Introductory Workshop on Bayesian Modeling and Inference

Afternoon Session

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Litters example

From BUGS Example Volume 1 (modified)

Two groups of sixteen pig litters

Number surviving recorded

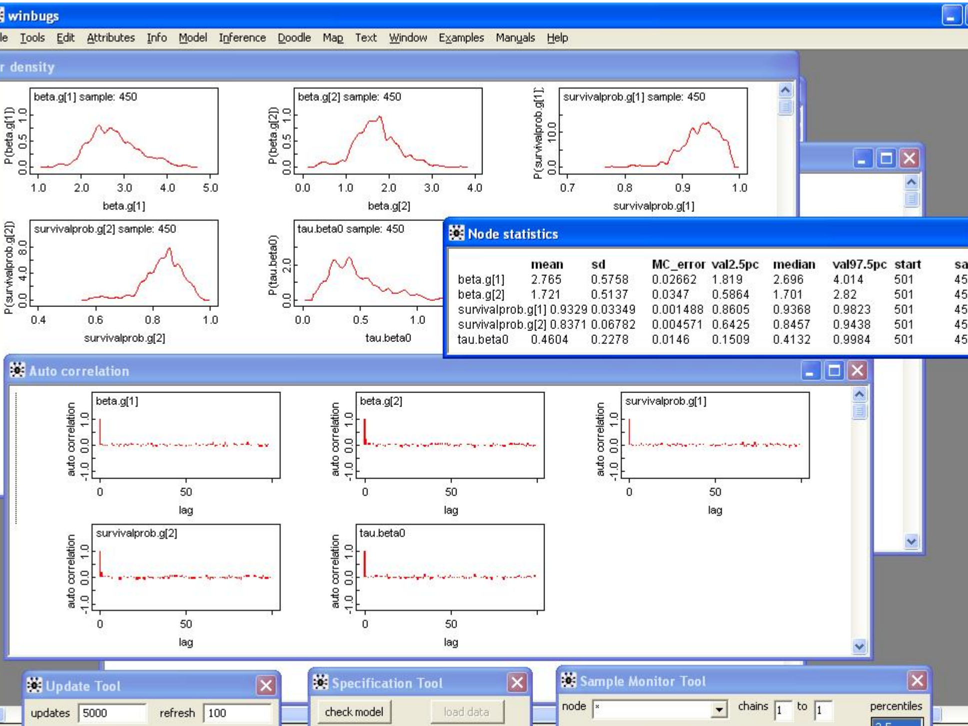
Data:

```
group[]  litter[] n[] r[]
  1         1      13  13
  1         2      12  12
  1         3      10   7
  1         4       9   9
  1         5       9   9
  1         6       8   8
  .
  .
  .
  2        13      10   5
  2        14       6   3
  2        15      10   3
  2        16       7   0
END
```

Litters example

Model: $logit(\pi_{ij}) = \beta_0 + \beta_1 I_i(group = 2) + \beta_{ij}, \quad \beta_{ij} \sim \mathrm{N}(0, \sigma^2)$

```
model{
 for(i in 1:32){
  logit(p[i])<- beta.g[group[i]]+beta.l[group[i],litter[i]]
  r[i]~dbin(p[i],n[i])
  }
 for(i in 1:2){
  beta.g[i]~dnorm(0,0.001)
  logit(survivalprob.g[i])<-beta.g[i]
 }
 for(i in 1:2){
  for(j in 1:16){
   beta.l[i,j]~dnorm(0,tau.beta0)
  }
 }
 tau.beta0~dgamma(0.01,0.01)
}
```


winbugs
le Tools Edit Attributes Info Model Inference Doodle Map Text Window Examples Manuals Help
r density
beta.g[1] sample: 450
beta.g[2] sample: 450
survivalprob.g[1] sample: 450
survivalprob.g[2] sample: 450
tau.beta0 sample: 450
Node statistics
mean sd MC_error val2.5pc median val97.5pc start sa
beta.g[1] 2.765 0.5758 0.02662 1.819 2.696 4.014 501 45
beta.g[2] 1.721 0.5137 0.0347 0.5864 1.701 2.82 501 45
survivalprob.g[1] 0.9329 0.03349 0.001488 0.8605 0.9368 0.9823 501 45
survivalprob.g[2] 0.8371 0.06782 0.004571 0.6425 0.8457 0.9438 501 45
tau.beta0 0.4604 0.2278 0.0146 0.1509 0.4132 0.9984 501 45
Auto correlation
beta.g[1]
beta.g[2]
survivalprob.g[1]
survivalprob.g[2]
tau.beta0
auto correlation
lag
Update Tool
updates 5000 refresh 100
Specification Tool
check model load data
Sample Monitor Tool
node * chains 1 to 1 percentiles

