

# You and Michael

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## Overview of Lesson

Roman writer, architect, and engineer Marcus Vitruvius proposed, among other relationships, that a person's height and their arm span (herein called "wingspan") are approximately equal. In this investigation, students will collect data to assess whether or not Vitruvius's proposal was reasonable. Scatterplots will be drawn to illustrate the data and a best-fit line will be overlain on the scatterplot. The equation of the best-fit line will be determined, and the slope interpreted in context.

## GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level B activity.

## Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.

## Common Core State Standards Grade Level Content (Grade 8)

8. SP. 1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
8. SP. 2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
8. SP. 3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

## **NCTM Principles and Standards for School Mathematics**

### **Data Analysis and Probability Standards for Grades 6-8**

**Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them:**

- formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
- select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.

**Select and use appropriate statistical methods to analyze data:**

- discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.

**Develop and evaluate inferences and predictions that are based on data:**

- make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;
- use conjectures to formulate new questions and plan new studies to answer them.

### **Prerequisites**

Students will have knowledge of making measurements (in centimeters) using a tape measure. Students will have knowledge of how to organize data in a table. Students will have knowledge of how to create a scatterplot of bivariate data. Students will have knowledge of how to draw a “best-fit” line on the scatterplot.

### **Learning Targets**

Students will be able to collect bivariate data and to create a scatterplot to display the data using technology. In addition, students will be able to analyze and interpret the data.

### **Time Required**

Approximately one 45-50 minute class period; some discussion time the following class period might be necessary.

### **Materials Required**

Graphing calculators or computers with spreadsheet or statistical software that can be used to create and print scatterplots. Centimeter tape measure at least 200 cm long.

## **Instructional Lesson Plan**

### **The GAISE Statistical Problem-Solving Procedure**

#### **I. Formulate Question(s)**

Begin the lesson by asking students if they think that student body dimensions (such as height, arm span) might be related.

Ask students to write some questions that they would be interested in investigating about students' height and arm span. Some possible questions might be:

1. What kinds of heights do typical students have? What is the shortest height a typical student might reasonably have? What is the tallest height a typical student might reasonably have?
2. What kinds of arm spans (herein called “wingspan”) might a typical student have? What is the shortest wingspan a typical student might reasonably have? What is the longest wingspan a typical student might reasonably have?
3. Do you think there is a relationship between students' height and wingspan?
4. If there is a relationship between height and wingspan, what might that relationship be?
5. What type of graph might be used to display the relationship? What would a graph of the data look like if a relationship between height and wingspan does exist?
6. Do you think athletes would follow the same relationship between height and wingspan as typical students? What athletes might follow this relationship? What athletes might not follow this relationship?
7. Are there other variables that might affect a relationship between height and wingspan? How might those variables affect the relationship?
8. How might we collect data to study the relationship between height and wingspan?

#### **II. Design and Implement a Plan to Collect the Data**

Begin the data collection phase by asking students what measurements should be made.

Continue the discussion asking questions such as who should make the measurements, how the student being measured should stand, and whether or not shoes should be worn. These questions form the basis of a measurement protocol. Ask the students why a measurement protocol is important. By developing a measurement protocol, there can be consistency from measurement to measurement.

Record the height and wingspan measurements in centimeters in a data table. Sample data are shown in the table below; a blank data table, including Michael Phelps' height and wingspan, is provided on the Activity Sheet on page 9.

Table 1. Sample class data.

<b>Student</b>	<b>Height</b>	<b>Wingspan</b>
Adam	152	151
Mallory	162	162
Marianne	160	158
Patrick	154	155
Darryl	149	147
Taylor	147	145
Tasha	162	162
Wes	159	160
Amanda	154	152
Dave	163	161
Jason	157	158
Jake	149	149
Darcy	163	162
Marissa	146	147
Paul	148	148
Teal	153	154
Zac	168	166
Ian	159	159
Alan	158	157
Ambreia	148	148
Davis	149	150
Greg	160	159
Michael	193	201

### III. Analyze the Data

Have students use appropriate technology (graphing calculator, Excel, statistical software) to create a scatterplot of height and wingspan data. Do not include Michael Phelps' data point on this scatterplot; it will be added later. A scatterplot for the sample data is shown below.

