	6	Defender	341
England	C	13	Defender	346
England	C	3	Defender	354
Slovenia	C	7	Forward	3
Slovenia	C	23	Forward	13
Slovenia	C	12	Goalkeeper	28
Slovenia	C	22	Defender	29
Slovenia	C	16	Goalkeeper	32
Slovenia	C	6	Defender	37
Slovenia	C	4	Defender	68
Slovenia	C	18	Midfielder	115
Slovenia	C	2	Defender	121
Slovenia	C	13	Defender	137
Slovenia	C	11	Forward	138
Slovenia	C	15	Midfielder	141
Slovenia	C	5	Defender	190
Slovenia	C	1	Goalkeeper	195
Slovenia	C	10	Midfielder	219
Slovenia	C	3	Defender	237
Slovenia	C	17	Midfielder	249
Slovenia	C	19	Defender	259
Slovenia	C	8	Midfielder	263
Slovenia	C	21	Midfielder	270
Slovenia	C	14	Forward	278
Slovenia	C	20	Midfielder	338
Slovenia	C	9	Forward	349
United States	C	22	Midfielder	19
United States	C	13	Midfielder	41
United States	C	6	Defender	50
United States	C	2	Defender	60
United States	C	10	Forward	63
United States	C	1	Goalkeeper	65
United States	C	8	Midfielder	68
United States	C	9	Forward	96
United States	C	19	Midfielder	108
United States	C	5	Defender	133

United States	C	21	Defender	137
United States	C	14	Forward	141
United States	C	7	Midfielder	144
United States	C	3	Defender	145
United States	C	23	Goalkeeper	166
United States	C	4	Midfielder	212
United States	C	11	Midfielder	213
United States	C	20	Forward	216
United States	C	18	Goalkeeper	252
United States	C	16	Midfielder	302
United States	C	17	Forward	310
United States	C	12	Defender	311
United States	C	15	Defender	338
Australia	D	12	Goalkeeper	31
Australia	D	23	Midfielder	42
Australia	D	7	Midfielder	53
Australia	D	14	Forward	58
Australia	D	2	Defender	68
Australia	D	22	Midfielder	98
Australia	D	18	Goalkeeper	163
Australia	D	17	Forward	173
Australia	D	6	Defender	215
Australia	D	15	Midfielder	215
Australia	D	20	Defender	216
Australia	D	5	Midfielder	217
Australia	D	16	Midfielder	226
Australia	D	9	Forward	232
Australia	D	19	Midfielder	247
Australia	D	10	Midfielder	265
Australia	D	8	Midfielder	275
Australia	D	13	Midfielder	278
Australia	D	1	Goalkeeper	279
Australia	D	21	Defender	334
Australia	D	4	Midfielder	340
Australia	D	3	Defender	346
Australia	D	11	Midfielder	364
Germany	D	18	Midfielder	4
Germany	D	4	Defender	14
Germany	D	9	Forward	25
Germany	D	21	Midfielder	72
Germany	D	14	Defender	72
Germany	D	15	Midfielder	81
Germany	D	19	Forward	86
Germany	D	1	Goalkeeper	86

Germany	D	6	Midfielder	94
Germany	D	5	Defender	114
Germany	D	22	Goalkeeper	148
Germany	D	3	Defender	149
Germany	D	10	Forward	155
Germany	D	11	Forward	160
Germany	D	23	Forward	191
Germany	D	7	Midfielder	213
Germany	D	20	Defender	246
Germany	D	13	Forward	256
Germany	D	17	Defender	272
Germany	D	8	Midfielder	288
Germany	D	2	Defender	308
Germany	D	16	Defender	315
Germany	D	12	Goalkeeper	351
Ghana	D	23	Midfielder	65
Ghana	D	20	Forward	105
Ghana	D	17	Defender	106
Ghana	D	9	Midfielder	122
Ghana	D	22	Goalkeeper	164
Ghana	D	4	Defender	166
Ghana	D	15	Defender	172
Ghana	D	2	Defender	179
Ghana	D	8	Defender	194
Ghana	D	6	Midfielder	202
Ghana	D	7	Defender	234
Ghana	D	11	Midfielder	239
Ghana	D	16	Goalkeeper	248
Ghana	D	19	Defender	269
Ghana	D	14	Forward	297
Ghana	D	12	Forward	313
Ghana	D	1	Goalkeeper	314
Ghana	D	3	Forward	326
Ghana	D	5	Defender	333
Ghana	D	18	Forward	333
Ghana	D	21	Midfielder	343
Ghana	D	13	Midfielder	351
Ghana	D	10	Midfielder	358
Serbia	D	21	Forward	23
Serbia	D	2	Defender	26
Serbia	D	4	Midfielder	26
Serbia	D	6	Defender	53
Serbia	D	19	Midfielder	67
Serbia	D	12	Goalkeeper	84

