Building Split-Plot Designs in Statistical Software

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CONTENTS

- **1. INTRODUCTION**
- **2. A BRIEF OVERVIEW OF SPLIT-PLOT DESIGNS**
- **3. KEY CONCEPTS TO BUILDING SPLIT-PLOTS IN SOFTWARE**
- **4. EXAMPLES**
- **5. CONCLUSION**
- **6. REFERENCES**

1. INTRODUCTION

- **Background:**
  - Trying to recreate the ANOVA table for a standard split-plot in Douglas Montgomery's Design and Analysis of Experiments.
  - Intuition was no help. Needed a set of rules.
  - There are many pitfalls awaiting the practitioner who attempts to specify models in software, especially split-plots.
  - Truly, the building of split-plot designs in statistical software is a nontrivial task (Adworth and Hoffman 2002).
- **Objective:** To establish a set of rules for correctly specifying split-plot designs in statistical software.

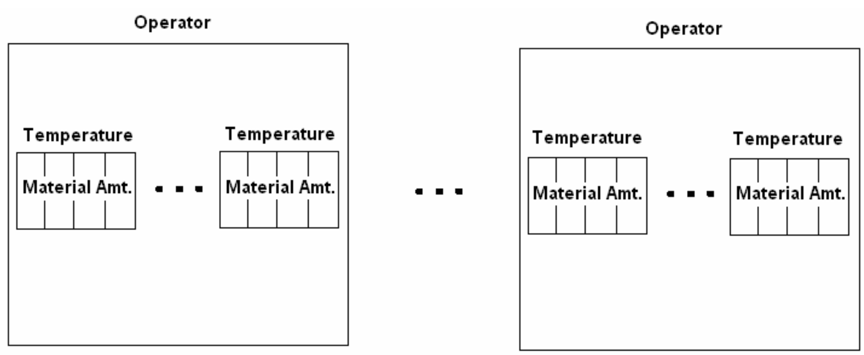
1. INTRODUCTION

- **The model:** A two-way model with one hard-to-change factor is considered.
- **More to come:** Rules for more complex models, including those with more than one hard-to-change factor.
- **Software:**
  - JMP 7.0.1 (using the Fit Model platform)
  - SAS 9.2 (using Proc Mixed)
  - Minitab 15.1 (using ANOVA→GLM)
- **Disclaimers:**
  - Many elements of split-plots are not addressed.
  - A basic knowledge of the software tools is assumed.

2. A BRIEF OVERVIEW OF SPLIT-PLOT DESIGNS

- **Split-plots:**
  - The *experimental unit* for some subset of factors is an *observational unit* for another subset of factors (Vining 2008).
  - Common in industrial experiments.
    - hard-to-change factors
- **Example:**
  - Suppose that factors are temperature and material amount, but it is difficult to change the temperature setting. If the blocking factor is operator, observations will be made at different temperatures with each operator, but the temperature setting is held constant until the experiment is run for all material amounts. In this example, the plots under operator constitute the whole-plots and temperatures constitute the subplots.

2. A BRIEF OVERVIEW OF SPLIT-PLOT DESIGNS



3. KEY CONCEPTS TO BUILDING SPLIT-PLOTS IN SOFTWARE

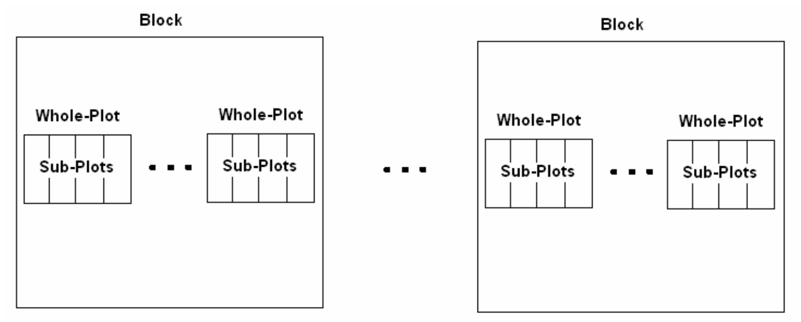
- Split-plots are in essence two designs combined into one. Schwarz (2007) notes,
  - "Recognizing the two types of designs that are combined is the key to analyzing [split-plots]. This combination of designs results in TWO different sizes of experimental units... Most standard statistical packages can be used to produce analysis of variance tables for the results, but the correct model must be specified… In my experience in statistical consulting, this design is likely the most common design to be analyzed incorrectly."

