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# The Interplay Between Spoken Language and Informal Definitions of Statistical Concepts

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## Abstract

Various terms are used to describe mathematical concepts, in general, and statistical concepts, in particular. Regarding statistical concepts in the Hebrew language, some of these terms have the same meaning both in their everyday use and in mathematics, such as Mode; some of them have a different meaning, such as Expected value and Life expectancy; and some have the opposite meaning, such as Significance level. Spoken language plays an important role in shaping how the informal statistical definitions taught in schools are remembered. In the present study we examine the impact of the everyday use of terms on the students' informal definitions of various statistical concepts. Though all the study participants were familiar with the concepts they were asked to define, a high percentage of them failed to provide correct definitions of the given statistical concepts. Analysis of the incorrect definitions revealed that the everyday use of the terms used to label the concepts, influenced the informal definitions provided by the students.

## 1. Contextual Framework

The formal definitions of mathematical concepts constitute a symbolic language which is independent of any spoken language. Since these definitions in their symbolic format are difficult to teach and understand, spoken language is used to informally define mathematical concepts. Within these informal definitions, mathematical symbols are replaced by proper words from the spoken language. Following this model, statistics has become language dependent and its informal definitions are also an integral part of its language. In addition, a basic understanding of statistics requires one to distinguish between samples and populations. On the one hand, while some of the concepts are clearly population quantities, such as Expected value, some of them, on the other hand, such as Standard deviation, require the addition of an adjective (population or sample) to distinguish the statistical definition from its everyday use.

In the case of Hebrew as well, various words from the spoken language are used to describe mathematical concepts. Regarding statistical concepts, some of these terms have the same meaning in their everyday use and in mathematics, such as Mode, some have a different meaning, such as Expected value and 'Life expectancy, and some have the opposite meaning such as Significance level.

Researchers have explored various aspects regarding the understanding of statistical concepts (i.e. Falk, 1986, Haas et al., 2003), in order to see how technology can help students understand, integrate, and apply fundamental statistical concepts (Chance et al., 2000). Pimm (1987) related understanding to the use of everyday words with particular mathematical sense. This connection is also relevant in the case of statistics. He also considered that analogies and metaphors, where everyday words can be given particular meanings, are very important for the construction of mathematical meaning.

Difficulties in the acquisition and understanding of mathematical concepts were explored by various researchers (e.g. Vinner, 1983; Vinner & Hershkovitz, 1980; Sfard, 1991). According to Vinner (1983), the representation and organization of many mathematical books are based on the following two assumptions: (a) the acquisition of concepts is done mainly via their definitions; (b) students will use definitions to solve problems and prove theorems. Hence, definitions should be minimal and elegant. Referring to the acquisition of mathematical concepts, Vinner & Hershkovitz (1980) distinguished between 'concept definition' and 'concept image.' 'Concept definition' is defined as a form of words used to specify an idea (Tall & Vinner, 1981). The term 'concept image' describes the total cognitive structure associated with the idea, including all mental pictures and associated properties and processes. Difficulties in attaining proper definitions can point to a gap between the concept definition and the concept image of a certain idea. According to Sfard (1991), mathematical conceptions have a dual nature. Namely, mathematical entities and concepts can be perceived as process (structurally), or as object. Moreover, the understanding of a concept structurally, precedes the understanding of a concept on the object level.

In the present study we examine the interplay between Hebrew as a spoken language and the informal definitions of specific statistical concepts. A questionnaire, consisting of several statistical concepts and everyday expressions bearing the same meaning as the statistical concepts, was given to second-year college students who had already completed courses in probability and statistics. The students were asked to define several statistical concepts informally, in their own words, and to include an example of the concept's applicability.

## 2. Theoretical framework

Though all the study participants were familiar with the concepts they were asked to define, a considerably high percentage of them failed to provide even a correct informal definition of the given statistical concepts. Some of the difficulties stem from the fact that informal definitions of statistical concepts are language dependent. Since some of the terms used in statistical concepts have different or opposite meanings in spoken Hebrew, we assumed that the common use of these terms had a strong impact on their perceived statistical meaning. Therefore we referred to past research on language and understanding.

