ESSAYS IN MONETARY POLICY AND ASSET PRICING

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Acknowledgements

Long before coming to Stanford, I audited a class in Paris taught by Nicole El Karoui. The class was much too advanced for me, but I continued to go to the lectures, more and more captivated by the subject of asset pricing. After the class was over, I bought a little blue book that should help me get started with the material. My fascination with the blue book was great, and it was a dream of mine to ever talk to its author. This dream became true at Stanford.

No words describe what a great advisor Darrell Duffie has been to me.

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Chapter 1

Introduction

In factor models of the term structure, bond yields are taken to be functions of a Markov state process which are restricted to satisfy no arbitrage conditions. The traditional approach is to assume that the factors are unobservable and to infer their dynamics from either selected conditional moments of the short rate or from a set of yields.[1]

This thesis presents a continuous-time factor model of the term structure that incorporates the effects of monetary policy on interest rates. Relative to the previous literature, the thesis makes three main contributions. First, it introduces a class of linear-quadratic jump-diffusions (LQJD) which is flexible enough to capture the main features of discontinuous movements in interest rates induced by the central bank's operating procedures, yet still yields an essentially closed-form solution for the term structure. Second, it proposes the first term-structure model that makes jumps induced by policy depend on macroeconomic variables. This means that at least part of the state vector consists of observables. The model can be used to understand the role of monetary policy and macroeconomic variables for interest rates, or vice versa, while respecting the discipline imposed on the dynamic behavior of yields by no-arbitrage.[2] Third, it estimates versions of the model with German and U.S. data.

[1]See, for example, Anderson and Lund (1996) for an estimation using short rate information only. For estimations with longer maturity yields, see, for example, Pearson and Sun (1994), Chen and Scott (1993), Duffie and Singleton (1997) and Dai and Singleton (2000).

[2]This goal could, of course, also be achieved in a general equilibrium framework such as Pennacchi

To include monetary policy into a model of the yield curve, two aspects of monetary policy must be taken into account. The first aspect is that a central bank has an objective function that depends on the state of the macroeconomy, which it attempts to influence by targeting the short rate. Many models of monetary policy capture this aspect by specifying a *policy rule*, a map from a set of indicators (typically past interest rates and other current and lagged macroeconomic variables) into current interest rates.

The second aspect, which is especially important in a model with high frequency interest rate movements, is how interest-rate targeting works in practice. In the United States, the Federal Reserve (Fed) fixes a target rate for the overnight rate on interbank lending, the federal funds rate. The target rate is used by the Fed to communicate its monetary policy goals to its agent, the Federal Reserve Bank of New York. As documented by Borio (1997), similar relationships between an official policy rate and a short-term market rate exists in many countries.

Central bank operating procedures differ, however, with respect to when the target rate may be changed. Two types of regimes can be distinguished: With *unscheduled announcements* of the target, monetary policy interventions (that is, changes in the target rate) may occur essentially on any given business day. This type of interest-rate targeting was conducted in the U.S. from October 1982 to 1993. With *scheduled announcements*, monetary policy actions occur at central bank meeting days. Apart from two exceptions, the Fed followed the example of the Bundesbank in changing its target rate only on FOMC meeting days since February 1994 (Thornton (1997), Meulendyke (1998)).[3]

The importance of the details of operating procedures for term-structure analysis stems from the fact that in a no-arbitrage framework, expected future short rates are always a significant component of the price of a discount bond. The way that bond markets form expectations about future short rates is different across regimes, since the probability of a large increase in the short rate caused by restrictive monetary

(1991), Sun (1992), den Haan (1995) and Berardi (1998). The advantage of the approach taken here is that there is no need to take a stand on the complete structure of the economy (such as agents' preferences).

[3]The two exceptions occurred on April 18, 1994 and October 15, 1998.

policy on any given business day is zero in a scheduled announcement regime, while it may be positive in an unscheduled announcement regime.

The model developed in the second chapter of this thesis can accommodate both types of policy regimes and general specifications for policy rules. More generally, it allows for a large class of joint stochastic processes of short rates and other state variables (such as macroeconomic factors), but restricts the system in such a way that monetary policy interventions are identified as *jumps* in the official rate. In order to capture unscheduled announcements, restrictive policy interventions (jumps of positive size) and expansive interventions (jumps of negative size) are timed by stochastic mean arrival rates (intensities, in the language of probabilistic modeling of stopping times). Since central banks base their policy decisions on a policy rule, the stochastic arrival rates will be linked to the macroeconomic variables that are indicator variables in the policy rule. In the second setup, the size of the jumps may depend on the state of the macroeconomy through the policy rule, while the policy arrival is predetermined. Despite the dependence of jumps in interest rates on the state of the macroeconomy, the model has a simple solution for the term structure of interest rates. The latter property is important for both empirical analysis and for the pricing of interest-rate derivatives.

The third chapter estimates a scheduled announcements regime with Euro-DM rate data from 1982 to 1997. During this time period, the Bundesbank has never changed official interest rates outside of its meetings. The following aspects complicate the estimation: (*i*) there are different data-observation frequencies of interest rates and macroeconomic variables; (*ii*) the targeted interest rates have seasonal maintenance-period effects; and (*iii*) yields are nonstationary due to meeting-day and release-day effects.

The fourth chapter estimates versions of the model by the method of simulated maximum likelihood, extended here to the case of jump-diffusions. Two classes of models are presented, one using as data the Fed target and yields at several maturities, the other exploiting additional macro variables such as nonfarm payroll employment and CPI inflation.

The first class of models is designed to explore the role of the target rate as an observable factor in the post-1994 U.S. policy environment. After 1994, the target was changed almost exclusively on FOMC meeting days, and changes in the target were always integer multiples of 0.25%. This estimation uses LIBOR and swap yields with maturities of up to five years. The most interesting variant is a 4-factor model that includes (*i*) the target rate, (*ii*) a spread factor that measures deviations of the short rate from the target, (*iii*) a traditional stochastic-volatility factor, and (*iv*) a *policy-inertia factor*.