3. KEY CONCEPTS TO BUILDING SPLIT-PLOTS IN SOFTWARE

- These two levels of design are built as follows:
  - First, the whole-plot treatment levels are randomly assigned to whole-plots. This is usually done following a Completely Randomized Design (CRD) or a Randomized Complete Block Design (RCBD).
  - Second, the subplot treatment levels are randomly assigned to the subplots within each whole-plot so that the whole-plots act as blocks.
- Getting the denominator degrees of freedom (DF) correct for each term in the design can be challenging.

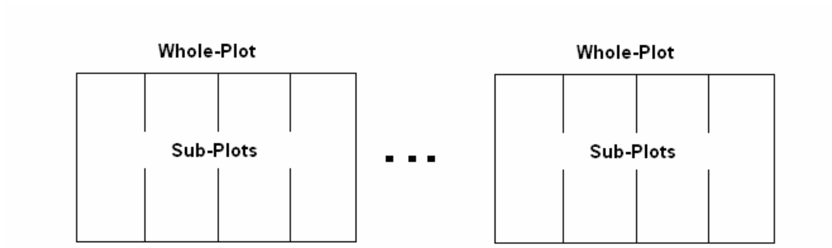
3. KEY CONCEPTS TO BUILDING SPLIT-PLOTS IN SOFTWARE

- Split-plot with the whole-plot factor assigned to the whole-plot following a RCBD.



3. KEY CONCEPTS TO BUILDING SPLIT-PLOTS IN SOFTWARE

- Split-plot with the whole-plot factor assigned to the whole-plot following a CRD.



3.1 - Experimental Units and How to Identify Them

- A split-plot model is built in software by including terms corresponding to the treatment structure and terms for the experimental units.
- All experimental units should be random effects as we think of them as being randomly selected from a much larger population.
- The smallest experimental unit is usually left out and combined into residual error.
- The Key Question: **How are the levels of the whole-plot factor assigned to the whole-plot?** Only two of the possible answers are considered: by a CRD or a RCBD.
  - **RCBD**: the whole-plot experimental unit is uniquely determined by the whole_plot_block*whole_plot_factor interaction.
  - **CRD**: nesting the factor with the experimental unit is often needed and always recommended. Use this syntax: *experiment_unit(factor)*.

3.1 - Experimental Units and How to Identify Them

- Whole-plot experimental units.

Whole-Plot Arrangement	Whole-Plot Experimental Unit
In Blocks	whole_plot_block*whole_plot_factor
In a CRD	whole_plot_unit(whole_plot_factor)

3.2 Denominator Degrees of Freedom

- The main tools we have at our disposal to control denominator DF:
  - (1) The inclusion or exclusion of terms in the model.
  - (2) The computational method of calculating denominator DF used by the software.
  - (3) Bounding or un-bounding variance components.

3.2 Denominator Degrees of Freedom

- **(1) The inclusion or exclusion of terms in the model.**
  - Consider the case where the whole-plots are arranged in blocks. Let A be the whole-plot factor and B be the subplot factor. The mean square for Block*A is the correct error term for testing factor A and the Block. As stated above it is also the experimental unit for the whole plot. When we make this random the software will use it correctly.
  - We can control DF for testing B and A*B by including or excluding the Block*B term.
    - If you enter the term Block*B in the model, Block*B is used to test factor B. The remaining error (which is Block*A*B – the smallest experimental unit) is used for testing A*B.
    - If you leave Block*B out it is pooled into error and this pooled error is used to test B and A*B.
      - This is done when one assumes that the Block*B and Block*A*B interactions do not exist. You might also pool the two terms if the mean square for Block*B is small relative to Block*A*B, or, in the case that Block is random, if the Block*B variance component is small. (Minitab Inc. 2007).

3.2 Denominator Degrees of Freedom

- The denominator used to test terms when whole-plots are in blocks depends on the inclusion of the Block*B term.

Block*B in the Model	Model Term (assuming fixed effects)	Denominator Used to Test
Yes	Block	Block*A
	A	Block*A
	B	Block*B
	A*B	error = Block*A*B
	Block*B	error = Block*A*B
No	Block	Block*A
	A	Block*A
	B	error = Block*B + Block*A*B
	A*B	error = Block*B + Block*A*B
	Block*B	N/A

3.2 Denominator Degrees of Freedom

- When the whole-plots are arranged in a CRD there is no Block*B term to deal with so the setup is easier. Let A and B be the same as above and the Whole-Plot unit that is nested with A be Whole_Plot_Unit.