The inability to provide even informal definitions is related to difficulties the students have in the acquisition and understanding of these concepts. These difficulties stem from the complex nature of mathematical concepts. We decided to interpret the research data using two theoretical frameworks which deal with the essence of the acquisition and understanding of mathematical concepts. According to the first theoretical framework, the understanding of a mathematical concept necessitates the construction of two mental ‘entities’ in the learner's mind: ‘concept image’ and ‘concept definition’ (Vinner & Hershkowitz, 1980; Tall & Vinner, 1981). According to the second theoretical framework, the complexity of mathematical concepts stems from their dual nature: they are both process and object. This means that the acquisition and understanding of mathematical concepts occurs gradually, whereby they change from being perceived as process and become mental objects (Sfard, 1991). Elaboration of the above frameworks will be presented in the next section.

### 2.1 Language and understanding

Mathematical understanding is both a linguistic and a conceptual matter (Vergnaud, 1998). To understand mathematics, one has to be able to not only identify relationships between mathematical symbols, but one has to be able to identify these symbols' relationships to natural everyday language. Learning and understanding of mathematical concepts are required to build associations between symbols and realities which exist independently of our minds (Stadler, 2004).

Language and the understanding of mathematical concepts and entities come to fruition in mathematical discourse. Mathematical discourse and its objects are mutually constituted (Sfard, 1998). An activity of mathematical discourse initiates the need for creating mathematical objects, and it is the mathematical objects that influence the mathematical discourse and point it in new directions. When more concepts are introduced, students try to fit them into familiar templates to use in the new discourse. Hence, the introduction of mathematical symbols can be considered an important part of the reification process. Reification is the ability to see the concept as a whole entity.

One of the central goals of mathematics teaching in school, is the "understanding" of mathematical concepts. Researchers have distinguished between different kinds of understanding: instrumental and relational (Skemp, 1987; Pesek & Kirshner, 2000). Instrumental understanding refers to developing skills to use algorithms and arriving at the correct answer, while relational understanding refers to the deeper understanding of the content associated with a certain concept. According to Sfard (1991), mathematical understanding can be viewed as a process whereby a mathematical object transforms from process to mental object. A profound understanding of a mathematical object entails not only the manipulation of difficult expressions, but additionally, involves the ability to create a mental picture of an abstract concept. This, apparently, is a major part of mathematical ability.

When students are engaged in a mathematical discourse, they use words and terms from the spoken language, while remaining aware that they have to relate to their mathematical meaning. However, since some of the terms used in statistical concepts have a different or an opposite meaning in Hebrew, when students are asked to define these concepts informally, the first denotation that comes to mind is the everyday use of these terms.

### 2.2. Concept image and concept definition

Many researchers have explored the difficulties regarding the acquisition and understanding of mathematical concepts (e.g. Vinner & Hershkovitz, 1980; Tall & Vinner, 1981; Vinner, 1983; Sfard, 1991; Stadler, 2004). Some of these difficulties might be a consequence of intuitive human thinking. Other difficulties might be related to visual representations or verbal descriptions of the concept that precede its definition. Intuitive thinking, visual intuitions, and the verbal descriptions of a concept are essential for its understanding. However, there might be a gap between the mathematical definition of a concept and the way one perceives the concept. In this case we may say that there is a gap between the concept definition and the concept image. Concept image describes the total cognitive structure that is associated with the idea, including all mental pictures, and associated properties and processes (Vinner & Hershkowitz, 1980). Concept definition is the words used to specify the idea (Tall & Vinner, 1981). In the present study, we examined the students' difficulties in providing intuitive definitions of basic statistical concepts. However, since they were expected to define these concepts, they provided definitions that described the everyday meaning of the words. Hence, we assume that the concept images of the statistics terms being tested are not independently or wholly structured in the students' minds.