The empirical results show that the short rate reverts quickly and continually to the target, while the target adjusts slowly toward the Fed's new desired target *only through jumps occurring at FOMC meeting days.* The likelihood of a target-rate move at an FOMC meeting depends crucially on two factors: the current target and the inertia factor. Persistence in the target holds the target near its old value (*interest-rate smoothing*), thereby introducing *positive autocorrelation in the target-rate level.* The cross-sectional response of yields at different maturities to target shocks is therefore monotonically decreasing in maturity. The inertia factor slowly pulls the target toward the new desired value of the target (*policy inertia*). Shocks to the policy-inertia factor increase the likelihood of a target move, not only at the next meeting, but also at subsequent meetings. This leads to *positive autocorrelation in target-rate changes.* As the anticipated cumulative effect of these pending target changes is largest for maturities of around 2 years, the cross-sectional impulse response of yields to shocks in the inertia factor has a hump at those intermediate maturities. The combined effect of money market shocks and inertia-factor shocks leads to a "snake-shaped" pattern in the term structure of responses of yields to changes in the target: high for very short maturities, rapidly decreasing until maturities of around 6 months, then increasing until maturities of up to 2 years, and finally decreasing again. As these shocks are important for yields, this "snake-shaped" pattern carries over to the "volatility curve," the standard deviation of yield changes as a function of maturity. Shocks to the target rate in the post-1994 environment happen mostly at FOMC meetings and thereby introduce a seasonality into the volatility of yields.

By incorporating the Fed's interest-rate targeting behavior, the estimated 4-factor model considerably improves the performance of existing 3-factor models, such as that of Dai and Singleton (2000), especially at the short end. Adding the target as an observable factor allows the remaining latent state variables to concentrate on explaining medium and long maturity yields. The Fed's target rate thus provides a tractable way to improve bond pricing, avoiding the use of additional latent variables.

Weekly yield information is used to identify, through the 4-factor model, a high-frequency policy rule of the Fed, according to which the Fed reacts to information contained in the yield curve. The policy rule describes the target better than several benchmarks, including estimated versions of Taylor-type rules in which the Fed reacts to current macroeconomic information (Taylor (1993)). An explanation for the good fit of the estimated policy rule is that the policy-inertia factor implied by yield data anticipates many target moves.

The second class of models is estimated for the purpose of capturing the behavior of yields around release days, and also a role for macroeconomic variables in bond pricing. An important consideration here is the identification of release "surprises." For a typical release day, market participants may have prior information about the release. This information would have already been incorporated into prices. Surprises are identified with analyst-forecast data by specifying the joint dynamics of analyst forecasts and actual macro variables (nonfarm payroll employment and CPI inflation) in a state-space system. Models in this class are estimated using LIBOR-rate data for maturities of up to 1 year.

Release surprises are found to be temporary components of macro variables, in the sense that the impulse-response of macro variables to these shocks dies off after one month. In a model in which the Fed reacts to current macroeconomic variables, this means that release surprises can affect the conditional probability of target moves at only those FOMC meetings that are scheduled before the next macro release. In other words, *release surprises are not inertia-type factors themselves.* In order to replicate the hump-shaped cross-sectional response of yields to release surprises, the propagation of these surprises would need to 'live longer.' This may be achieved by allowing for correlation between the release surprises and the policy-inertia factor. Here, the macroeconomic news have an impact on the new desired target, but the FOMC only gradually implements this desired target over a number of meetings.

Chapter 2

Theoretical Model

2.1 The Yield Curve Model

After an overview (Section 2.1.1), we will provide a number of technical results about arbitrage-free pricing in a state-space model for the yield curve that includes both latent and macroeconomic variables (Sections 2.1.2 and 2.1.3) These results will later be used to price the assets used in the estimations (Section 2.1.4).

2.1.1 Overview

The state of the economy at time t is described by the vector $X(t)$. The state includes the target rate $\theta(t)$, some macro variables $m(t)$, analyst forecasts $m_F(t)$ of these macro variables, and certain latent variables such as the spread $s(t) = r(t) - \theta(t)$ between the riskless short-rate r and the target θ. The dynamics of X are described by a stochastic differential equation (SDE) of the form

$$dX(t) = \mu(X(t), t)\, dt + \sigma(X(t), t)\, dW(t) + dJ(t), \tag{2.1}$$

whose components will be explained shortly. In the absence of jumps J, this system may be thought of as a vector-autoregression with a linear mean rate of change μ. The Gaussian process W is responsible for continuous "small" shocks to X. These small shocks may translate into a non-Gaussian distribution of X if the volatility

$\sigma(X(t), t)$ depends on $X(t)$. The pure-jump component J of (2.1) is responsible for discontinuous moves in X, *macroeconomic jump effects*. These jumps can be caused, for example, by macroeconomic releases and monetary-policy events. In the example, the short-rate process r is a linear function of the state, in that $r = \theta + s$. More generally, r can be linear-quadratic.

Arbitrage-free pricing can be done though an exogenous risk-adjustment specified in the form of a 'density process' ξ. Asset prices are then given by the conditional expected value of their payoff, weighted by ξ, and discounted at the riskless rate. In particular, the time-t price of a zero-coupon bond that matures at time T is

$$P(t,T) = E_t \left[\frac{\xi(T)}{\xi(t)} \exp\left(- \int_0^t r(u)\, du \right) \right], \tag{2.2}$$

where E_t denotes expectation given the information available to bond investors at time t. In a Lucas (1978) economy, for example, the term inside the expectation is just the marginal rate of substitution of a representative agent. We can use the weight ξ to define a *risk-neutral* probability measure $\mathcal{Q}$ which satisfies $E_t\left(Z\xi(\bar{T})/\xi(t)\right) = E_t^{\mathcal{Q}}(Z)$ for any random variable[1] Z known at time $\bar{T}$. By specifying the dynamics of X and the switch to $\mathcal{Q}$ carefully, the bond-pricing formula (2.2) can be computed in closed form. This will be the objective of the remainder of this section.

2.1.2 Linear-Quadratic Jump-Diffusions

We now specify a particular parametric model for the dynamics of X. Uncertainty in the economy is described by a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$. The resolution of uncertainty over time is given by a filtration $\{\mathcal{F}(t) : t \geq 0\}$ satisfying the usual conditions (Protter (1990)). The process X satisfying (2.1) lives in some state space $D \subset \mathbb{R}^N$. For the SDE (2.1), W is an N-dimensional standard Brownian motion on $(\Omega, \mathcal{F}, \mathcal{P}, \{\mathcal{F}(t)\})$, $\mu : D \times [0, \infty) \to \mathbb{R}^N$ is the drift of X, $\sigma : D \times [0,\infty) \to \mathbb{R}^{N\times N}$ is its "volatility," and J is an $\{\mathcal{F}(t)\}$-adapted pure-jump process further described below. The value of X 'just before' the jump at t is denoted $X(t-) = \lim_{s\uparrow t} X(s)$.

[1] Here, Z is $\mathcal{F}(\bar{T})$-measurable and $E^{\mathcal{Q}}(|Z|) < \infty$.

The jump of X at t is $\Delta X(t) = X(t) - X(t-)$. For each fixed t, both $\mu(x,t)$ and $\sigma(x,t)\sigma(x,t)^\top$ are affine (constant-plus-linear) in the state, in a manner to be made precise shortly.