Model Term	Denominator Used to Test
A	Whole_Plot_Unit(A)
B	error
A*B	error

3.2 Denominator Degrees of Freedom

- **Here we explore (2) and (3) together: software choice and bounding or un-bounding variance components.**
  - SAS gives you several different options for calculating DF with the default being the Containment method (SAS Institute, Inc. 2007).
  - JMP always uses the Kenward and Roger (1997) method (SAS Institute, Inc. 2007).
  - Minitab always uses Satterthwaite's method (Minitab Inc. 2008) which gives similar results to the Containment method.
  - **The Key Point:** When using the Kenward-Roger method any terms with zero variance will be pooled into the error term. Zero variance estimates most often occur when variances are bounded.
    - Consider the case where whole-plots are arranged in blocks and Block is random. The Block*B interaction will also be random. If the Block*B interaction is zero the Kenward-Roger method will automatically pool that term into the error just as if it was never entered into the model as summarized in (1).
  - The Containment method will not pool terms that have zero variance.

3.2 Denominator Degrees of Freedom

- Method summary of pooling variances.

DF Method	Pool Random Terms with Zero Variance
Kenward-Roger	Yes
Containment	No

3.3 Other Considerations

- EMS may be needed to recreate what is in some text books.
- In practice, REML is preferred.
- When using REML in JMP and SAS you have the option to bound or unbound the variance component estimates.

3.4 Key Concept Summary

- The model for the split-plot design is built by including terms corresponding to the treatment structure and terms corresponding to the experimental units.
- The smallest experimental unit is not usually entered into the model.
- Specify all experimental units terms as random effects.
- If the levels of the whole-plot factor are assigned to the whole-plot following a RCBD then the whole_plot_block*whole_plot_factor interaction is the whole-plot experimental unit.
- If the levels of the whole-plot factor are assigned to the whole-plot following a CRD then the nested term: whole_plot_exp_unit(whole_plot_factor) is the whole-plot experimental unit.
- Use REML over EMS.
- Leaving the Block*B term out of the model will pool Block*B with error.
- If you bound variance components in with the Kenward-Roger method then any zero variance components will be pooled into error.

4. EXAMPLES

- The following examples show the application the above key concepts. The common “Paper Tensile Strength” data set is used.
- The first example sets up the problem with the whole-plot factor assigned to whole-plots following a RCBD.
- The second sets up the problem slightly different so that it follows a CRD whole-plot structure.
- We consider all the possible combinations of the points we have discussed.

4. EXAMPLES

- Summary of examples.

Whole-Plot Structure	Block*B Included in the Model	Variance Components Bounded	Example
RCBD	Yes	No	4.1.1
RCBD	Yes	Yes (Only available in SAS and JMP)	4.1.2
RCDB	No	No	4.1.3
RCDB	No	Yes	**N/A – You get the same results as 4.1.3 since all variances > 0.**
CRD	Yes	No	**N/A – There is no Block*B**
CRD	Yes	Yes	**N/A – There is no Block*B**
CRD	No	No	4.2
CRD	No	Yes	**N/A – You get the same results as 4.2 since all variances > 0.**

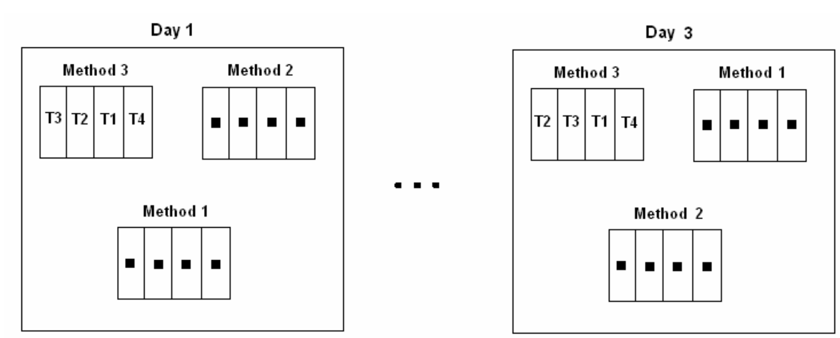
4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Step 1 - State The Problem:** "Consider a paper manufacturer who is interested in three different pulp preparation methods and four different cooking temperatures for the pulp and who wishes to study the effect of these two factors on the tensile strength of the paper. Each replicate of a factorial experiment requires 12 observations, and the experimenter has decided to run three replicates. However, the pilot plant is only capable of making 12 runs per day, so the experimenter decides to run one replicate on each of the three days and to consider the days or replicates as blocks. On any day, he conducts the experiment as follows. A batch of pulp is produced by one of the three methods under study. Then this batch is divided into four samples, and each sample is cooked at one of the four temperatures. Then a second batch of pulp is made up using another of the three methods. This second batch is also divided into four samples that are tested at the four temperatures. The process is then repeated, using a batch of pulp produced by the third method."