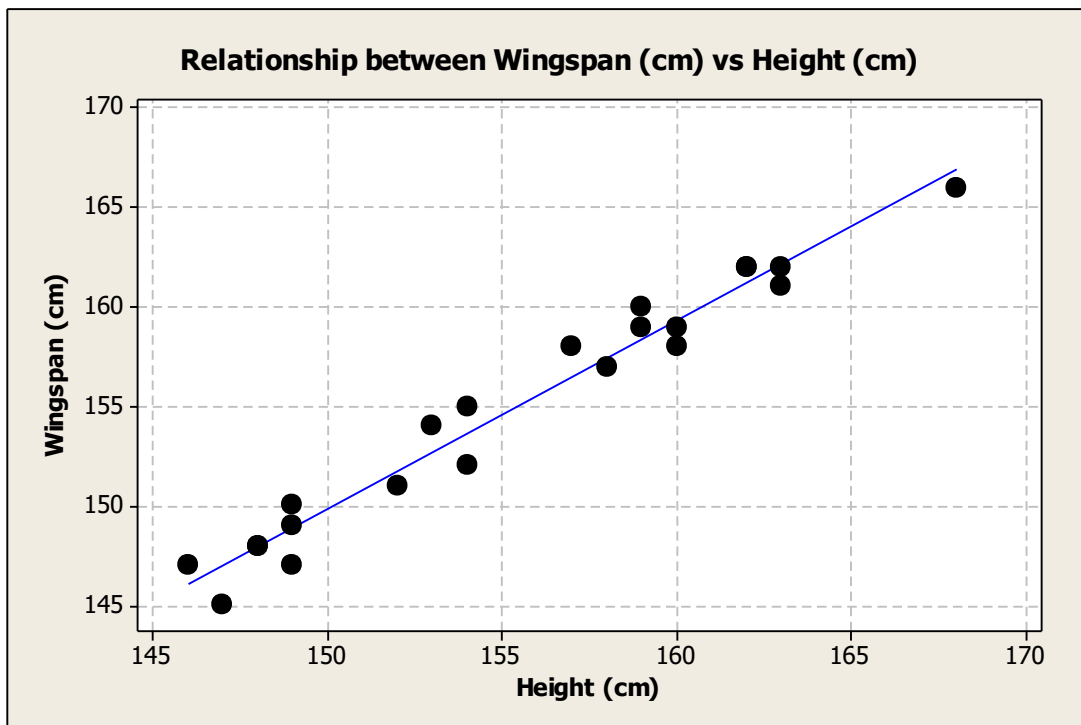


Figure 1. Scatterplot of sample class data.

Have students look carefully at the scatterplot. Ask students if there appears to be evidence of a relationship between height and wingspan. If there seems to be a relationship, what type of relationship is it? How would you describe the relationship (is it linear or non-linear)? Discuss what constitutes a “best fit” line, and have students draw a best fit line through the data, and write the equation of their best-fit line. Note that “best-fit” lines for this scatterplot may vary somewhat from student to student; they should all have a positive slope and the line should be positioned such that approximately an equal number of points lie above and below the line.

### IV. Interpret the Results

Ask the students to consider the slope of their best fit line and interpret that value in context. The slope determined for the least-squares regression line on the sample data is approximately 0.95, which indicates that for a one-centimeter increase in height, the wingspan increases by approximately 0.95 cm on average. Note that the slope may differ from this value depending on the choice of the best-fit line.

Have students consider what would happen to the best fit line if Michael Phelps' height and wingspan were added to the dataset. Without changing the best fit line on the scatterplot, add Michael's data point, and discuss where Michael's data point is located on the scatterplot. The new scatterplot, including Michael's data point, is shown below.