Rats example

From Gelfand and Smith (1990)

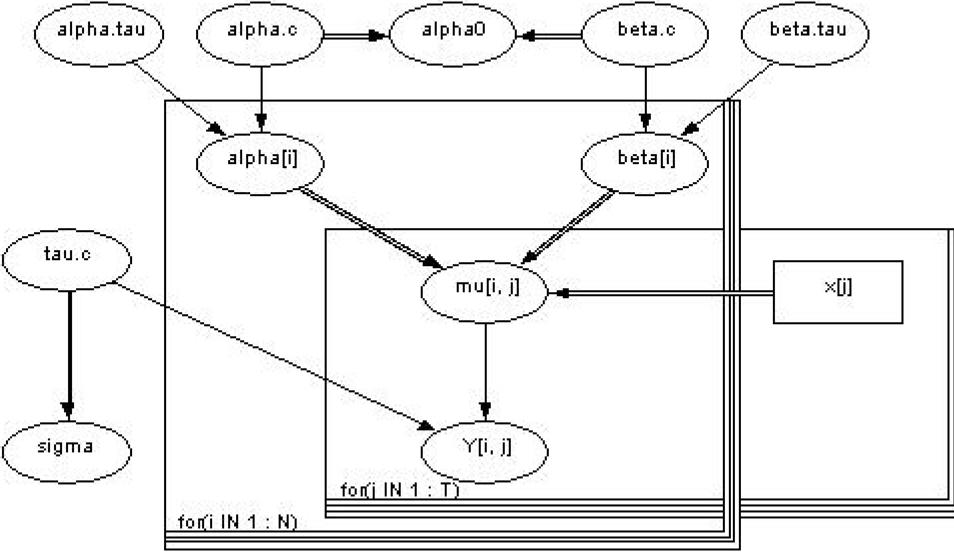
Thirty rats weighed over 5 weeks after birth

Repeated measurements; growth curves

Data:

```
list(x = c(8.0, 15.0, 22.0, 29.0, 36.0),
xbar = 22, N = 30, T = 5,
Y = structure(
.Data =   c(151, 199, 246, 283, 320,
 145, 199, 249, 293, 354,
 147, 214, 263, 312, 328,
 155, 200, 237, 272, 297,
                   .
                   .
                   .
 157, 205, 248, 289, 316,
 137, 180, 219, 258, 291,
 153, 200, 244, 286, 324),
.Dim = c(30,5)))
```

Rats example model graph



Rats example

Model: $y_{ij} = \alpha_i + \beta_i(x_j - \bar{x}) + \epsilon_{ij}, \quad \epsilon_{ij} \sim \mathrm{N}(0, \sigma^2)$
$\alpha_i \sim \mathrm{N}(\alpha_c, \sigma^2_\alpha), \beta_i \sim \mathrm{N}(\beta_c, \sigma^2_\beta)$

```
model{
 for(i in 1:N){
  for(j in 1:T){
   Y[i,j] ~ dnorm(mu[i,j],tau.c)
   mu[i,j]<-alpha[i]+beta[i]*(x[j]-xbar)
  }
  alpha[i] ~ dnorm(alpha.c,alpha.tau)
  beta[i] ~ dnorm(beta.c,beta.tau)
 }
 tau.c ~ dgamma(0.001,0.001)
 sigma <- 1 / sqrt(tau.c)
 alpha.c ~ dnorm(0.0,1.0E-6)
 alpha.tau ~ dgamma(0.001,0.001)
 beta.c ~ dnorm(0.0,1.0E-6)
 beta.tau ~ dgamma(0.001,0.001)
 alpha0 <- alpha.c - xbar * beta.c
}
```