Except when there is a jump caused by J, the state X has continuous sample paths driven by W. Two types of jumps contribute to the pure-jump process J. First, there are jumps (of different types) arriving at deterministic dates counted by a vector N_d of counting processes. Second, there are jumps arriving at random dates counted by a vector N_p of Poisson processes with stochastic intensity λ. Heuristically, the $\mathcal{F}(t)$-conditional probability that there is a Poisson jump in the small interval $[t, t+\Delta]$ is $\lambda(t)\Delta$. More formally, stochastic intensities are characterized by the fact that the compensated process $\{M_p(t) = N_p(t) - \int_0^t \lambda(t)\, dt,\ t \geq 0\}$ is a martingale. (See Brémaud (1981) for further details.)

We can now define linear-quadratic jump-diffusions (LQJDs) by choosing particular functional forms for the coefficients μ and σ of the SDE (2.1), together with additional restrictions on the jump process J. In describing these parametric specifications, we can, without loss of generality, partition the state as $X = (X_1, X_2)$ so that X_2 is a k_2-dimensional process, with $k_1 + k_2 = N$. Assumption 1 will restrict X_2 to be Gauss-Markov. It will be convenient to define the set

$$\begin{aligned} C \;=\; & \{(c_0, c_1, c_2) \in \mathbb{R} \times \mathbb{R}^N \times \mathbb{R}^{N\times N} : c_2 \text{ is symmetric positive semidefinite} \\ & \text{and consists of zeros except possibly the lower right } k_2 \times k_2 \text{ partition}\} \end{aligned}$$

of coefficients. We will make repeated use of linear-quadratic (LQ) functions of the state of the form $g : D \times C \to \mathbb{R}_+$, with

$$g(x,c) = c_0 + c_1\, x + x^\top c_2 x. \tag{2.3}$$

We are now in the position to specify the LQJD as follows.

Assumption 1 (Characterization of LQJD processes)

(a) (Functional Form of Drift and Volatility)

The drift and 'volatility' of X are given by are given by

$$\mu(x,t) = K(t)\ (\bar{x}(t) - x) \tag{2.4}$$

$$\sigma(x,t) = \Sigma(t)\, S(x,t), \tag{2.5}$$

where $S(x,t)$ is a $N \times N$ diagonal matrix with i-th diagonal element $[S(x,t)]_{i,i} = \sqrt{s_{0i}(t) + s_{1i}(t) \cdot x}$, and where the coefficients $s_{0i}(t) \in \mathbb{R}$, $s_{1i}(t), \bar{x} \in \mathbb{R}^N$ and $K(t), \Sigma(t) \in \mathbb{R}^{N \times N}$ are deterministic functions of time.

(b) (Functional Form of Stochastic Intensities)

The jumps J are counted by a p-dimensional counting process N_p with stochastic intensity, and by a d-dimensional deterministic counting process N_d without explosions,[2] and with no common jump times.[3] The stochastic intensity $\{\lambda_i(t) : t \geq 0\}$ of N_p^i is given by

$$\lambda_i(t) = g\left(X(t-), l^i(t)\right), \tag{2.6}$$

for time-dependent coefficients $l^i(t) \in C$. The coefficients $l^i(t)$ and the domain D satisfy joint conditions to ensure that $\lambda_i(t) \geq 0$, as required for any intensity process.

(c) (Conditional Jump-Size Distributions)

For any Poisson jump time τ, the $\mathcal{F}(\tau-)$-conditional distribution $v_{p,\tau}$ of the jump size $\Delta X(\tau)$ is independent of $X(\tau-)$. For any deterministic jump time t, the $\mathcal{F}(t-)$-conditional distribution $v_{d,t}$ of the jump size $\Delta X(t)$ has a Laplace transform which is an exponential LQ function of $X(t-)$. More precisely, for all $a \in \mathbb{R}^N$, we have that

$$E\left[\exp(a \cdot J^d(t))\right] = \exp\left(g\left(X(t-), c(t;a)\right)\right) \tag{2.7}$$

for some $c(t;a) \in C$.

[2] For all t, $N_d(t) < \infty$ almost surely.

[3] This means that $\Delta N_p^i \cdot \Delta N_p^j = 0$ and $\Delta N_d^i \cdot \Delta N_d^j = 0$, $i \neq j$ almost surely.

(d) (Parameter Restrictions)

(i) All of the time-dependent coefficients are bounded and piece-wise constant functions of time.[4]

(ii) Joint restrictions on $(\mu, \sigma, v_p, v_d, l)$ and the domain D apply that guarantee a unique (strong) solution to (2.1).

(iii) Gaussianity of X_2: The lower left $k_2 \times k_1$ partitions of the matrices $K(t)$ and $\Sigma(t)$, labeled $K_{21}(t)$ and $\Sigma_{21}(t)$, consist of zeros only. Also, $s_{1i}(t)$ is an N-vector of zeros for all $i \in \{k_1 + 1, \dots, N\}$.

This definition of the state process X generalizes in two directions the concept of an affine jump-diffusions introduced by Duffie and Kan (1996). First, jumps are allowed to occur at deterministic points in time. The associated jump size, or mark, may have a state-dependent conditional distribution provided its Laplace transform is an ELQ function in the state. For a deterministic jump time t, an example of an $\mathcal{F}(t-)$-conditional jump-size distribution that satisfies this requirement is a Gaussian distribution with a conditional mean that is a LQ function in the state $X(t-)$ and with a constant variance. Another example is a jump size that is an LQ function in $X(t-)$ plus a random variable that has any given state-independent distribution subject to technical integrability conditions.

Second, the intensity of Poisson jumps may be quadratic in a Gaussian state vector. This allows jumps to arrive at negatively correlated jump intensities, a property that cannot be accommodated in an affine setting. Negatively correlated state variables with a positive domain would force a violation of assumption Condition A in Duffie and Kan (1996). In the absence of jumps, the condition is sufficient for the existence of a solution to the stochastic differential equation (2.1) describing the state. As already noted by Duffie and Liu (2000), it is possible to square two Gaussian processes so that each variable takes only positive values, while allowing for arbitrary correlation. This idea is applied here to the case of jumps.

[4] This particular type of time-dependence of the parameters determining the dynamics of X is sufficient for the seasonality effects studied this paper. Alternatively, the parameters may be bounded continuous functions of time.

Even without jumps, the state process may exhibit rich dynamics such as conditional heteroscedasticity (through s_1).