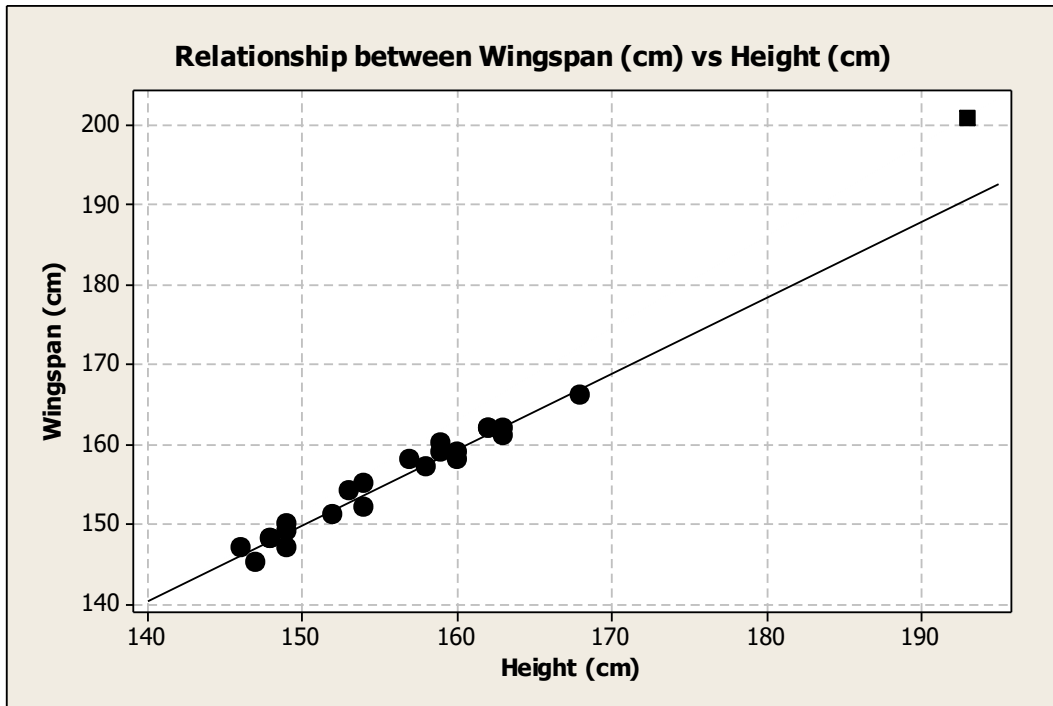


Figure 2. Scatterplot of sample class data including data for Michael Phelps.

## Assessment

1. How well does a line fit the wingspan vs. height data? What does that mean?
2. Can we claim that the scatterplot represents the relationship between height and “wingspan” in the general population? Why or why not?
3. What about Michael Phelps – is he like us or is he different? How?
4. How do your measurements compare to Michael Phelps?

## Answers

1. The data points do seem to cluster closely to the best-fit line; there is not a lot of deviation between the line and the points.
2. No, we cannot claim that the scatterplot represents the relationship between height and wingspan in the general population. These data values were collected for students; there is no guarantee that as adults or younger children this same relationship between height and wingspan holds true.
3. Although Michael seems to follow the same general trend, his wingspan seems to be somewhat longer compared to his height than typical students.
4. Answers may vary. One possible answer is “My height and wingspan are closer to each other than are Michael’s height and wingspan.”

## Possible Extensions

Vitruvius postulated other relationships between body measurements, such as the length of the hand is one-ninth of the height or that the kneeling height is three-fourths of the standing height. Research other relationships postulated by Vitruvius, and design an activity to verify or refute his postulates.

## References

Adapted from an activity created by Paul J. Fields, Ph.D. for the *American Statistical Association Meeting Within a Meeting Program for Middle School Teachers* (2008).