mont_splitplot.TXT

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Step 2 - Draw The Model:**



4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Step 3 - Identify the Experimental Units and Factors:** We ask ourselves how the levels of the whole-plot factor are assigned to the whole-plot. In this example it is by a RCBD so the Day*Method interaction is the whole-plot experimental unit. The units and factors are as follows.
- Whole-plot Block = **Day**
- Whole-plot Experimental Unit = **Day*Method**
- Whole-plot Factor = **Method** – 3 levels randomized to whole-plots grouped within blocks
- Subplot Experimental Unit = **Day*Method*Temp**
- Subplot Factor = **Temperature** – 4 levels randomized to subplots within whole-plots

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Step 4 - Build The Model:**
  - Day, the whole-plot block, is considered random.
  - The two factors Method and Temp and their interaction are considered fixed.
  - As always, the experimental unit term (Day*Method) is random.
  - Leave out the sub-plot experimental unit term (Day*Method*Temp).

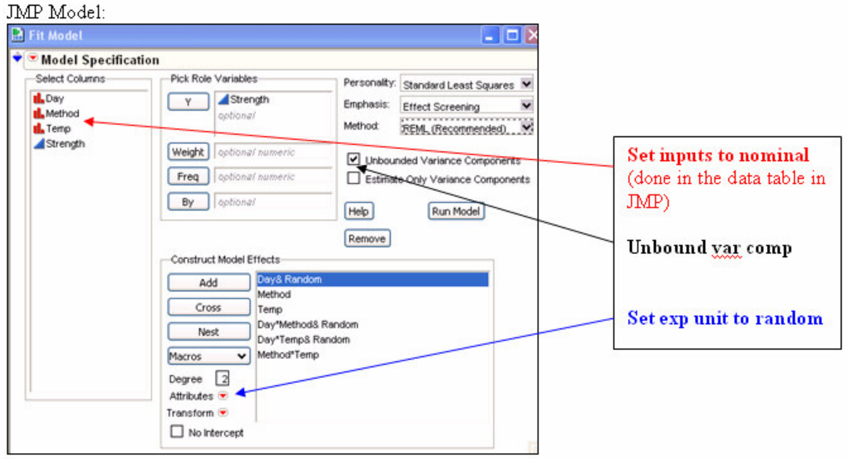
4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Analysis 4.1.1**

Whole-Plot Structure	Block*B Included in the Model	Variance Components Bounded
RCBD	Yes	No

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

JMP Model:



4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

SAS Model

```
proc mixed data=temp.mont_splitplot nobound;
      class day method temp;
      model strength = method temp method*temp ;
      random day day*method day*temp;
run;
```

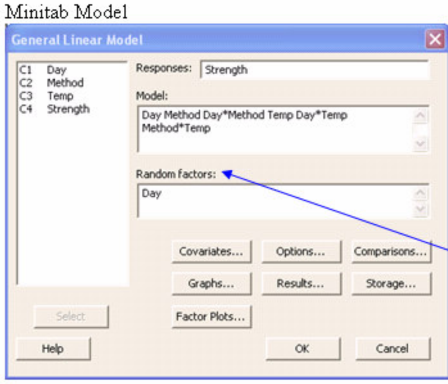
Set inputs to nominal (use class statement in SAS)

Unbound var comp

Set exp unit to random (only put random terms in the random statement not the model statement in SAS)

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

Minitab Model



Set inputs to nominal (use Data→Code on the spreadsheet)

Unbound var comp (only option in Minitab)

Set exp unit to random (all terms are put in Model then any terms with the Random factors in them will be random in Minitab)

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

Analysis 4.1.1 Output: All variance component estimates, denominator degrees of freedom, and p-values match between the three software programs. Note that the Day*Temp (Block*B) interaction is negative. Some may argue that this term should be pooled into the error. This will be shown in Analyses 4.1.2 (bound variances) and 4.1.3 (exclude Block*B).