2.1.3 Change of Measure

We assume that there exists a nominal "short-term" riskless-rate process r, at which agents can borrow and lend, in the sense that r is adapted and jointly measurable, with $\int_0^T |r(t)|\,dt < \infty$. We consider the existence of an equivalent probability measure $\mathcal{Q}$ under which all security prices, $\{F^i\}_{i=1}^I$, normalized by the price $F^0(t) = \exp(\int_0^t r(u)\,du)$ of 1 Dollar invested at time 0 and rolled over at the riskless rate, are martingales, in that

$$\frac{F^i(t)}{F^0(t)} = E_t^{\mathcal{Q}}\left[\frac{F^i(T)}{F^0(T)}\right] = \frac{E_t^{\mathcal{P}}\left[\xi(T)F^i(T)/F^0(T)\right]}{\xi(t)}, \tag{2.8}$$

where ξ denotes the "density" of $\mathcal{Q}$. If such a "risk-neutral" (or equivalent martingale) measure $\mathcal{Q}$ exists, there is no arbitrage, at least under reasonable restrictions on trading strategies (Harrison and Kreps (1979), Harrison and Pliska (1981)). Conversely, the absence of arbitrage, and some technical conditions, implies the existence of such a "risk-neutral" measure (Delbaen and Schachermayer (1994)).

Consider as a candidate for the density process ξ of an equivalent martingale measure the solution of the SDE

$$\frac{d\xi(t)}{\xi(t-)} = -\sigma_\xi(t)\,dW(t) + J_\xi^d(t)\,dN_d(t) + J_\xi^p(t)\,dM_p(t), \tag{2.9}$$

with the initial condition $\xi_0 = 1$. The construction of exogenous risk premia proceeds in three steps. First, we show, under specific assumptions on the coefficients of SDE (2.9) (Assumption 2 in Appendix A), that ξ is a square-integrable $\mathcal{P}$-martingale. This means that we can use $\xi(\bar{T})$, for some fixed time $\bar{T}$, as the Radon-Nikodym derivative $d\mathcal{Q}/d\mathcal{P}$ of an equivalent probability measure $\mathcal{Q}$. Allowing for a jump J_ξ^d at a deterministic jump time (such as a scheduled announcement date) is unusual in the term-structure literature. Appendix B provides an example. Second, a generalized Girsanov theorem (Proposition 2 in Appendix A) provides a representation of the dynamics of the state process X under $\mathcal{Q}$. For econometric convenience, only parameterizations that make X a LQJD under both $\mathcal{P}$ and $\mathcal{Q}$ are considered in this paper.[5] Third, we establish restrictions on the coefficients of ξ (Proposition 3) that ensure the absence of arbitrage by virtue of condition (2.8).

In order to proceed with this 3-step construction of risk-neutral pricing, suppose we have I asset prices $\{F^i(t)\}_{i=1}^I$ of the form $F^i(t) = \exp(f(X(t),t))$, for smooth $f : D \times [0,T] \to \mathbb{R}$. (In our setting, we will see that zero-coupon bond prices are of just this form.) By Ito's Lemma,

$$\frac{dF^i(t)}{F^i(t-)} = \mu_{F^i}(t)dt + \sigma_{F^i}(t)\, dW(t) + J^d_{F^i}(t)\, dN_d(t) + J^p_{F^i}(t)\, dM_p(t), \quad (2.10)$$

with $\mu_{F^i}(t) = F^i(t-)^{-1}\mathcal{A}F^i(t)$, where $\mathcal{A}$ is the infinitesimal generator[6] of X, the volatility is $\sigma_{F^i}(t) = f_x(X(t),t)\sigma(X(t),t)$, and the jump size is $J^j_{F^i}(t) = \exp\left[f(X(t) + J^j(t), t) - f(X\right.$ 1 for $j = p, d$.

For notational simplicity, the following result is stated for one-dimensional versions of the counting processes N_p and N_d. A proof can be found in Appendix **??**.

Proposition 3 (Equivalent Martingale Measure): Suppose Assumption 2 (stated in Appendix A) holds. Suppose the normalized asset price $\{F^i(t)/F^0(t) : t \geq 0\}$ is square-integrable, where F^i solves (2.10). Then, for any fixed time $\bar{T} > 0$, the discounted asset price is a martingale under the equivalent probability measure $\mathcal{Q}$

[5] There is considerable evidence that the dynamics of the short rate is nonlinear, at least in a one or two-factor setting (Ait-Sahalia (1996), Boudoukh, Richardson, Stanton, and Whitelaw (1998), Ang and Bekaert (1998)). In the present framework, this nonlinearity may be introduced in two ways: quadratic terms under the data-generating measure $\mathcal{P}$ and market prices of uncertainty that, while preserving a LQJD structure under $\mathcal{Q}$ and therefore tractable pricing formulas, take the state dynamics outside the LQJD class under $\mathcal{P}$. The latter approach is explored by Duffee (1999) in an affine term-structure model.

[6] For a function $f : D \times [0,T] \to \mathbb{R}$, the infinitesimal generator $\mathcal{A}$ of X is a function $\mathcal{A}f : D \times [0,T] \to \mathbb{R}$ given by

$$\begin{aligned}\mathcal{A}f(x,t) &= f_t(x,t) + f_x(x,t)\mu(x,t) + \frac{1}{2} f_{x\,x}(x,t)\sigma(x,t)\sigma(x,t)^\top \\ &\quad + \sum_{i=1}^{p} g(x, l^i(t))\, E\left[f(x + J_i^p(t), t) - f(x,t)\right],\end{aligned}$$

using the fact that the jump $J_i^p(t)$ is independent of the state.

defined by $d\mathcal{Q}/d\mathcal{P} = \xi(\bar{T})$ provided:

(i) For any t that is not a deterministic jump time,[7]

$$\mu_{F^i}(t) - r(t) = \sigma_{F^i}(t)\sigma_\xi(t)^\top - \lambda(t)E^{\mathcal{P}}\left[J^p_{F^i}(t)\left(1 + J^p_\xi(t)\right)\right].$$

(ii) For any deterministic jump time t,

$$E^{\mathcal{P}}_{t-}\left[J^d_{F^i}(t)\right] = -E^{\mathcal{P}}_{t-}\left[J^d_{F^i}(t)J^d_\xi(t)\right].$$

Proposition 3 provides an interpretation of the coefficients of the SDE (2.9) for ξ in terms of *market prices of uncertainty* that compensate investors for different sources of risk.[8] In order to interpret these risk premia, suppose first that there are no Poisson jumps. Then (i) says that on 'normal days' (not deterministic jump times) the instantaneous expected excess rate of return is the "market price of Brownian motion uncertainty," σ_ξ, multiplied by the "factor loading" σ_{F^i}. In other words, the expected excess rate of return is proportional to the 'conditional covariance' of the return and the density, or pricing kernel. This is along the lines of the Intertemporal CAPM by Merton (1993). In the presence of Poisson jumps, there is an additional premium which, loosely speaking, is the conditional probability $\lambda(t)$ of a Poisson jump in the next "small" time period multiplied by the expectation of the product of the "market price of jump uncertainty" $-(1 + J^p_\xi)$ weighted by the jump-conditional

[7]The notation "$E^{\mathcal{P}}\left[J^p_{F^i}(t)\left(1 + J^p_\xi(t)\right)\right]$" actually means the unconditional mean over the joint distribution of the jump $\Delta X(t)$ of the state and the jump $\Delta\xi(t)$ of the density ξ at an arbitrary jump time τ of N^p. Because these jumps are of a distribution independent of $X(t-)$, and because the number of jumps during any time interval is finite almost surely, the expectation is unambigous, despite the abuse of notation.