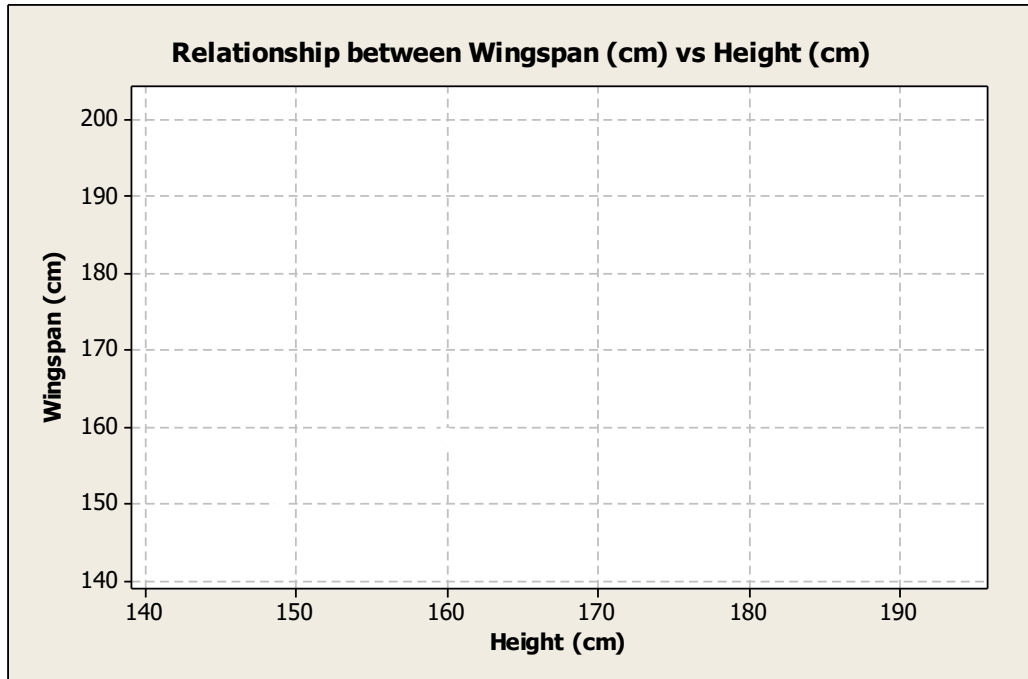
## **You and Michael Activity Sheet**

Roman writer, architect, and engineer Marcus Vitruvius proposed, among other relationships, that a person's height and their arm span (herein called "wingspan") are approximately equal. In this investigation, you will collect data to assess whether or not Vitruvius's proposal was reasonable.

1. What kinds of heights do typical students have? What is the shortest height a typical student might reasonably have? What is the tallest height a typical student might reasonably have?
  
2. What kinds of arm spans (herein called "wingspan") might a typical student have? What is the shortest wingspan a typical student might reasonably have? What is the longest wingspan a typical student might reasonably have?
  
3. Do you think there is a relationship between students' height and wingspan?
  
4. If there is a relationship between height and wingspan, what might that relationship be?
  
5. What type of graph might be used to display the relationship? What would a graph of the data look like if a relationship between height and wingspan does exist?
  
6. Do you think athletes would follow the same relationship between height and wingspan as typical students? What athletes might follow this relationship? What athletes might not follow this relationship?
  
7. Are there other variables that might affect a relationship between height and wingspan? How might those variables affect the relationship?



10. Using appropriate statistical software, make a scatterplot of wingspan vs. height (not including Michael Phelps' data point). Either print your scatterplot or reproduce it on the axes below.



11. Does there appear to be evidence of a relationship between height and wingspan? How would you describe the relationship (is it linear or non-linear)?

12. Draw a best-fit line through the data plotted on the scatterplot. What is the equation of that best-fit line?

13. Interpret the slope of the best-fit line in context.

**14.** What do you think would happen to the best-fit line if Michael Phelps' data point was added to the scatterplot? Add his data point to your scatterplot above.

**15.** Does Michael seem to have the same relationship between wingspan and height as the students in your class? How is it similar? How is it different?