### 2.3 The dual nature of mathematical conceptions

The difficulty in the acquisition and the understanding of mathematical concepts stems from their complex structure. According to Sfard (1991), mathematical entities and concepts have a dual nature. Referring to the differences between structural and object perception, Sfard said:

> "*Whereas the structural conception is static, instantaneous and integrative, the operational is dynamic, sequential and detailed … The former* [object] *is more abstract, more integrated and less detailed than the latter."* (Sfard, 1991, p.4)

>There are people who conceptualize mathematical concepts as real objects which exist outside the human brain. Indeed, most mathematicians discuss properties of functions and groups the same way scientists discuss the structure of physical materials. It can be said that the mathematicians' perception of mathematical concepts is structural perception. However, we can find in mathematical textbooks definitions which uncover a different kind of perception: process based. The latter is based on a dynamic imagery of the mathematical concepts. Sfard (1991) also said that the structural perception precedes the object perception. Sfard described the development from an operational to a structural conception as a process of reification. In order to internalize the concept meaning, the student becomes acquainted with it by computing in single steps. These steps connect with each other in the next, condensation phase. Reification is the ability to see the concept as a whole. It is a static state where "the concept becomes semantically unified by this abstract and purely imaginary construct" (Sfard, 1991, p.20).

Some of the study participants, in spite of providing an intuitive verbal definition, also described the computational steps needed to calculate the defined concepts, which may possibly imply that the students' perception of these concepts exists at a structural level.

## 3. The Study

In this section we describe the research goal, the study participants, the research tools, and the process of the data analysis.

### 3.1 Goal

The goal of our study was to examine the influence of spoken language (in this case Hebrew) on the informal definitions of statistical concepts among second year college students.

### 3.2 About the participants.

In order to fulfill their academic requirements, most students Emek Yezreel Valley College in Israel have to take one statistics course. Only behavioral science students are required to take two courses in which statistical concepts are explored extensively. Since the focus of our research was to examine the influence of spoken language on the statistical meaning of concepts, we choose to include behavioral science students in our study, assuming they were familiar with the statistical concepts being examined. Hence, 96 college students in the beginning of their second year of the behavioral science program participated in this research. In fact, 110 students received the questionnaire but only 96 of them completed it. All the study participants had already completed two courses in statistics and probability. The material covered in these courses included the following topics: descriptive statistics (frequency distributions, central tendency, and variability), probability and inferential statistics (hypothesis testing, normal distribution, t-test for single sample, two dependent or independent samples). In order to circumvent significant memory loss, the students were given the questionnaire immediately following their second course in statistics. Both the language of academic instruction and the language of the questionnaire was Hebrew, the native tongue of the study participants. During their studies in the aforementioned courses, the students became acquainted with the concepts under examination. All study participants passed their final exams in the above mentioned courses.

### 3.3 The research tool

The main research tool was a written questionnaire. As was previously mentioned, the study participants had taken two statistical courses successively before participating in the present research. The questionnaire was distributed at the end of the second course and the students were acquainted with the research goals. Being aware of the fact that the study participants are behavioral science students, we did not expect them to be able to provide mathematically rigorous definitions. However since we aimed at examining the influence of the spoken language on the statistical meaning of concepts, the students were asked to define in *their own words* several statistical concepts and to provide an example of their own choosing. We did not ask for formulae or relevant illustrations since the *words* used in the informal definitions were the object of our research.

We chose several concepts to examine the influence of spoken Hebrew on the statistical meaning of concepts, First, we selected concepts whose meaning in spoken Hebrew was the same in statistics, such as Mode. Second, we chose concepts that have a different meaning in the spoken language and statistics, such as Expected value and Life expectancy. We decided to include the latter in the questionnaire, since in Hebrew the concept Life expectancy is derived from the statistical concept Expected value. We wanted to examine what the influence was, if any, between these two related terms. And third, we chose concepts that have an opposite meaning in spoken language and statistics, such as Significance level. In Hebrew, the word *significant* is an adjective which reinforces a statement (whether it is positive or negative), while the statistical use of Significance level is a noun which refers to error probability.

The questionnaire was anonymous. The concepts (in the following order) were: Life expectancy [1], Mode, Standard deviation, Expected value, and Significance level. All the questionnaires were ordered similarly since we did not anticipate that differences would arise from any given question sequence. In order to understand the students' difficulties in defining the given concepts, informal interviews were conducted with several study participants who agreed to be interviewed. During the interviews, students were asked to refer to the fact that the majority of their informal definitions were incorrect.