[8]In an economy with an endowment that follows a diffusion process and time-additive utility (Duffie and Zame (1989)), the market price of uncertainty equals minus the coefficient of relative risk aversion of a representative agent multiplied by the volatility of the growth rate of the aggregate endowment process. In this setting, it is usually called market price of risk. A different structural interpretation is offered by recent papers on uncertainty aversion (Chen and Epstein (1999), Anderson, Hansen, and Sargent (2000)), in which market prices of uncertainty consist of two terms. The first term is the standard risk adjustment just mentioned, while the second represents a measure of distance between the true data-generating measure and the probability measure underlying max-min behavior by the agent.

"factor loading" J^p_{Fi}. A similar interpretation holds in (*ii*) for deterministic jumps which occur on a deterministic schedule.

2.1.4 Linear-Quadratic Short Rate and Bond Pricing

The affine structure of Duffie and Kan (1996) and the quadratic structure of the SAINTS model (Constantinides (1992), El Karoui, Myneni, and Viswanathan (1993)) are combined by the following assumption.

Assumption 3 (Linear-Quadratic Short Rate of Interest)

Fixing a linear-quadratic jump diffusion X, the short-rate process $\{r(t);\ t \geq 0\}$ is assumed to have the linear-quadratic form $R(x,t) = g(x, \delta(t))$ for some given coefficients $\delta(t) \in C$.

The rest of this section is concerned with the computation of a solution $P(t,T)$ to (2.2). If, for example, r is Gaussian under the risk-neutral probability measure $\mathcal{Q}$, then this just involves taking the expectation of the exponential of a sum of Gaussians, which can be computed directly (Vasicek (1977)). For the general case in which X is a LQJD under $\mathcal{Q}$ and r is a LQ function of X, the idea is first to guess that bond prices are given by the exponential LQ form

$$P(t,T) = \exp\left(g\left(X(t), c(t,T)\right)\right), \tag{2.11}$$

for some $c(t,T) \in C$, which depends on the particular ordering of deterministic jump dates between t and T. This guess is verified by calculating $c(t,T)$ using the method of undetermined coefficients and equations (2.2) and (2.11). Note that (2.11) describes a *linear-quadratic model of the term structure of yields.*[9]

The computation of $c(t,T)$ proceeds recursively, starting at the time T of maturity with the boundary condition $c(T,T) = 0$, imposed from the fact that $P(T,T) = 1$, and from the assumption that D contains an open set. Two steps are needed along

[9] The continuously compounded yield $Y(t,T)$ at time t of a bond maturing at time T is defined by $Y(t,T) = -\ln(P(t,T))/(T-t)$.

the way. Roughly speaking, the first step (Lemma 1 in Appendix C) is to show that if the bond price at the next deterministic jump date is an exponential LQ function in the state vector, as in (2.11), then the price of a bond "just before" the jump date is of the same form. The second step (Lemma 2 in Appendix D) is to demonstrate that if the bond price "just before" the next deterministic jump date is given by the exponential LQ form (2.11), then the price during the entire interim period between two deterministic jump dates is also an exponential LQ function. Together, these two steps guarantee that for every t, the price $P(t,T)$ inherits the postulated form.

Proposition 4: Suppose that Assumptions 1 and 3 hold under $\mathcal{Q}$. Let the coefficient vector $c\,(t,T)$ be calculated recursively using the algorithm shown in Appendix E. If Assumption 4 (Appendix D) holds at all deterministic jump times $(t_i, c(t_i,T))$, $i \in \{1,\ldots,n\}$, and also at $(T,0)$, then $P(t,T) = \exp\left(g\left(X(t), c(t,T)\right)\right)$ for all $t \leq T$.

Proof: The proof is by induction over the deterministic jump dates $t_1,\ldots,t_n$ between t and T. By assumption, $P(T,T)=1$. Applying Lemma 2 with $s=T$, $\bar{c}=0$ and $\psi(X(T),\bar{c})=1$, we see that $P(t,T)$ satisfies (2.11) for $t \in [t_n, T)$. We can then apply Lemma 1 to obtain the desired property for $P(t_n-,T)$. Now suppose, for any deterministic jump time t_i, that $P(t_{i+1}-,T)$ is given by (2.11). We can apply Lemma 2 to establish the desired property for any time $t \in [t_i, t_{i+1})$ and then Lemma 1 to get it for $P(t_i-,T)$. By induction, $P(t,T)$, $t \in [0,T]$, has the desired property. (Note that Lemma 2 can also be applied to the interval $[0,t_1)$).■

2.2 Modeling 'Economic' Jumps

This section explains, using a series of examples, how the linear-quadratic jump-diffusion (LQJD) framework can be applied to model the factors underlying interest rates. The main theme is that, while time evolves continuously in the economy, events related to monetary policy occur at discrete points in time. The information released at these events is conveniently modeled as *jumps* in the state variables.

We distinguish between two broad classes of such events. First, many operating procedures features discrete *policy events*, recorded as changes in the central bank's desired rate. This suggests modeling the desired rate r^* itself as a pure-jump process. Subsection 2.2.1 will show how the LQJD framework accommodates interest-rate targeting under different operating procedures. A second important class of events are *releases of macroeconomic variables* modeled in Subsection 2.2.2.

2.2.1 Interest-Rate Targeting

The LQJD framework offers two alternatives for specifying the dependence of the central bank's desired rate on macroeconomic variables through a policy rule. The central bank's indicator variables (variables in the central bank's policy rule) can have an impact through the jump amplitude of the desired rate or through the timing of jumps.

Policy Rule in Jump Amplitude

For jumps at deterministic dates, the jump size can depend on the state vector in an LQ way. This feature is particularly useful for modeling scheduled announcements of desired-rate changes, such as those occurring at central-bank meeting days. The main modeling issue is then to pick a suitable jump-size distribution that satisfies the Laplace transform condition A(1.c.). For Taylor-type policy rules, the jump size $J_*(t)$ is of the form $\ln \psi(X(t-, a)$ plus possibly a white noise monetary policy shock $\epsilon(t)$.