Serbia	D	14	Midfielder	108
Serbia	D	7	Midfielder	118
Serbia	D	8	Forward	137
Serbia	D	16	Defender	206
Serbia	D	1	Goalkeeper	209
Serbia	D	11	Midfielder	242
Serbia	D	10	Midfielder	254
Serbia	D	9	Forward	258
Serbia	D	22	Midfielder	265
Serbia	D	15	Forward	268
Serbia	D	5	Defender	294
Serbia	D	13	Defender	296
Serbia	D	17	Midfielder	305
Serbia	D	3	Defender	314
Serbia	D	23	Goalkeeper	325
Serbia	D	20	Defender	344
Serbia	D	18	Midfielder	359
Cameroon	E	15	Forward	20
Cameroon	E	23	Forward	22
Cameroon	E	1	Goalkeeper	49
Cameroon	E	22	Goalkeeper	59
Cameroon	E	17	Forward	67
Cameroon	E	9	Forward	69
Cameroon	E	13	Forward	82
Cameroon	E	18	Midfielder	82
Cameroon	E	2	Defender	83
Cameroon	E	3	Defender	86
Cameroon	E	12	Defender	115
Cameroon	E	19	Defender	140
Cameroon	E	11	Midfielder	149
Cameroon	E	10	Midfielder	156
Cameroon	E	14	Midfielder	171
Cameroon	E	4	Defender	182
Cameroon	E	5	Defender	190
Cameroon	E	21	Defender	220
Cameroon	E	6	Midfielder	252
Cameroon	E	16	Goalkeeper	326
Cameroon	E	7	Midfielder	332
Cameroon	E	20	Midfielder	343
Cameroon	E	8	Defender	354
Denmark	E	11	Forward	16
Denmark	E	23	Defender	28
Denmark	E	21	Midfielder	45
Denmark	E	5	Midfielder	55

Denmark	E	2	Midfielder	59
Denmark	E	12	Midfielder	79
Denmark	E	3	Defender	85
Denmark	E	22	Goalkeeper	114
Denmark	E	1	Goalkeeper	163
Denmark	E	7	Midfielder	176
Denmark	E	14	Midfielder	188
Denmark	E	19	Midfielder	203
Denmark	E	20	Midfielder	208
Denmark	E	13	Defender	212
Denmark	E	8	Midfielder	224
Denmark	E	9	Forward	241
Denmark	E	18	Forward	249
Denmark	E	6	Defender	263
Denmark	E	10	Midfielder	279
Denmark	E	15	Midfielder	280
Denmark	E	17	Midfielder	297
Denmark	E	16	Goalkeeper	330
Denmark	E	4	Defender	346
Japan	E	17	Midfielder	18
Japan	E	15	Defender	25
Japan	E	7	Midfielder	28
Japan	E	13	Defender	30
Japan	E	22	Defender	56
Japan	E	21	Goalkeeper	79
Japan	E	6	Defender	86
Japan	E	12	Forward	95
Japan	E	11	Forward	101
Japan	E	1	Goalkeeper	105
Japan	E	9	Forward	106
Japan	E	4	Defender	114
Japan	E	19	Forward	127
Japan	E	8	Midfielder	131
Japan	E	16	Forward	160
Japan	E	18	Midfielder	164
Japan	E	10	Midfielder	175
Japan	E	3	Defender	206
Japan	E	23	Goalkeeper	227
Japan	E	2	Midfielder	249
Japan	E	5	Defender	255
Japan	E	20	Midfielder	261
Japan	E	14	Midfielder	304
Netherlands	E	18	Midfielder	11
Netherlands	E	11	Forward	23