## 3.4 The process of the data analysis

The data gathered from the students' responses in the questionnaires were analyzed in three main phases. In the first phase we divided the students' responses into correct and incorrect definitions, according to each concept. Since we aimed at examining the influence of spoken Hebrew on the statistical meaning of concepts, in the second phase, we categorized the incorrect responses according to the types of errors. Then we labeled each category using "c<n>". The "c" stood for the word "category" and the number (<n>) stood for the number of the category. Each researcher did her categorization separately. Then the results (which turned out to be very similar) were discussed, and one accepted system of categorization was arrived at:

1. Using the meaning of each word separately (c1). Namely, the definition of a concept which consisted of two or more words was constructed using the meaning of each word separately. For example, as regards the concept of Significance level, one of the definitions was: "*The probability that the results received in the sample are indeed correct in the population."*

2. Confusing related concepts (c2). Namely, the given definition for a certain concept was correct only for the related concept. For example, as regards the concept of Standard deviation, one of the definitions was: "*The difference between means in the sampling distribution.*"

3. Failing to connect the meaning of the statistical and the everyday use of the concept (c3). Namely, when the everyday meaning of the concept was similar to the statistical one, the students failed to connect them. For example, as regards the concept of Life expectancy, one of the definitions was: "*The period of time a person lives*."

4. Using the word's root or tone (c4). Namely, the definition given by the students was constructed from meanings derived from the word's tone or root. For example, as regards the concept of Mode, one of the definitions was: "*The frequency that something occurs.*" In this example the student mixed with the word *shehihut* in Hebrew with the word *shahiach*, which has a similar tone and is the Hebrew term for mode.

5. Relating the definition to only part of the concept (c5). For example, as regards the concept of Life expectancy one of the definitions was: "*mean.*" In this definition the student referred only to the first part of the concept *tohelet hayim*, although the first part refers to the mean of the population.

6. Describing the implementation of the concept (c6). Namely, the definition refers to the operation that has to be undertaken in order to realize the concept. For example, as regards the concept of Significance level, one of the examples was: "*it is the level in which we reject $H_o$ and accept $H_1$.*" We will refer to this example later in our discussion.

The category "others" included incorrect responses that did not fit into categories c1-6. The percentage of definitions that fit under this category was negligible. In fact, definitions with no statistical meaning were classified in this category. For example, as regards the concept of Expected value, one of the definitions was: "*number of something.*"

In the third phase, based on the Process-Object theory, and the terms of concept image and concept definition, we present our interpretation of the results obtained and include excerpts from the interviews for support. It is important to note that most of the study participants did not add an example of the concept's use, as was requested.

## 3.5 Results and Discussion

In this section, each concept will be examined according to the most common category applicable to the students' given definitions. Then analysis of our findings according to Process-Object theory and concept image and concept definition will be presented.

## 3.6 The concepts of *'Life expectancy'* and *'Expected value'*

Table 1 summarizes the distribution of categories regarding the concepts of Life expectancy and Expected value.

| Concept/ category | Correct | C1 | C2 | C3 | C4 | C5 | C6 | Partial definition | others |
|---|---|---|---|---|---|---|---|---|---|
| **Life expectancy** | 26% | 3% | 0% | 59% | 2% | 6% | 0% | 0% | 4% |
| **Expected value** | 19% | 0% | 7% | 31% | 9% | 0% | 5% | 15% | 14% |

**Table 1: Distribution of categories regarding the concepts Life expectancy and Expected value**

Although the concept of Life expectancy is not a pure statistical concept, we incorporated it into our questionnaire in order to determine whether or not the students connect it to the concept of Expected value. In Hebrew these two concepts share the word *tohelet*. The concept of Life expectancy is *tohelet hayim* and the concept of Expected value is *tohelet*. This fact might strengthen our conclusion that the similarity between concepts caused the high number of incorrect definitions.

In relation to the concept of Life expectancy, Table 1 shows that the most common category of incorrect definitions was c3 (failing to connecting the meaning of the statistical and the everyday use of the concept). Approximately 60% of the students' definitions related to the everyday **faulty use** of the concept of Life expectancy. Though the meaning of Life expectancy is the average life span in relation to a certain population, in conversation, there is an erroneous tendency to use the term Life expectancy when referring to the lifespan of an individual. For example, the students defined Life expectancy as: (*)*"The period of time a person lives."* In terms of intuitive thinking, definitions such as (*) are the result of utilizing the most accessible explanation. Since the meaning of the concept in the spoken language is fast, automatic, and associative, the students selected the first definition which came to their minds which reflected the faulty use of the concept of Life expectancy in the spoken language. The students' definitions were the result of what was easily accessible to them, and they also happened to be more or less appropriate.