Example 1: Gaussian Changes in the Desired Rate

Suppose central bank meetings are counted by a deterministic counting process N_M. Given an initial value r^*_0, the desired rate r^* solves

$$dr^*(t) = (J_*(t) - r^*(t-))\, dN_M(t). \tag{2.12}$$

The original rule recommended by Taylor (1993) is linear in quarterly inflation and quarterly percentage deviations from trend of real output. In this framework, the Taylor rule uses the level $\pi(t-)$ of inflation and real output deviations $y(t-)$ 'just before' the central bank meeting at time t, so that we get

$$J_*(t) = \pi(t-) + r_R + 0.5y(t-) + 0.5(\pi(t-) - \pi^*) + \epsilon(t),$$

where the real rate r_R and the inflation target π^* are assumed to be constants equal to 2. The Gaussian shock ϵ may be interpreted as a monetary policy shock.

The Gaussian assumption is not appropriate if a large fraction of central bank meetings do not result in policy moves. For example, the Fed has left its target unchanged in 70% of its meetings since 1994. Any distribution, however, that assigns positive probability to a zero move does not satisfy the Laplace condition A(1.c.), so we extend the LQJD setup to accommodate this feature.

Example 2: Non-Gaussian Changes in the Desired Rate

Suppose, for a central-bank meeting date t, that with $\mathcal{F}(t-)$-measurable probability $p(t)$, the central bank moves its desired rate at that meeting. Conditional on a move, the central bank applies the Taylor rule. Consider a bond of maturity date T, with only one central bank meeting scheduled before maturity. Suppose that the price of the bond at any $s > t$ satisfies $P(s,T) = \psi\left(X(s), u(s,T)\right)$ for some $u(s,T) \in C$. We then have

$$P(t-,T) = p(t)P(t,T) + (1 - p(t))\, E_t^Q\left[\psi\left(X(t-) + J(t), u(t,T)\right)\right]. \tag{2.13}$$

If we let $p(t) = \ln\psi(X(t-), \bar{p})$ for some coefficient vector $\bar{p} \in C$, then the bond price is a linear combination of terms of the form $p(t)\psi(X(t), u(t,T))$. If the lifetime of the bond spans more than one central bank meeting, the same reasoning can be applied recursively, so that bond prices are state-dependent combinations of ELQ functions.

More generally, discrete distributions can be specified by setting the probability that $\Delta r^*(t) = J_*^i$ for given outcomes $J_*^1, \ldots, J_*^I$ to be LQ functions $p^i(t) = \ln\psi(X(t-), \bar{p}_i)$ for some $\bar{p}_i \in C$. By restricting the jump-size distribution to be state-independent, it is possible to incorporate the central bank's policy rule entirely into $p(t)$, so that it enters only through the timing of policy moves. This general type of characterization of monetary policy will be discussed next.

Policy Rule in Jump Timing

Stochastic intensities for the random arrival of policy events can be used to capture, for example, the timing of U.S. target-rate moves before 1994, as well as macroeconomic 'Peso events' since 1994 period, at which the Fed decided to move the target at times other than those of FOMC meetings. In addition, stochastic intensities can be used to characterize policy moves at central bank meetings by introducing time-dependence, so that stochastic intensities are only active during the central-bank meeting interval $[\tilde{t}_M, t_M]$, or during some small subinterval.

For jumps at random times, tractability calls for the jump size to be independent of the state (see Assumption A(1.c.)). It is convenient to introduce Poisson processes, N^U and N^D, counting upward and downward jumps with stochastic intensities λ^U and λ^D. The jump size J_* is taken to be the same for upward and downward jumps. Specifically, the desired rate r^* solves

$$dr^*(t) = J_* \left(dN^U(t) - dN^D(t) \right). \tag{2.14}$$

The conditional probability of, say, a net desired-rate increase in the small interval $[t, t + \Delta]$ is approximately $\lambda^U(t)\Delta$. An attractive feature of this approach is that a non-Gaussian distribution of rate changes is easily accommodated. The next example specifies the functional form of stochastic intensities to be LQ to ensure that the intensities remain positive.

Example 3: Quadratic Intensities

The intensities, $\lambda^U(t)$ and $\lambda^D(t)$, are assumed to be of the form $\ln \psi(X(t-), a)$, for suitable $a \in C$. As the central banks tend to move their desired rates procyclically, movements in λ^U and λ^D would be negatively correlated. Quadratic dependence on the state may be used to allow for negative correlation. For example, we can let

$$\begin{aligned} \lambda^D(t) &= (a_X X(t-) - \underline{a})^2, \\ \lambda^U(t) &= (\bar{a} - a_X X(t-))^2, \end{aligned} \tag{2.15}$$

for some range $[\underline{a}, \bar{a}]$ specified such that the probability that $a_X X$ moves outside this range is small.

This setup offers an intuitively appealing model of a central bank that adjusts its desired rate toward $a_X X$, which might be given by the Taylor rule. The formulation (2.15), however, presents two difficulties. First, Assumption A(1.c.) restricts the quadratic terms in ψ to depend only on Gaussian variables. In particular, this precludes dependence of λ on r^* itself, so that the desired rate cannot exhibit mean reversion.[10] The second difficulty is that, in the presence of latent states, the model cannot be estimated with methods based on inverting the map from factors to yields, including maximum likelihood.

Example 4: Peso Events

To model rare policy events outside of meeting days, the counters N^U and N^D can be made 2-dimensional, with one component counting policy events at central bank meetings and the second counting 'Peso Events' that arrive at, say, a constant intensity for policy events that occur outside of meetings. Without expanding the state vector too much, more complex arrival rates may be accommodated by constructing a 'Peso indicator' based on variables such as stock return indices, exchange rates and foreign trade numbers. The idea would be that the central bank changes its stance on monetary policy outside of meetings, if the 'Peso indicator' $X_P(t)$ exits some target corridor $[\underline{X}_P, \bar{X}_P]$, so that the arrival rate of Peso events is given by (2.15), with $\underline{a} = \underline{X}_P$, $\bar{a} = \bar{X}_P$, $a = 1$, and $X = X_P$.

To sum up, a variety of theoretical and econometric tradeoffs should be considered when constructing a model with 'economic' jump effects based on the LQJD framework. In particular, the computational requirements associated with the map from factors to yields must be considered. In this paper, the policy rule will be incorporated into the jump amplitude, as in (2.12), to model the implied desired rate

[10] In a setting in with continuously compounded yields, this is particularly important. For example, if the short rate is given by $dr(t) = \sigma_r dB(t)$ and, for simplicity, market prices of uncertainty are all zero, Jensen inequality drives yields negative as maturity $\tau \to \infty$. By contrast, in discrete time, the specification $r_t = r_{t-1} + \epsilon_t$ where ϵ is white noise, together with the assumption that long yields are conditional expected values of averaged future short rates leads to a flat term structure.

of the Bundesbank. In this context, the Gaussian assumption is natural, the Taylor rule being a good description of the Bundesbank's policy rule, as will be shown in later sections. Compared with this approach, the state-dependent extension of (2.13) is computationally more intensive, especially for long-maturity yields. For instance, the lifespan of a 1-year yield typically covers 24 Bundesbank meetings, implying that the number of different exponential terms to be computed is 2^{24}.