Netherlands	E	2	Defender	34
Netherlands	E	5	Defender	36
Netherlands	E	23	Midfielder	42
Netherlands	E	17	Forward	44
Netherlands	E	20	Midfielder	92
Netherlands	E	4	Defender	95
Netherlands	E	15	Defender	98
Netherlands	E	6	Midfielder	112
Netherlands	E	14	Midfielder	146
Netherlands	E	10	Midfielder	160
Netherlands	E	13	Defender	192
Netherlands	E	7	Forward	203
Netherlands	E	9	Forward	218
Netherlands	E	21	Forward	224
Netherlands	E	1	Goalkeeper	265
Netherlands	E	22	Goalkeeper	293
Netherlands	E	16	Goalkeeper	293
Netherlands	E	3	Defender	319
Netherlands	E	8	Midfielder	334
Netherlands	E	19	Forward	353
Netherlands	E	12	Defender	362
Italy	F	8	Midfielder	9
Italy	F	22	Midfielder	18
Italy	F	15	Midfielder	19
Italy	F	1	Goalkeeper	28
Italy	F	18	Forward	31
Italy	F	12	Goalkeeper	38
Italy	F	2	Defender	42
Italy	F	19	Defender	50
Italy	F	14	Goalkeeper	85
Italy	F	23	Defender	121
Italy	F	21	Midfielder	139
Italy	F	11	Forward	186
Italy	F	6	Midfielder	205
Italy	F	20	Forward	214
Italy	F	4	Defender	226
Italy	F	7	Forward	242
Italy	F	5	Defender	256
Italy	F	17	Midfielder	268
Italy	F	16	Midfielder	277
Italy	F	10	Forward	286
Italy	F	9	Forward	325
Italy	F	13	Defender	334
Italy	F	3	Defender	364

New Zealand	F	15	Midfielder	7
New Zealand	F	3	Defender	12
New Zealand	F	16	Midfielder	15
New Zealand	F	12	Goalkeeper	19
New Zealand	F	2	Defender	34
New Zealand	F	8	Midfielder	65
New Zealand	F	14	Forward	79
New Zealand	F	17	Defender	83
New Zealand	F	19	Defender	90
New Zealand	F	21	Midfielder	142
New Zealand	F	7	Midfielder	161
New Zealand	F	23	Goalkeeper	181
New Zealand	F	4	Defender	184
New Zealand	F	5	Defender	246
New Zealand	F	18	Defender	261
New Zealand	F	9	Forward	272
New Zealand	F	22	Midfielder	280
New Zealand	F	10	Forward	281
New Zealand	F	6	Defender	291
New Zealand	F	20	Forward	341
New Zealand	F	1	Goalkeeper	347
New Zealand	F	11	Midfielder	354
New Zealand	F	13	Midfielder	358
Paraguay	F	22	Goalkeeper	20
Paraguay	F	14	Defender	32
Paraguay	F	3	Defender	33
Paraguay	F	13	Midfielder	69
Paraguay	F	15	Midfielder	84
Paraguay	F	7	Forward	140
Paraguay	F	17	Defender	167
Paraguay	F	1	Goalkeeper	181
Paraguay	F	8	Midfielder	196
Paraguay	F	12	Goalkeeper	197
Paraguay	F	2	Defender	207
Paraguay	F	21	Defender	211
Paraguay	F	9	Forward	228
Paraguay	F	4	Defender	241
Paraguay	F	6	Midfielder	275
Paraguay	F	5	Defender	278
Paraguay	F	20	Midfielder	280
Paraguay	F	16	Midfielder	289
Paraguay	F	11	Midfielder	292
Paraguay	F	10	Forward	312
Paraguay	F	19	Forward	317