Analysis 4.1.1 Output Summary

	JMP	SAS	Minitab
Day*Temp Variance Component	-0.263889	-0.2639	-0.2639
Method Denominator DF	4	4	4
Temp Denominator DF	6	6	6
Method*Temp Denominator DF	12	12	12

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

JMP Output

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Day	0.6	2.5416667	3.2827294	-3.892483	8.9758163	32.914
Day*Method	0.2852459	1.2083333	1.660538	-2.046321	4.4629878	15.647
Day*Temp	-0.062295	-0.263889	0.8784776	-1.985705	1.4579271	-3.417
Residual		4.2361111	1.7293851	2.1782605	11.543091	54.856
Total		7.7222222				100.000

-2 LogLikelihood = 139.27872392

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Method	2	2	4	7.0781	0.0485*
Temp	3	3	6	42.0081	0.0002*
Method*Temp	6	6	12	2.9574	0.0520

SAS Output

Cov Parm	Estimate
Day	2.5417
Day*Method	1.2083
Day*Temp	-0.2639
Residual	4.2361

Fit Statistics

-2 Res Log Likelihood	122.2
AIC (smaller is better)	130.2
AICC (smaller is better)	132.3
BIC (smaller is better)	126.6

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
3	8.18	0.0425

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Method	2	4	7.08	0.0485
Temp	3	6	42.01	0.0002
Method*Temp	6	12	2.96	0.0520

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

Minitab Output

Analysis of Variance for Strength, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Day	2	77.556	77.556	38.778	4.68	0.126 x
Method	2	128.389	128.389	64.194	7.08	0.049
Day*Method	4	36.278	36.278	9.069	2.14	0.138
Temp	3	434.083	434.083	144.694	42.01	0.000
Day*Temp	6	20.667	20.667	3.444	0.81	0.580
Method*Temp	6	75.167	75.167	12.528	2.96	0.052
Error	12	50.833	50.833	4.236		
Total	35	822.972				

x Not an exact F-test.

Error Terms for Tests, using Adjusted SS

	Source	Error DF	Error MS	Synthesis of Error MS
1	Day	2.85	8.278	(3) + (5) - (7)
2	Method	4.00	9.069	(3)
3	Day*Method	12.00	4.236	(7)
4	Temp	6.00	3.444	(5)
5	Day*Temp	12.00	4.236	(7)
6	Method*Temp	12.00	4.236	(7)

Variance Components, using Adjusted SS

Source	Estimated Value
Day	2.5417
Day*Method	1.2083
Day*Temp	-0.2639
Error	4.2361

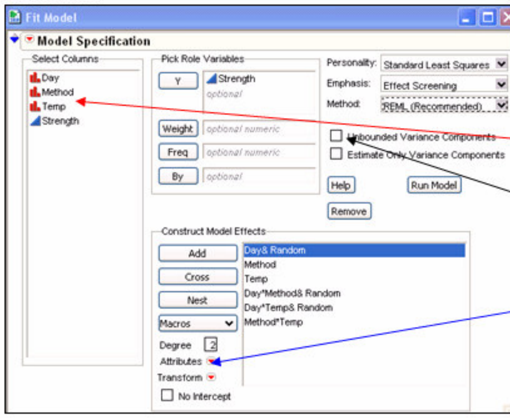
4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Analysis 4.1.2**

Whole-Plot Structure	Block*B Included in the Model	Variance Components Bounded
RCBD	Yes	Yes (Only available in SAS and JMP)

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

JMP Model:



Set inputs to nominal (done in the data table in JMP)

Bound var comp

Set exp unit to random

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

SAS Model

```
proc mixed data=temp.mont_splitplot;
      class day method temp;
      model strength = method temp method*temp ;
      random day day*method day*temp;
run;
```

Set inputs to nominal (use class statement in SAS)

Bound var comp (default in SAS)

Set exp unit to random (only put random terms in the random statement not the model statement in SAS)

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

Analysis 4.1.2 Output: Note the difference in the degrees of freedom between the JMP and SAS output. This is because the Kenward-Roger method (JMP) pools terms with variance equal to zero. SAS uses the Containment method by default so the Day*Temp interaction is not pooled. The choice of method does affect the p-values of Temp and Method*Temp, almost switching significance at the 0.05 level.