A basic understanding of statistics requires making the distinction between samples and populations. Both in Hebrew and in English the concept of Expected value is clearly a population quantity. Namely, we use a different concept ('mean' in English and "*memuza*" in Hebrew) when we refer to a sample. Although there is a clear distinction between population and sample as regards this concept, only 19% of the students provided the correct definition.

As was previously mentioned, the majority of the study participants did not add an example illustrating the use of the defined concept. However, from the examples that were written by some of the participants we could gain the following understandings: Regarding the concept of Life expectancy, it was revealed that the students' examples could be divided into two groups: (a) incorrect definition – correct example, and (b) incorrect definition- incorrect example.

**Incorrect definition – correct example**: several students defined Life expectancy incorrectly as "*the span of one's life*." However, the examples given for this definition were correct and were similar to the following - "*the average life span of women is 75*." In these cases we can conclude that the students understood the concept of Life expectancy but failed when defining it in words.

**Incorrect definition - incorrect example** – we found several variations. (1) With regards to the incorrect definition: "*how long a life time is expanded*," the example was: "*instead of living only 60 years the life expectancy of a person extended to 80 years*." In this case the students switched between two Hebrew terms: *tohelet hayim* (life expectancy) and *ha'arachat hayim* (extending a person's life). (2) In relation to the incorrect definition "span of life," some of the students wrote the example: "*the average life expectancy of a person*." This example implies that the students did not refer to Life expectancy as an average duration of life. Specifically, adding the word 'average' shows that the students did not notice that it is already part of the concept meaning of Life expectancy and in fact, this addition was unnecessary. (3) With regards to the definition: "*the number of years a person lives*," the example was: "*a person dies of old age – at the age of 75*." In this case, the student referred to life expectancy as the life span of a person without any clue that this is an average period. And finally, (4) concerning the definition: "*the life span of a person – his age*," there was the example: "*I have lived 26 years*." In this case the student defined Life expectancy as the actual age of a person.

Yet, the results presented in Table 1 do not indicate that the students do not have the knowledge to produce the right definition, had they only stopped to consider what they know. This explanation causes us to recall Kahneman’s (2002) words: "People are not accustomed to thinking hard, and are often content to trust a plausible judgment that quickly comes to mind" (pp. 451-452).

When we interviewed some of the students, we asked them to reflect on their attitudes towards the questionnaire. Most of them expressed ideas similar to the following:

> "*Throughout our statistics studies at the college, we were never asked to define statistical concepts; however, we have implemented these concepts in computational problems. So this kind of activity was unfamiliar to us. We know how to use these concepts in exercises, but we have never practiced defining them using words."*

The above excerpt refers to the way statistical concepts are usually taught. In most cases the definition of a concept is given without any reference to the everyday meaning of the words comprising the concept, and then teachers immediately start implementing use of the new concept via the presentation of computational tasks.

Regarding the concept of Expected value, 31% of the study participants formulated definitions using meanings which were influenced by the every day faulty use of the concept of Life expectancy. Specifically, their definitions included words such as ‘span’ ‘a certain time’ ‘a period of time,’ etc. Since the concept of Life expectancy appeared before the concept of Expected value in the questionnaire, it may explain the above phenomenon. In addition, 15% of the participants answered using partial definitions. That is, they wrote ‘mean’ as a definition to the concept of Expected value. It should be noted that this definition is partly correct since the concept of Expected value refers to the mean of the population. It is also interesting to note that 9% of the study participants defined Expected value as: "*the content of something."* Expectancy = *tohelet* and content = tehula as one can see (or more precisely, hear) in the Hebrew language have the same tone. Many of the interviewees expressed ideas similar to the following: "*When I sat in front of the questionnaire, I did not remember the exact definition of some concepts. The first thing that came to my mind was the everyday meaning of the words that comprise the concept so I tried to use it and make sense of it.*"