Paraguay	F	18	Forward	332
Paraguay	F	23	Forward	344
Slovakia	F	13	Forward	17
Slovakia	F	21	Defender	24
Slovakia	F	4	Defender	26
Slovakia	F	14	Forward	57
Slovakia	F	19	Midfielder	57
Slovakia	F	11	Forward	91
Slovakia	F	8	Midfielder	112
Slovakia	F	20	Midfielder	138
Slovakia	F	23	Goalkeeper	141
Slovakia	F	6	Midfielder	160
Slovakia	F	17	Midfielder	208
Slovakia	F	10	Midfielder	212
Slovakia	F	5	Defender	259
Slovakia	F	15	Midfielder	292
Slovakia	F	18	Forward	299
Slovakia	F	2	Defender	303
Slovakia	F	22	Defender	306
Slovakia	F	12	Goalkeeper	332
Slovakia	F	7	Midfielder	334
Slovakia	F	1	Goalkeeper	339
Slovakia	F	16	Defender	344
Slovakia	F	3	Defender	349
Slovakia	F	9	Forward	350
Brazil	G	11	Forward	25
Brazil	G	4	Defender	32
Brazil	G	14	Defender	44
Brazil	G	12	Goalkeeper	46
Brazil	G	18	Midfielder	83
Brazil	G	23	Forward	92
Brazil	G	10	Midfielder	112
Brazil	G	16	Defender	115
Brazil	G	13	Defender	126
Brazil	G	3	Defender	128
Brazil	G	7	Midfielder	165
Brazil	G	20	Midfielder	170
Brazil	G	21	Forward	195
Brazil	G	17	Midfielder	200
Brazil	G	2	Defender	207
Brazil	G	6	Defender	214
Brazil	G	5	Midfielder	238
Brazil	G	1	Goalkeeper	246
Brazil	G	15	Defender	265

Brazil	G	19	Midfielder	274
Brazil	G	8	Midfielder	280
Brazil	G	22	Goalkeeper	295
Brazil	G	9	Forward	312
Ivory Coast	G	22	Defender	13
Ivory Coast	G	6	Defender	39
Ivory Coast	G	17	Defender	53
Ivory Coast	G	11	Forward	70
Ivory Coast	G	12	Midfielder	74
Ivory Coast	G	4	Defender	78
Ivory Coast	G	3	Defender	92
Ivory Coast	G	19	Midfielder	133
Ivory Coast	G	10	Forward	147
Ivory Coast	G	21	Defender	155
Ivory Coast	G	13	Midfielder	155
Ivory Coast	G	20	Defender	164
Ivory Coast	G	9	Midfielder	172
Ivory Coast	G	8	Forward	217
Ivory Coast	G	18	Midfielder	218
Ivory Coast	G	23	Goalkeeper	317
Ivory Coast	G	15	Forward	330
Ivory Coast	G	2	Defender	332
Ivory Coast	G	5	Midfielder	348
Ivory Coast	G	16	Goalkeeper	364
Ivory Coast	G	1	Goalkeeper	364
Ivory Coast	G	14	Midfielder	365
Ivory Coast	G	7	Forward	365
North Korea	G	12	Forward	40
North Korea	G	19	Midfielder	49
North Korea	G	9	Forward	61
North Korea	G	16	Defender	127
North Korea	G	10	Forward	142
North Korea	G	23	Defender	150
North Korea	G	7	Forward	178
North Korea	G	4	Midfielder	183
North Korea	G	20	Goalkeeper	196
North Korea	G	15	Midfielder	200
North Korea	G	21	Defender	229
North Korea	G	3	Defender	236
North Korea	G	5	Defender	247
North Korea	G	13	Defender	248
North Korea	G	1	Goalkeeper	252
North Korea	G	2	Defender	268
North Korea	G	11	Midfielder	272