Analysis 4.1.2 Output Summary

	JMP	SAS	Minitab
Day*Temp Variance Component	0	0	N/A
Method Denominator DF	4	4	N/A
Temp Denominator DF	18	6	N/A
Method*Temp Denominator DF	18	12	N/A

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

JMP Output:

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Day	0.6232517	2.4756944	3.2753747	0.5235796	1102.7251	32.059
Day*Method	0.3208042	1.2743056	1.6370817	0.2767716	408.68905	16.502
Day*Temp	0	0	0	0	0	0.000
Residual		3.9722222	1.3240741	2.2679421	8.6869402	51.439
Total		7.7222222				100.000

-2 LogLikelihood = 139.36226272

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Method	2	2	4	7.0781	0.0485*
Temp	3	3	18	36.4266	<.0001*
Method*Temp	6	6	18	3.1538	0.0271*

SAS Output:

Cov Parm	Estimate
Day	2.4757
Day*Method	1.2742
Day*Temp	0
Residual	3.9723

Fit Statistics

-2 Res Log Likelihood	122.3
AIC (smaller is better)	128.3
AICC (smaller is better)	129.5
BIC (smaller is better)	125.6

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Method	2	4	7.08	0.0485
Temp	3	6	36.43	0.0003
Method*Temp	6	12	3.15	0.0428

If the following SAS code were run the results would match JMP.

```
proc mixed data=temp.mont_splitplot;
       class day method temp;
       model strength = method temp method*temp / DDFM=KENWARDROGER;
       random day day*method day*temp;
run;
```

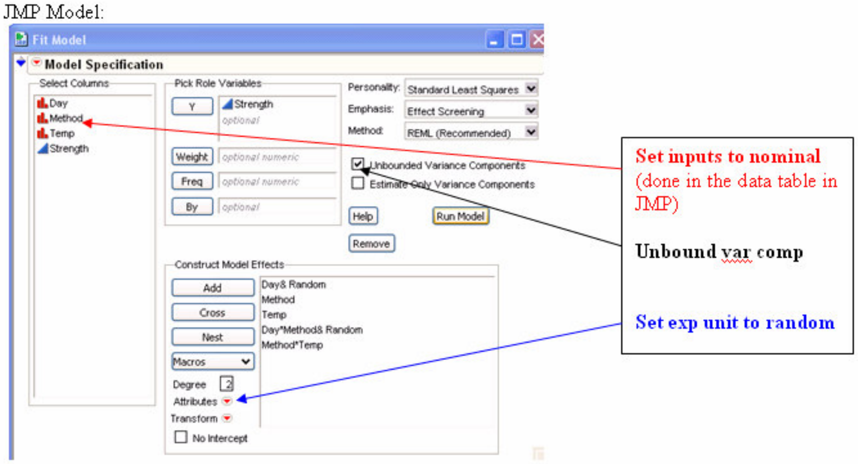
4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Analysis 4.1.3**

Whole-Plot Structure	Block*B Included in the Model	Variance Components Bounded
RCDB	No	No

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

JMP Model:



4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

SAS Model

```
proc mixed data=temp.mont_splitplot nobound;
      class day method temp;
      model strength = method temp method*temp ;
      random day day*method;
run;
```

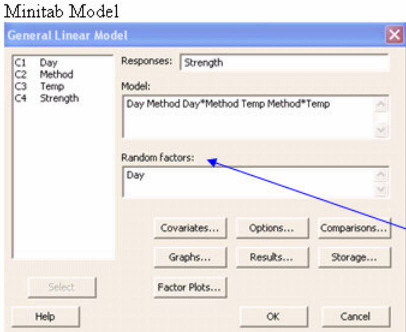
Set inputs to nominal (use class statement in SAS)

Unbound var comp

Set exp unit to random (only put random terms in the random statement not the model statement in SAS)

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

Minitab Model



Set inputs to nominal (use Data→Code on the spreadsheet)

Unbound var comp (only option in Minitab)

Set exp unit to random (all terms are put in Model then any terms with the Random factors in it in them will be random in Minitab)

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

Analysis 4.1.3 Output: All variance component estimates, denominator degrees of freedom, and p-values match between the three software programs. Note that the Day*Temp interaction is pooled into error. Also note that all variance components are positive. Thus there will be no difference in the results in JMP (Kenward-Roger) or SAS (Containment) were we to re-run this analysis with bounded variance components.