## 3.7 The concept of *Significance level*

| | **Correct** | **C1** | **C2** | **C3** | **C4** | **C5** | **C6** | **Others** |
|---|---|---|---|---|---|---|---|---|
| **Significance level** | 29% | 45% | 0% | 0% | 0% | 0% | 13% | 14% |

**Table 2: Distribution of categories regarding the concept Significance level**

The Hebrew term for significance level, *ramat muvhakut*, is only used in statistical contexts. While the adjective ‘significant’ *muvhak* is used in everyday Hebrew, the noun *muvhakut* is not. This is in contrast to English, where ‘significance’ is a common everyday word. Table 2 shows that the most common incorrect definition given by the students was c1 (using the meaning of each word separately). Instead of trying to define the concept as a whole, the students gave their interpretation regarding each word (according to its meaning in Hebrew) of the concept separately. For example: *"The probability that the results received in the sample are indeed correct in the population"; "Our level of confidence in achieving a result."* The above examples demonstrate the case in which the given definition relates to the meaning of each word separately, and the outcome is a definition which has an opposite meaning to the statistical concept of Significance level. In Hebrew the word ‘significant’ is an adjective which reinforces a statement, while the statistical concept of Significance level is a noun which refers to error probability. Both aforementioned definitions refer to the meaning of the adjective ‘significant’ as it is used in Hebrew and not as a noun in statistics. Naturally, the meaning of the adjective ‘significant’ in Hebrew is more accessible than its statistical meaning, so the results are not surprising. Moreover, most of the examples given for these definitions were similar to the following: "*we have to examine whether the percentage of the population who watches football is higher among children than adults with a significance level of 95%*." The above example demonstrates the fact that reference to each word separately, made the students switch the concept of Significance level with the concept of ‘confidence level’ since they refer to the meaning of the word ‘significant’ separately.

These findings can be interpreted according to the terminology of Vinner and Hershkovitz (1980) regarding concept image and concept definition. This may indicate that the students' definitions of Significance level were influenced by the concept image of the familiar, everyday meaning of the term ‘significant.’

Thirteen percent of the students’ definitions were phrased in an operational way. Specifically, instead of writing the definition of Significance level, the definition was comprised of the operations that have to be executed in order to obtain the Significance level. For example, "*it is the level in which we reject $H_0$ and accept $H_1$."* This case can be interpreted according to Sfard (1991) who argued that mathematical concepts have a dual nature. Sfard noted that a mathematical concept can be understood both as a process and as a mathematical object. Moreover, the understanding of a mathematical concept is built gradually and the process perception of a concept precedes the object perception of it. In the case of the concept of Significant level the definition "*it is the level in which we reject $H_0$ and accept $H_1$,"* demonstrates the perception of the concept on a process level, that is, the students described the operations which underlie this concept. That means that these students still have to undergo a process perception of the concept Significance level and have not yet gained object perception of it.

### The concept of '*Standard deviation*'

**Table 3: Distribution of categories regarding the concept Standard deviation**

| | Correct | C1 | C2 | C3 | C4 | C5 | C6 | others |
|---|---|---|---|---|---|---|---|---|
| **Standard deviation** | 38% | 37% | 15% | 0% | 0% | 0% | 3% | 7% |

Both in English and in Hebrew and in contrast to the concept of Expected value, the concept of Standard deviation requires the addition of an adjective (population or sample) to make the definition unique. By including the concept of Standard deviation, we expected students to make the above distinction between population and sample. However, most of them referred only to the sample Standard deviation. Table 3 shows that 37% of the research participants defined the concept of Standard deviation referring to the meaning of these words in the spoken language. For example, "*The distance from the mean*"; "*The distance of the variables in the experiment from the mean"; "The difference from a value that is determined to be an acceptable standard value."* In these examples it can be understood that the students referred to the meaning of the term Standard deviation in the spoken language and not to the statistical meaning of this concept. Specifically, they referred to the meaning of the term Standard deviation as a distance of a *certain value* from the mean and not to the **average** distance of the data from the mean.