2.2.2 Macro Variables

If the policy rule depends on macroeconomic variables, one must be careful about the informational assumptions regarding these variables, as they have important consequences for both the lag structure and the estimation procedure. In general, one might want to distinguish among three information sets: those of the market investors, those of the central bank and finally those of the econometrician, who typically observes a macro variable, or some time-aggregate of it, only once a month. In what follows, we look at specifications for which all agents in the economy have the same information set.[11]

We distinguish between two setups here. In the first, agents are assumed to continually observe macroeconomic aggregates, while the econometrician observes only releases made by the relevant government statistical office at discrete points in time. This is important because macro variables are typically measured at coarser time intervals than are yields.

Example 5: Filtering continuously observed macro variables

Here, we consider the use of the instantanously expected inflation rate $\hat{\pi}$ in the policy rule. Letting p denote the CPI, we suppose that

$$\frac{dp(t)}{p(t)} = \hat{\pi}(t)\, dt + \sigma_p\, dB(t), \tag{2.16}$$

where B is a standard Brownian motion. Assuming that $\hat{\pi}$ is a Gauss-Markov process

[11] Evidence presented by Romer and Romer (1996) indicates that the Fed has superior information about the macroeconomic state relative to other market participants. Incorporating this asymmetry is an interesting topic for future research.

we can apply the Kalman filter to obtain a monthly estimate of the filtered state $\hat{\pi}$ from CPI data alone.[12] In a two-step procedure, the recovered estimate of $\hat{\pi}$ may then be part of, for example, a GMM estimation of the remaining parameters. Moment conditions can be set up such that they exploit yield data sampled at a higher-than-monthly frequency. Based on a model of the yield curve, a higher-frequency series of expected inflation can be backed out from a set of yields with different maturities. For higher-order autoregressive lags, the state space can be expanded by unobservable Gaussian processes $(x_1, \ldots, x_K)$, so that $\hat{\pi} = \sum_{i=1}^{K} x_i$.

The setup in Example 5 assigns no particular significance to the day on which macro variables are released. This setup is therefore not suitable for analyzing the 'seasonal' effects on release days that have been noted in the literature. A simple way to introduce such effects into the LQJD model is to make exactly the opposite informational assumption, supposing that agents in the model learn the realization of the macro variable only at release dates.

Example 6: 'Measured Variables'

Suppose inflation measured at the end of the month π is released at days counted by N_π, so that we have

$$d\pi(t) = (J_\pi(t) - \pi(t-))\, dN_\pi(t),$$

were the jump size $J_\pi(t)$ is, for example, Gaussian with an LQ-conditional mean. In this case, the econometrician has no need to resort to filtering or yield data, as the continuous-time process π is observed from CPI data directly, at each instant t.

Higher-order autoregressive lags can be easily incorporated by defining the (observed) variable π_L, the last released value of inflation, which solves

$$d\pi_L(t) = (\pi(t-) - \pi_L(t-))\, dN_\pi(t).$$

Additional lags, π_{LL} and so on, can be defined along the same lines. Correlation

[12] Kalman filtering to recover expected inflation has been used extensively in the literature. For a recent application, see Berardi (1998).

among macro variables can also be introduced by specifying a bivariate system (π, y) of measured variables in discrete time. This system can be converted into a bivariate system in continuous time with Gaussian jumps. This only requires adjusting the conditional distributions of the timing of releases. For example, one could suppose that π is released before y every month. The (Gaussian) distribution of y, conditional on the observation of π of the same month, can be calculated explicitly.

2.3 Conclusion

A flexible bond-pricing framework has been presented that accommodates various operating procedures of monetary policy that rely on scheduled and unscheduled announcements of changes in the stance of monetary policy. Moreover, macroeconomic variables can be incorporated in a way that respects the timing of their releases.

Chapter 3

Application to German Data

3.1 Example with Scheduled Announcements

This section reviews some institutional features of the German interest rates and macro variables used, specifies the model, and provides maximum likelihood estimates of the parameters.

3.1.1 Operating Procedures in Germany

The Bundesbank conducts monetary policy by adjusting its two key refinancing rates, the discount and lombard rates, as well as through open-market operations on the interbank market centered in Frankfurt, the German analogue of the U.S. Federal Funds market. In this market, commercial banks trade unsecured, mostly overnight, funds to match short-term liquidity needs that arise from reserve management, as banks try to minimize the opportunity costs of holding reserves (Schnadt (1994)).

Banks must hold minimum reserves of "good funds" in the form of cash or in accounts at the Bundesbank on which no interest is paid. Regarding the determination of their reserve requirements, banks may choose between two calculation methods. The first method averages over requirements at the end of each day (including holidays) from the 16th of the previous month to the 15th of the current month. The second method is based on the average requirement at the end of four specific days ('bank-week return days'): 23rd and the end of the previous months plus the 7th and 15th of the current month. The Bundesbank may opt to suspend the second method for particular banks. Reserves must be maintained on average over the current month. In order to effectively manage their reserves, banks estimate their reserve requirements during the first part of the month, until the computation period ends. Together with other factors such as transaction costs, this leads to a period of high volatility in money-market interest rates around the end of each month, similar to the spikes in the federal-funds rate on "settlement Wednesdays," which mark the end of the biweekly maintenance period in the U.S. (Hamilton (1996)). Banks pay a penalty rate on the amount by which they fall short. (The penalty rate is currently 3% over the 30-day lombard rate (Bundesbank (1998), pp. 125, hereafter DB)).

In addition to borrowing reserves in the money market, banks may borrow funds from the Bundesbank, which offers collateralized loans at two different rates, the discount and lombard rates. Since the discount rate is usually lower than the prevailing interbank market rate, borrowing at this rate is limited by re-discount quotas. While not constraint by quotas, borrowing at the lombard rate is expensive, as the lombard rate is usually 100–200 basis points higher than the discount rate. Together, these two rates form a corridor for the overnight interbank rate.[1] Up to the mid 80s, these two refinancing rates constituted the Bundesbank's most important monetary-policy tool. More recently, their role in the monetary-policy process has been taken over by open-market operations. The bulk of these operations consists of repo auctions, in which the Bundesbank buys domestic bills from banks that simultaneously repurchase them forward. Repurchase agreements are thus a form of collateralized loans, and account today for about 70% of total bank borrowing.[2]

A feature of the operating procedures that is important for term-structure modeling is that the Central Bank Council decides upon all official interest rates (discount,

[1] This corridor is soft, in the sense that the overnight interbank rate can be lower than the discount rate when the liquidity of banks is high, or above the lombard rate when liquidity is low. Strict arbitrage in the latter situation is ruled out because loans at the lombard rate are collateralized and have a different maturity.