Analysis 4.1.3 Output Summary

	JMP	SAS	Minitab
Method Denominator DF	4	4	4
Temp Denominator DF	18	18	18
Method*Temp Denominator DF	18	18	18

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

JMP Output

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Day	0.6232517	2.4756944	3.2753747	-3.94404	8.8954289	32.059
Day*Method	0.3208042	1.2743056	1.6370817	-1.934375	4.4829857	16.502
Residual		3.9722222	1.3240741	2.2679421	8.6869402	51.439
Total		7.7222222				100.000

-2 LogLikelihood = 139.36226272

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Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Method	2	2	4	7.0781	0.0485*
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SAS Output

Cov Parm	Estimate
Day	2.4757
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Residual	3.9722

Fit Statistics

-2 Res Log Likelihood	122.3
AIC (smaller is better)	128.3
AICC (smaller is better)	129.5
BIC (smaller is better)	125.6

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
2	8.10	0.0175

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Method	2	4	7.08	0.0485
Temp	3	18	36.43	<.0001
Method*Temp	6	18	3.15	0.0271

4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

Minitab Output

Analysis of Variance for Strength, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Day	2	77.556	77.556	38.778	4.28	0.102
Method	2	128.389	128.389	64.194	7.08	0.049
Day*Method	4	36.278	36.278	9.069	2.28	0.100
Temp	3	434.083	434.083	144.694	36.43	0.000
Method*Temp	6	75.167	75.167	12.528	3.15	0.027
Error	18	71.500	71.500	3.972		
Total	35	822.972				

S = 1.99304   R-Sq = 91.31%   R-Sq(adj) = 83.11%

Error Terms for Tests, using Adjusted SS

	Source	Error DF	Error MS	Synthesis of Error MS
1	Day	4.00	9.069	(3)
2	Method	4.00	9.069	(3)
3	Day*Method	18.00	3.972	(6)
4	Temp	18.00	3.972	(6)
5	Method*Temp	18.00	3.972	(6)

Variance Components, using Adjusted SS

Source	Estimated Value
Day	2.476
Day*Method	1.274
Error	3.972

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

- **Step 1 - State The Problem:** "A paper manufacturer is interested in the effect of three different pulp preparation methods and four different cooking temperatures for the pulp on the tensile strength of the resulting paper. The equipment that is used for the pulp preparation methods only works on large amounts of pulp. The equipment that is used to cook the pulp can work on smaller batches of material. On any one day, the experiment is conducted as follows. A batch of pulp is produced by one of the three methods under study. The method of pulp preparation is randomized among 9 days available for the experiment. Within a day, a batch is divided into four sub-batches, and cooked at one of the four temperatures. The resulting tensile strength of the paper is measured."
- **Data:** http://www.stat.sfu.ca/~cschwarz/Stat-650/Notes/MyPrograms

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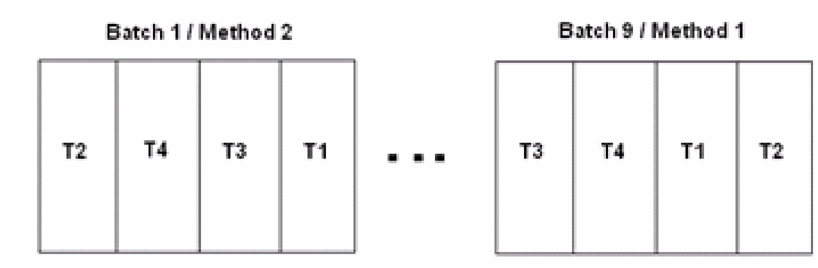
4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

- **Step 2 - Draw The Model:**



4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

- **Step 3 - Identify the Experimental Units and Factors:** We ask ourselves how the levels of the whole-plot factor are assigned to the whole-plot. Is it by a CRD or RCBD? In this example it is by a CRD so the nested term Batch(Method) is the whole-plot experimental unit. The units and factors are as follows.
- Whole-plot Experimental Unit = **Batch(Method)**
- Whole-plot Factor = **Method** – 3 levels randomized to the whole-plots
- Subplot Experimental Unit = **Batch(Method)*Temp**
- Subplot Factor = **Temperature** – 4 levels randomized to subplots within each whole-plot

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

- **Step 4 - Build The Model:**
  - The two factors Method and Temperature and their interaction are considered fixed effects.
  - The experimental unit nested term is random.
  - There is no Block*B term to deal with making the analysis simpler.
  - The sub-plot experimental unit term is left out.