The students' examples of this concept can be divided into two groups: (1) using the concept in the example without any further explanation. For example: "*The mean score of a class was 80 with a standard deviation 5.*" (2) Using the concept in the example while adding an incorrect explanation. For example: "*It is known that the average weight of a woman in the population is 58 kg with a Standard deviation of 10, namely, an error of 10 kg (48-68 kg)."* In the latter example the student referred to the range in the data and not to the concept of Standard deviation.

The students' difficulties in properly defining the above concept might be a consequence of intuitive thinking. These difficulties also might be related to verbal descriptions of the concept that precede its definition. Intuitive thinking and the verbal description of a concept are necessary for its understanding. However, there might be a gap between the mathematical definition of a concept and the way one perceives it in everyday use. Hence, we may say that there is a gap between the concept definition and the concept image (Vinner & Hershkowitz, 1980; Tall & Vinner, 1981). We may say that where there is a gap between the meanings of a concept as it is used in the spoken language (the concept image), and the mathematical definition of the concept (the concept definition), the students defined the concept according to the concept image.

In addition, 15% of the students' definitions included confusion between related concepts. Specifically the answers of these students indicated confusion among the concepts 'Standard Score' (or Z-score) or Standard Error and the concept Standard Deviation. For example, part of the given definitions was: "*The distance of a point from the mean in S* [Standard deviation] *units*." In this example the student's definition refers to the concept of Standard Score. In other cases the definition of standard deviation was: *Standard Error*.

The following excerpts from the informal interviews with some of the study participants can shed light on the above results. "*We found it* [defining statistical concepts] *difficult since we have not had to do so before"*. In response to our question, of how they think their ability to define mathematical concepts in general and statistical concepts in particular can be improved, most of the interviewees responded similar to the following: "*We think that when a teacher presents a new statistical concept, s/he has to initiate a discussion in which the relation of the new concept and its everyday meaning in the spoken language is discussed."* We also think that the quick transition to implementation of the concept in computational exercises does not help students internalize the new concept.

## Concluding remarks

Since some statistical concepts are difficult to understand, we suggest that students be presented with both their formal and informal definitions. Informal definitions include words of the spoken language which sometimes bring their own meanings to the definition. In the case of Hebrew, some concepts have the same, different, or opposite meanings as their statistical counterparts. In the present study we focused on a few statistical concepts which reflect various configurations of understanding that arose in relation to this phenomenon.

(a) When the statistical meaning is *similar* to the meaning in the spoken language – most of the definitions given by the students were correct. (i.e., the concept of Mode).

(b) When the statistical meaning is *different* from the meaning in the spoken language, the majority of the students' definitions were influenced from the everyday meaning of the terms that comprised the concept. (i.e., the concepts of Life expectancy, 'Expected value).

(c) When the statistical meaning is the *opposite* of the meaning in the spoken language, the majority of the students' definitions included the opposite meaning of the concept, meaning, as it is used in the spoken language. (i.e., the concept of Significance level).

From the informal interviews with some of the study participants, it can be concluded that the above misconceptions can be addressed by the following actions:

1. When presenting a new statistical concept informally in class, a discussion in which the relation between the concepts' meaning in statistics and its meaning in every day use should be initiated. Namely, nuances comprising the new concepts, both in their every day use and in their statistical meaning, need to be compared and discussed. For example, when the students study the concept of Significance level (in Hebrew: *ramat muvhakut*), the teacher should ask them what the meaning is of the adjective 'significant' and discuss the different meanings between the statistical concept and the everyday use of 'significant.' Although this discussion is in fact part of mathematical discourse, it should include references to the meanings of the target concepts in the spoken language. It should be noted that the students learn in their native language (in this case, Hebrew), hence, problems which can arise as a result of studying statistical concepts in a foreign language are avoided.
2. In addition to computational exercises given after learning new concepts, we recommend engaging the students with exercises that might foster a deeper understanding of the new concepts by connecting their meaning to previous ones. In fact, the traditional method for learning statistics devoted a significant portion of lessons to computational processes concerning the learned concepts. Therefore we recommend enlarging that portion of the lesson in which time is dedicated to understanding the spoken language meaning of concepts and the inter-relations between them.

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## End Notes

[1] Although the concept of 'life expectancy' is not a 'pure' statistical concept, we included it in the questionnaire in order to examine whether or not the students connected it to the statistical concept of 'expected value.'

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