What is the Probability of "Pigging Out"

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Published: April 2012

Overview of Lesson
This activity has students use the 'dice' game, *Pass the Pigs*, to design a simulation experiment in order to estimate a probability.

GAISE Components
This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level A activity.

Common Core State Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Common Core State Standards Grade Level Content (Grade 7)
7. SP. 5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
7. SP. 6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability Standards for Grades 6-8

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them:

- select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.

Select and use appropriate statistical methods to analyze data:

- find, use, and interpret measures of center and spread, including mean and interquartile range;
- discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.

Develop and evaluate inferences and predictions that are based on data:

- use conjectures to formulate new questions and plan new studies to answer them.

Understand and apply basic concepts of probability:

- use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.

Prerequisites

Prior to completing this activity students should have experience

- constructing dot plots and calculating measures of center, including the mean, median, and mode
- using technology to generate random integers
- representing probability outcomes with an area model

Learning Targets

Students will be able to devise and carry out a simulation in order to help determine a probability.

Time Required

One class period.

Materials Required

Several sets of *Pass the Pigs* pig dice or internet access to play *Pass the Pigs* online at:
http://www.softcom.net/users/mikey719/pass_the_pigs/PassThePigs.html.

Instructional Lesson Plan

The GAISE Statistical Problem-Solving Procedure

I. Formulate Question(s)

Begin by providing students with some background on the *Pass the Pigs* dice game. The game involves two tiny rubber pigs that players throw like dice. One side of each pig is marked with a single black dot. "The object of the game is to throw the pigs and score as many points in one turn as you can. The first player to score 100 points is the winner." (*Pass the Pigs* Instructions) Each pig can land in six different ways resulting in various point values for the tosser. Figure 1 displays the different landing positions along with their corresponding point values.

Position	Point Value	Position	Point Value
Razorback & or	5 points	Double Razorback	20 points
Trotter & or	5 points	Double Trotter	20 points
Snouter & or	10 points	Double Snouter	40 points
Leaning Jowler & or	15 points	Double Leaning Jowler	60 points
Pig Out	0 points	Sider or	1 point
Oinker	Back to zero	Mixed Combo	Combined score

Figure 1. *Pass the Pigs* pig dice landing positions and point values.

A turn consists of a player taking as many rolls as he or she desires until: (1) deciding to stop and record the total score for that turn, (2) a Pig Out is rolled and the player records a score of zero for that turn, or (3) an Oinker is thrown and all points accumulated in the game thus far by the player are lost.

In the activity, students will explore the experimental probability of obtaining a Pig Out. Students will work in groups of two. Each pair should have one *Pass the Pigs* dice game and a

flat table or desktop on which to work. Each student should have a sticky note and a copy of the Activity Worksheet (page 10).

Ask students to examine their pig dice. Discuss the possible outcomes if the pig dice are simultaneously tossed. Ask each student to estimate the probability that a Pig Out will occur. Students should write down and share their estimates and the reasoning behind them. This can lead into a discussion about the different types of probability. In addition to the two types of probability traditionally discussed in middle school (theoretical and experimental), this activity provides the opportunity to introduce students to subjective probability. Students recognize that this is a situation for which there is no theoretical probability and no data on hand for them to use to calculate an experimental probability. Therefore, they must assign probabilities based upon what they think will happen.

II. Design and Implement a Plan to Collect the Data

After students share their subjective estimates, they are ready to test them by developing an experiment. Students are asked to create possible plans for determining an estimate of the chances of a Pig Out. Once plans have been suggested, the class discusses the plans, agrees on a plan to use, and then carries out the plan. One possibility would be to have each pair of students toss the pig dice several times in order to see how often a Pig Out occurs.

Within each pair of students, the following tasks are assigned: one member tosses the pigs, and the other member records whether or not the pigs landed in the Pig Out position. After one student has completed ten trials, the students switch tasks and ten more trials are conducted. This provides student pairs with a total of twenty trials that can be used in determining an experimental probability. Table 1 shows typical class results.

Table 1. Example class results.