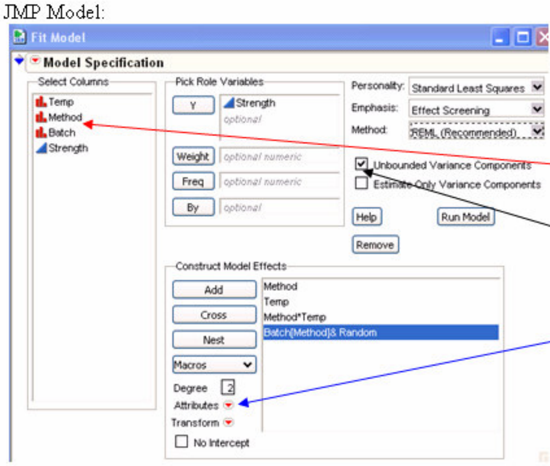
4.1 Paper Tensile Strength – Whole-Plots in RCBD, Montgomery (1991).

- **Analysis 4.2**

Whole-Plot Structure	Block*B Included in the Model	Variance Components Bounded
CRD	No	No

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

JMP Model:



Set inputs to nominal (done in the data table in JMP)

Unbound var comp

Set exp unit to random

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

SAS Model

```
proc mixed data=temp.paper nobound;
      class batch method temp;
      model strength = method temp method*temp;
      random batch(method);
run;
```

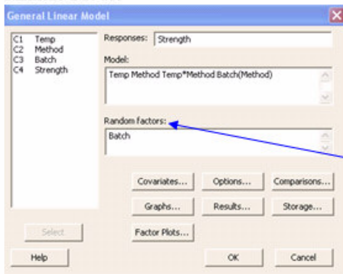
Set inputs to nominal (use class statement in SAS)

Unbound var comp

Set exp unit to random (only put random terms in the random statement not the model statement in SAS)

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

Minitab Model



Set inputs to nominal (use Data→Code on the spreadsheet)

Unbound var comp (only option in Minitab)

Set exp unit to random (all terms are put in Model then any terms with the Random factors in it in them will be random in Minitab)

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

Analysis 4.2 Output: All variance component estimates, denominator degrees of freedom, and p-values match between the three software programs. Given that the estimate of Batch(Method) is positive there would be no difference in output were we to re-run this analysis with bounded variance components.

Analysis 4.2 Output Summary

	JMP	SAS	Minitab
Method Denominator DF	6	6	6
Temp Denominator DF	18	18	18
Method*Temp Denominator DF	18	18	18

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

JMP Output

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Pct of Total
Batch[Method]	0.9440559	3.75	2.7583386	-1.656344	9.1563437	48.561
Residual		3.9722222	1.3240741	2.2679421	8.6869402	51.439
Total		7.7222222				100.000

-2 LogLikelihood = 140.88477951

Iterations

Fixed Effect Tests

Source	Nparm	DF	DFDen	F Ratio	Prob > F
Method	2	2	6	3.9985	0.0788
Temp	3	3	18	22.6853	<.0001*
Method*Temp	6	6	18	1.0559	0.4236

SAS Output

Cov Parm	Estimate
Batch(Method)	3.7500
Residual	3.9722

Fit Statistics

-2 Res Log Likelihood	123.8
AIC (smaller is better)	127.8
AICC (smaller is better)	128.3
BIC (smaller is better)	128.2

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
1	6.57	0.0104

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
Method	2	6	4.00	0.0788
Temp	3	18	22.69	<.0001
Method*Temp	6	18	1.06	0.4236

4.2 Paper Tensile Strength – Whole-Plots in CRD (Schwarz 2007).

Minitab Output

Analysis of Variance for Strength, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Temp	3	270.333	270.333	90.111	22.69	0.000
Method	2	151.722	151.722	75.861	4.00	0.079
Temp*Method	6	25.167	25.167	4.194	1.06	0.424
Batch(Method)	6	113.833	113.833	18.972	4.78	0.004
Error	18	71.500	71.500	3.972		
Total	35	632.556				

S = 1.99304 R-Sq = 88.70% R-Sq(adj) = 78.02%

	Source	Error DF	Error MS	Synthesis of Error MS
1	Temp	18.00	3.972	(5)
2	Method	6.00	18.972	(4)
3	Temp*Method	18.00	3.972	(5)
4	Batch(Method)	18.00	3.972	(5)

Variance Components, using Adjusted SS

Source	Estimated Value
Batch(Method)	3.750
Error	3.972

5. CONCLUSIONS

- With the keys to building split-plot models in hand and several examples to refer to the reader will hopefully approach their next split-plot analysis with confidence.

57 Intel Confidential

6. REFERENCES

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58 Intel Confidential