RatsPlus example

Example enhanced by simulation

Rats are from three strains

And fed one of two diets, assigned randomly

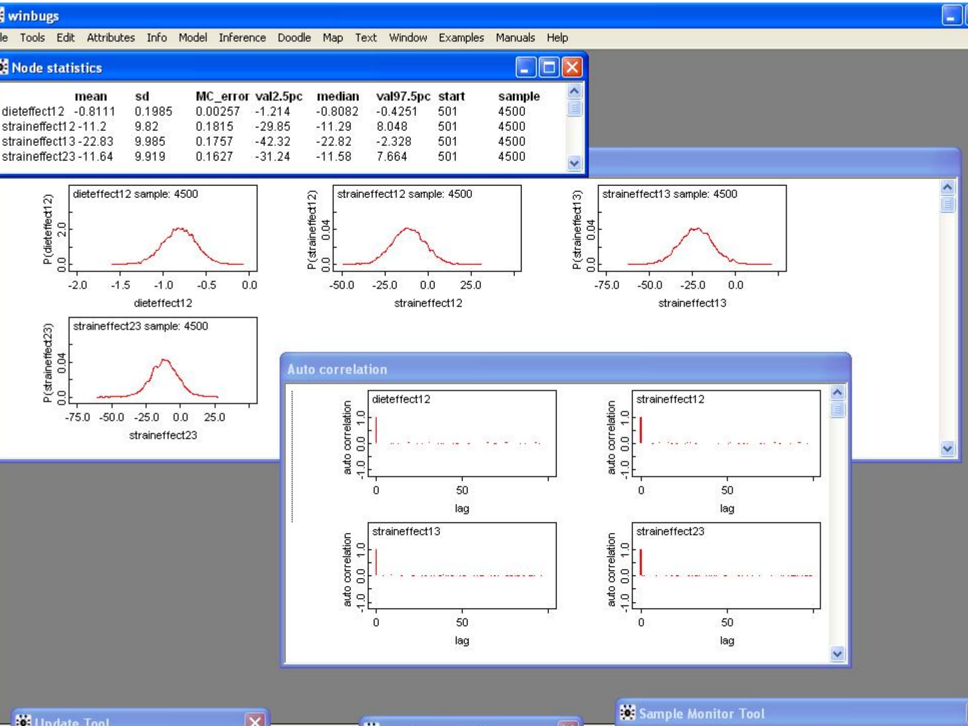
Data:

```
list(x=c(8.0,15.0,22.0,29.0,36.0),xbar=22,N=30,T=5)
strain[] diet[]  Y[,1]     Y[,2]     Y[,3]     Y[,4]     Y[,5]
 1          1 140.00673 179.28600 216.66904 255.70110 295.04636
 1          1 134.02741 174.07836 216.71992 258.08470 301.66891
 .
 1          2 159.21748 205.07470 250.99684 297.17496 345.72812
 1          2 154.39442 204.76790 254.62188 307.01217 357.67762
 2          1 162.66781 209.80742 258.23275 302.90084 350.41266
 .
 2          2 177.59712 225.59307 275.63673 326.51941 375.95313
 3          1 152.45878 191.97838 230.25669 271.07449 309.29888
 .
 3          2 188.44721 231.96011 276.55321 317.59955 361.89207
END
```

Rats example

Model: $y_{ij} = \alpha_i + \beta_i(x_j - \bar{x}) + \epsilon_{ij}, \quad \epsilon_{ij} \sim \mathrm{N}(0, \sigma^2)$
and linear models for α_i and β_i

```
{for( i in 1 : N ) {
  for( j in 1 : T ) {
   Y[i,j]~dnorm(mu[i,j],tau.c)
   mu[i,j]<-alpha[i]+ beta[i]*(x[j] - xbar)}
  alpha[i] ~ dnorm(mualpha[i],alpha.tau)
  beta[i] ~ dnorm(mubeta[i],beta.tau)
  mualpha[i]<-mualpha.st[strain[i]]
  mubeta[i]<-mubeta.dt[diet[i]]}
 tau.c ~ dgamma(0.001,0.001); sigma<-1/sqrt(tau.c)
 for(j in 1:3){mualpha.st[j] ~ dnorm(0.0,1.0E-6)}
 alpha.tau ~ dgamma(0.001,0.001)
 for(j in 1:2){mubeta.dt[j] ~ dnorm(0.0,1.0E-6)}
 beta.tau ~ dgamma(0.001,0.001)
 straineffect12<-mualpha.st[1]-mualpha.st[2]
 straineffect13<-mualpha.st[1]-mualpha.st[3]
 straineffect23<-mualpha.st[2]-mualpha.st[3]
 dieteffect12<-mubeta.dt[1]-mubeta.dt[2]
}
```



	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
dieteffect12	-0.8111	0.1985	0.00257	-1.214	-0.8082	-0.4251	501	4500
straineffect12	-11.2	9.82	0.1815	-29.85	-11.29	8.048	501	4500
straineffect13	-22.83	9.985	0.1757	-42.32	-22.82	-2.328	501	4500
straineffect23	-11.64	9.919	0.1627	-31.24	-11.58	7.664	501	4500

Let's go back to the lab