Student	Pig Out	No Pig Out	Student	Pig Out	No Pig Out	Student	Pig Out	No Pig Out
1	1	9	11	3	7	21	2	8
2	2	8	12	3	7	22	3	7
3	2	8	13	1	9	23	5	5
4	3	7	14	3	7	24	0	10
5	1	9	15	3	7	25	5	5
6	1	9	16	2	8	26	3	7
7	2	8	17	1	9	27	3	7
8	1	9	18	4	6	28	1	9
9	1	9	19	2	8	29	3	7
10	0	10	20	3	7	30	2	8

III. Analyze the Data

After students have conducted the experiment, ask them to refine their original estimates. Ask them how they could be more certain in their estimate. Most will respond that more tosses would result in more certainty. However, students recognize that it is unrealistic to toss the pigs a very large number of times. Instead, the results from the entire class can be used to get a better estimate for the probability of a Pig Out. Draw a number line on the whiteboard. Ask each student to write his/her own result from their tosses on a sticky note and post the note appropriately on the line in order to form a dot plot. Figure 2 shows a dotplot constructed from the example class results.
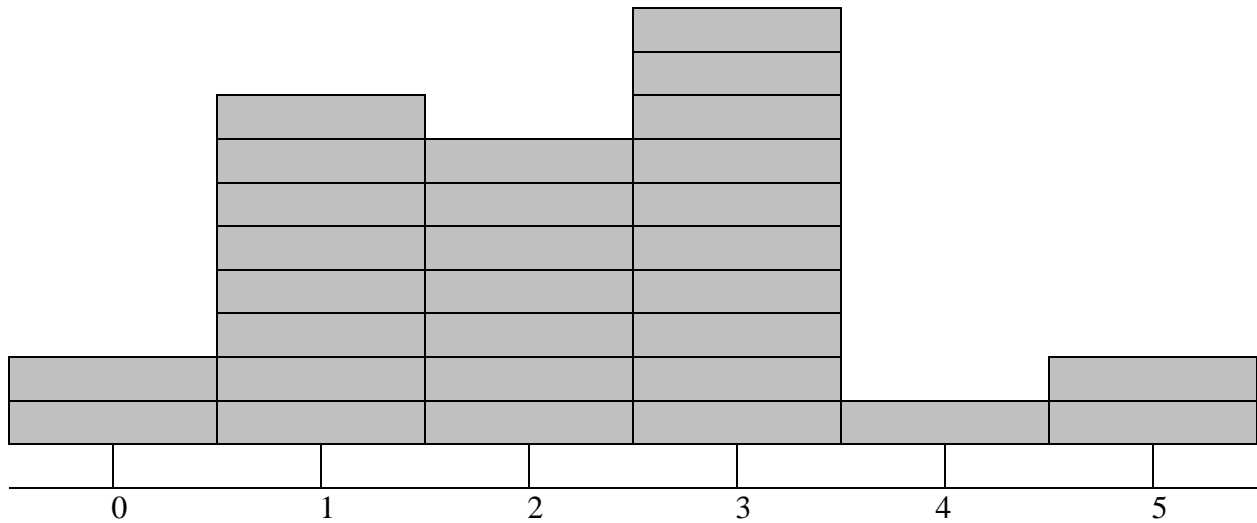

Figure 2. Example of class dot plot.

IV. Interpret the Results

After discussing the data for the whole class, ask everyone to make a new estimate for the probability and to share their new estimates. Most students select the class median or class mode as their new estimate. Figure 3 shows the measures of center for the class results.
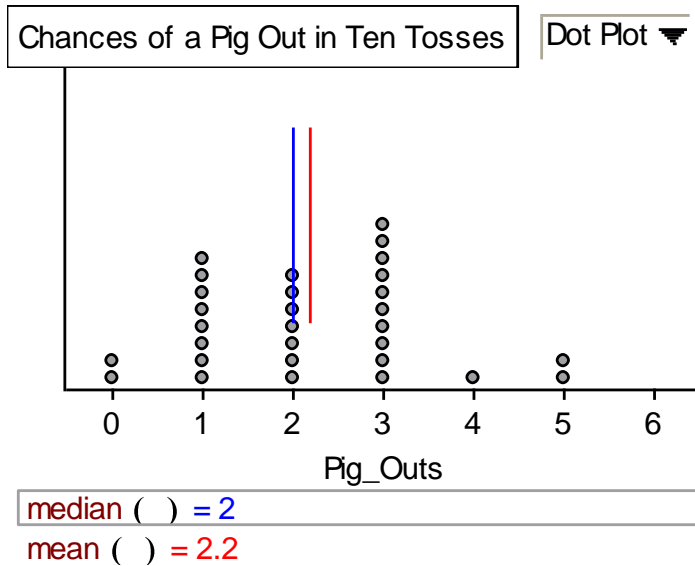

Figure 3. Example of central tendencies for class data.

Using the results from Figure 3, one could estimate that a Pig Out is rolled one time in every five rolls.

Through completing this activity, students will recognize that that they cannot simply count possible outcomes for every object and assign a theoretical probability to any given outcome. By exploring objects that cannot be assigned theoretical probabilities, students seem to have a greater understanding of when a theoretical probability can be assigned. Furthermore, students recognize the need to generate data in order to obtain experimental probabilities.