North Korea	G	14	Defender	276
North Korea	G	6	Forward	283
North Korea	G	18	Goalkeeper	289
North Korea	G	17	Midfielder	298
North Korea	G	8	Midfielder	324
North Korea	G	22	Midfielder	345
Portugal	G	13	Defender	4
Portugal	G	3	Defender	18
Portugal	G	17	Midfielder	27
Portugal	G	7	Forward	36
Portugal	G	8	Midfielder	57
Portugal	G	15	Midfielder	57
Portugal	G	23	Defender	70
Portugal	G	16	Midfielder	76
Portugal	G	12	Goalkeeper	121
Portugal	G	19	Midfielder	122
Portugal	G	14	Midfielder	131
Portugal	G	21	Defender	136
Portugal	G	6	Defender	138
Portugal	G	18	Forward	143
Portugal	G	5	Defender	178
Portugal	G	10	Forward	219
Portugal	G	20	Midfielder	239
Portugal	G	4	Defender	243
Portugal	G	1	Goalkeeper	262
Portugal	G	22	Goalkeeper	268
Portugal	G	11	Midfielder	303
Portugal	G	2	Defender	331
Portugal	G	9	Forward	351
Chile	H	16	Forward	27
Chile	H	6	Midfielder	52
Chile	H	19	Midfielder	80
Chile	H	1	Goalkeeper	103
Chile	H	9	Forward	130
Chile	H	14	Midfielder	135
Chile	H	23	Goalkeeper	138
Chile	H	8	Defender	142
Chile	H	13	Midfielder	148
Chile	H	15	Midfielder	152
Chile	H	4	Defender	163
Chile	H	12	Goalkeeper	185
Chile	H	11	Midfielder	191
Chile	H	22	Forward	213
Chile	H	17	Defender	215

Chile	H	2	Defender	216
Chile	H	18	Defender	241
Chile	H	5	Defender	254
Chile	H	21	Midfielder	287
Chile	H	10	Forward	292
Chile	H	20	Midfielder	307
Chile	H	3	Defender	338
Chile	H	7	Forward	353
Honduras	H	17	Midfielder	18
Honduras	H	22	Goalkeeper	34
Honduras	H	6	Midfielder	54
Honduras	H	12	Forward	68
Honduras	H	15	Forward	83
Honduras	H	3	Defender	122
Honduras	H	20	Midfielder	122
Honduras	H	18	Goalkeeper	123
Honduras	H	19	Midfielder	128
Honduras	H	21	Defender	130
Honduras	H	23	Defender	143
Honduras	H	5	Defender	144
Honduras	H	1	Goalkeeper	150
Honduras	H	8	Midfielder	210
Honduras	H	2	Defender	210
Honduras	H	14	Defender	247
Honduras	H	9	Forward	282
Honduras	H	13	Midfielder	298
Honduras	H	10	Forward	305
Honduras	H	11	Forward	309
Honduras	H	16	Defender	315
Honduras	H	7	Midfielder	318
Honduras	H	4	Defender	354
Spain	H	21	Midfielder	8
Spain	H	12	Goalkeeper	14
Spain	H	17	Defender	17
Spain	H	8	Midfielder	25
Spain	H	3	Defender	33
Spain	H	11	Defender	34
Spain	H	19	Forward	57
Spain	H	9	Forward	79
Spain	H	15	Defender	89
Spain	H	5	Defender	103
Spain	H	13	Midfielder	118
Spain	H	10	Midfielder	124
Spain	H	6	Midfielder	131

Spain	H	1	Goalkeeper	140
Spain	H	16	Midfielder	197
Spain	H	18	Midfielder	209
Spain	H	4	Defender	212
Spain	H	23	Goalkeeper	243
Spain	H	20	Midfielder	245
Spain	H	2	Defender	247
Spain	H	22	Midfielder	325
Spain	H	14	Midfielder	329
Spain	H	7	Forward	337
Switzerland	H	2	Defender	16
Switzerland	H	17	Defender	16
Switzerland	H	22	Defender	24
Switzerland	H	4	Defender	45
Switzerland	H	15	Midfielder	53
Switzerland	H	20	Midfielder	68
Switzerland	H	13	Defender	89
Switzerland	H	11	Midfielder	109
Switzerland	H	3	Defender	110
Switzerland	H	7	Midfielder	142
Switzerland	H	10	Forward	145
Switzerland	H	5	Defender	161
Switzerland	H	19	Forward	163
Switzerland	H	8	Midfielder	178
Switzerland	H	21	Goalkeeper	181
Switzerland	H	6	Midfielder	188
Switzerland	H	9	Forward	196
Switzerland	H	12	Goalkeeper	234
Switzerland	H	16	Midfielder	245
Switzerland	H	1	Goalkeeper	251
Switzerland	H	23	Midfielder	283
Switzerland	H	18	Forward	333
Switzerland	H	14	Midfielder	342