Teaching Bits: "Random Thoughts on Teaching"

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"Letting Go to Grow: Independent vs. Mutually Exclusive"

Teachers often get caught up in the discussion of how to teach this concept or that concept, or how to explain this connection or that connection, but sometimes we should just stand back and be bold enough to ask questions: "Should we even be teaching this?"; "Is it really relevant to the modern statistics course?"; "Does it follow the GAISE guidelines (http://www.amstat.org/education/gaise/)?"; Do we ever use this idea again later in our course?" As we contemplate the future of teaching statistics, now is a good time to stop, think, and ask the hard questions. The theme of USCOTS 2009 (The United States Conference on Teaching Statistics; http://www.causeweb.org/uscots/) is "Letting Go to Grow". In that spirit I'd like to throw out some ideas regarding the classic 'independent vs. mutually exclusive' discussion that is still included in many statistics textbooks and courses.

Two events A and B are defined to be independent if P(A) = P(A|B). In other words, knowing that event B has occurred does not change the chance that event A will occur. One traditional approach to illustrate this (for better or worse) is to use card decks. For example, the chance of getting a black card out of a 52 card deck is 26/52 = 0.5. If you know the card is a two, the chance of the card being black is 2/4, which is still 0.5. In other words, knowing the card is a two doesn't help you with the probability that the card is black. These two events are independent. Two events A and B are defined to be mutually exclusive if P(A and B) = 0. In other words, A and B cannot occur at the same time. As an example, the chance of a card being a two is 4/52 = 0.077. If you know the card is a three, the chance of it being a two is of course zero. That means a card being a two and a three are mutually exclusive events.

Once the concepts of independent and mutually exclusive events are defined, the question that typically follows is "If two events are independent, can they be mutually exclusive?" And the answer is no. This one statement can derail the discussion of probability. Many teachers get frustrated trying to come up with ways of thinking that help students separate and compare these two concepts, and in the end students are often just as confused as they were before, if not more.

Why does this happen? My theory is that students see the terms ‘mutually exclusive’ and ‘independent’ as meaning the same thing from a layman’s point of view. The most common reasoning I have heard from students who confuse these concepts is that independence to them means ‘separate’, as in two people being ‘independent’. In their minds, independent people (hence independent events) don’t overlap at all (ergo, they are mutually exclusive). (Is there a lesson here regarding the terminology that we use in our courses and how it affects our students’ ability to learn?) Strike one.

Suppose your student asks you to clarify the relationship between independent and mutually exclusive events using a Venn diagram. After all, Venn Diagrams are often touted as a great way to visualize events, relationships and probabilities. Now a Venn diagram makes mutually exclusive events easy to understand – two circles are drawn separately with no overlap. But how do you show that two events are independent with a Venn diagram? We know that two independent events do overlap, and the "nice version" of the multiplication rule confirms this: P(A and B) = P(A)*P(B). But to illustrate independence with a Venn diagram, all you can do is draw a picture of two overlapping circles; this adds nothing at all to a student's understanding. Two Venn diagrams depicting independent and dependent events (with no probabilities shown) are indistinguishable unless the events are mutually exclusive, so the Venn diagram actually does more harm than good in this case. (Venn Diagrams are a whole other story, which I’ll save for another day!) Strike two.

To be clear, I am not against the idea of including independent events and mutually exclusive events in a statistics course or textbook; each of these concepts has their place in probability. Mutually exclusive events are important for solving complex probabilities by breaking down the original problem into small, non-overlapping pieces and using the "nice version" of the addition rule to put the pieces together: P(A or B) = P(A) + P(B). Independence is a must for finding complex probabilities involving simultaneous events, such as those involved in the binomial distribution. Independence is also a major part of sampling. However, these important applications can easily get buried in the confusion and anxiety-raising issue of why independent events cannot be mutually exclusive. Strike three.

Students and teachers are faced with difficult topics, concepts, and connections in a statistics course, that’s a fact. But we need not make the journey even harder by concentrating on ideas that have no bearing on the real issues. What’s important in this case is that two events are either independent or not, and two events are either mutually exclusive or not. If they are independent, we get to use the "nice version" of the multiplication rule to find probabilities; if they are mutually exclusive (or made to be) then we get to use the "nice version" of the addition rule to find probabilities. These two concepts should be presented separately as tools that can be used to solve the probability problems at hand; their comparison need not (and indeed should not) be featured when presenting the material. (Of course, if a student is confused by these concepts and tries to make his or her own comparison, that would be handled on a case-by-case basis.)

So instead of asking "How do we explain the relationship between mutually exclusive and independent events?", what we really should be asking is "Why discuss this relationship at all?" This idea applies to other concepts we teach in the traditional version of a statistics course. The question then becomes, "What should we let go of in order to grow?

Those are my random thoughts on teaching for this time around. Now, what do YOU think?