Assessment

1. Jason wants to know the probability of getting a Pig Out the next two times he tosses the pigs. He forgot his Pigs at School and wants to use the RandInt output on his graphing calculator to generate random numbers to simulate the tossing of the pigs.
(a) What intervals should be used for the pig tosses?
(b) What integer(s) indicate a 'success' (tossing a Pig Out)?
(c) How many random integers should be generated on each trial?
(d) How many trials should Jason conduct?
(e) Use Jason's simulation to find the probability of tossing a Pig Out the next two times the pigs are tossed.

2. Melissa also wants to know the probability of getting a Pig Out the next two times she tosses the pigs. She wants to make an ordered list of the outcomes (Pig Out or no Pig Out) when tossing the pigs two times.
(a) Make an ordered list of the outcomes when tossing the pigs two times.
(b) Is each outcome equally likely? If not, which outcome is least likely? Most likely?

3. The area model below shows the outcomes when tossing the pigs twice.

Toss one

	No Pig Out				Pig Out
No Pig Out					
Pig Out					

Toss two

(a) According to the table, what is the probability of tossing a single Pig Out?
(b) According to the table, what is the probability of tossing two Pig Outs?
(c) According to the table, what is the probability of not tossing any Pig Outs on two tosses?
(d) According to the table, what is the probability of first tossing a Pig out, then not tossing a Pig Out?
(e) According to the table, what is the probability of not tossing a Pig Out, then tossing a Pig Out?
(f) If the pigs are tossed twice, what is the probability that at least one of the tosses will be a Pig Out?

Answers

1.

(a) Answers will vary here. The two most likely answers are (1,10) and (1,5).

(b) Answers will vary here as well. If the answer in part (a) is (1,10), there should be two integers within that interval that will indicate a success. If the interval is (1,5), then one integer within that interval will indicate a success.

(c) Two integers should be generated on each trial, each one representing the outcome of one of the two tosses.

(d) Answers will vary. One hundred is reasonable.

(e) Answers will vary, but overall should be very close to $\frac{4}{100}$ or $\frac{1}{25}$.

2.

(a) (Pig Out, no Pig Out), (Pig Out, Pig Out), (no Pig Out, no Pig Out), (no Pig Out, Pig Out)

(b) No - (no Pig Out, no Pig Out) is most likely, (Pig Out, Pig Out) is least likely.

3.

(a) $\frac{1}{5}$

(b) $\frac{1}{25}$

(c) $\frac{16}{25}$

(d) $\frac{4}{25}$

(e) $\frac{4}{25}$

(f) $\frac{9}{25}$

Possible Extensions (to GAISE Level B)

1. The students can choose another pig position (Sider, Razorback, or Trotter) and conduct a simulation to find the probability of that outcome.

2. The teacher can have students conduct a simulation to find out how many tosses are needed for each student to get the first Leaning Jowler or the first Double Trotter. The teacher can then use this data to discuss mean, median, mode, five-number summary, box plots, range, and interquartile range.

3. The teacher can have students conduct an investigation to determine whether the points assigned to each of the different landing positions are reasonable or whether it is fairer to have different points assigned to each landing position.

References

1. *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) Report, ASA, Franklin et al., ASA, 2007 http://www.amstat.org/education/gaise/.

2. Adapted from: Richardson, M. and Coffey, D. (2004). What are the Chances of a "Pig Out"? *Mathematics in Michigan*, a publication of the Michigan Council of Teachers of Mathematics, Volume 43, Number 1.

3. Pig dice images taken from: http://www.softcom.net/users/mikey719/pass_the_pigs/PassThePigs.html.

What is the Probability of "Pigging Out"? Activity Sheet

Understanding the Problem
In the dice rolling game, *Pass the Pigs©*, players are always on the lookout for the dreaded "Pig Out." If it is tossed before a player passes the pigs to an opponent, the player loses all the points for the round. Therefore, it would be good to know the chance of rolling a Pig Out. A Pig Out occurs when the pigs land on opposite sides – dot and no dot (as seen below).

Question:
What percent of the time will a Pig Out be rolled when playing *Pass the Pigs*?

initial guess = _____

Create a Plan
How might you determine the chances of a Pig Out? Describe several possible ways in the space provided below.

Carry Out a Plan
Carry out the plan decided on by the class.

Reflection
Make a new guess as to the percent of the time that a Pig Out will be rolled when playing *Pass the Pigs*.

updated guess = _____

How can you become more confident in your guess?