[2] DB, pp. 114.

lombard, and minimum bid repo rates[3]) during its meetings, which are usually scheduled every other Thursday, with a few exceptions such as national holidays and at the end of July (Nimmerrichter (1998)). Like the Federal Open Market Committee, the Central Bank Council consists of the presidents of the regional central banks and the members of the Bundesbank Board ("Directorate"), the executive body of the Bundesbank. Similar to the Federal Reserve Board, the Bundesbank Board has a president, a vice-president, and currently six other members. The Council's decisions concerning discount and lombard rate changes are announced that afternoon or the next morning. The minimum bid rate for the weekly repo auctions is announced every Tuesday morning at the beginning of each auction. In case of emergencies such as large foreign-exchange inflows, the Board would have the authority to act outside of the Council's meeting days (Nimmerrichter (1998)), but has never done so to the author's knowledge.

3.1.2 The Policy Rule of the Bundesbank

After the breakdown of Bretton Woods, the Bundesbank regained control over the monetary base and announced a monetary aggregate to be its primary long-run policy target. Annual target corridors were published starting in December 1974. While the definition of the measure of monetary aggregate changed somewhat over time, target levels were always derived from an estimate of the quantity-theory equation that links money, prices, velocity and potential output. In this equation, inflation is not estimated but set equal to the Bundesbank's maximum tolerance level of 2%.[4] In practice, the Bundesbank has had problems in meeting its own target corridor. Between 1975 and 1994, the Bundesbank has accomplished its goal in only 11 out

[3]There are two different kinds of repo auctions. In fixed-rate tenders, banks submit volume bids at a fixed interest rate. In variable-rate tenders, banks bid for both interest rate and volume. Until 1988, the Bundesbank usually announced an interest rate guideline and allotments were then made at uniform rates (Dutch auction). After 1988 they occurred at the individual bidding rate (U.S. auction). The term "minimum bid repo rate" used here refers to the interest rate set by the Bundesbank in fixed-rate tenders, and to the Bundesbank's interest rate recommendation in Dutch auctions.

[4]DB, pp. 81.

of 20 years.[5] On the one hand, this reflects the difficulties of targeting a monetary aggregate in the short run during this particular period which contains, for example, the unification year 1990. On the other hand, this also shows that the monetary target is ultimately sacrificed as long as the inflation goal is met,[6] which is what happened, for example, in 1986/87.

This raises the question of whether the Bundesbank, even though it claims to be targeting a monetary aggregate, is primarily targeting some other variable such as inflation (Bernanke and Mihov (1996), Clarida and Gertler (1996)). Supporting evidence for this view is the fact that the Bundesbank sets interest rate targets just as does the Federal Reserve. In official statements,[7] these interest-rate targets are called the Bundesbank's "short-term operational goals." Clarida and Gertler (1996) and Clarida, Gali, and Gertler (1997) find that the Bundesbank's policy rule seems to be well described by a modified version of the Taylor rule.[8] This indicates that the Bundesbank is at least acting as if it was targeting interest rates.

This paper follows this view, treating the Bundesbank as an interest rate targeting central bank. Setting up a policy rule is made difficult by German unification in 1990, which shifted the Bundesbank's attention to new macroeconomic aggregates. However, since GDP in Eastern Germany was about 7.5 % of that of Western Germany in 1991, preference is given to a large data sample over explicitly modeling the unification as a structural break in the policy rule. More concretely, two possible policy rules are investigated. The first rule, labeled the *original Taylor rule*, determines the target rate for overnight interbank borrowing as a function of Western German inflation and unemployment.[9] The second rule, called the *extended Taylor rule*, also reacts to the lagged target rate, in a manner to be specified shortly.[10]

[5] DB, Table 5 on pp. 79.

[6] DB, pp. 86. Also see Bernanke and Mihov (1996).

[7] DB, pp. 93.

[8] The modification consists of replacing current inflation in the Taylor rule with expected inflation, for which such instruments as current and lagged values of inflation are used.

[9] This is an abuse of terminology since the term 'original Taylor rule' usually refers to the link between the federal funds rate to current inflation and deviation of output from trend at a quarterly frequency. To be able to work with monthly macroeconomic series, I replace the output gap with unemployment by invoking Okun's Law.

[10] This terminology is used for the corresponding rules in the U.S. (see, for example, Rotemberg

3.1.3 Specification of State Variables

As the exact timing of policy moves is important given daily sampling frequency, I will assume that moves in the central bank's target rate are common knowledge on the morning after the Central Bank Council's meeting. This assumption does not apply literally, since the Bundesbank does not follow the policy of the Federal Reserve in announcing an interest rate target. This means that policy moves have to be inferred from changes in a combination of official rates. While changes in all official rates are made at meetings, minimum bid repo rates are not announced immediately after a meeting, as mentioned in Section 3.1.1. Nevertheless, the assumption that interest rate moves are known to all agents can be justified as follows. First, repo auctions were explicitly introduced as a way to provide liquidity without signaling effects.[11] Second, changes in minimum bid repo rates are small (less than 10 basis points) and their announcement has been found not to have had a significant effect on other short-term interbank interest rates (Hardy (1996)).

For modeling purposes, policy moves are assumed to only occur immediately after decisions made at Bundesbank meetings. As described in Section 3.1.1, this assumption ignores emergency situations in which the Bundesbank Board might decide upon a change in policy even outside of a regular meeting. Since such an event has never occurred during the sixteen-year sample period, the probability of a policy move outside a meeting is apparently small and ignored in the present model.[12]

Bundesbank meetings are stopping times that are counted by the deterministic counting process N_M. At the i-th meeting, the Bundesbank sets the target rate $r^*(\tau_M(i))$ according to a policy rule. Between meeting dates, r^* is constant. The

and Woodford (1998)).

[11] DB, pp. 113.

[12] Random policy moves could in principle be modeled by introducing Poisson processes with stochastic intensities that govern the arrival of policy moves in addition to the moves at meeting days. With no data observations on the occurrence of such random moves, the estimated arrival intensities would in any case turn out to be zero.

jump at the i-th meeting date $\tau_M(i)$ is

$$r^*(\tau_M(i)) = \beta_0 + \beta_1\ \pi(\tau_M(i)) + \beta_2\ u(\tau_M(i)) \\ + \beta_3\ r^*(\tau_M(i)-) + \sigma_*\ \varepsilon_*(\tau_M(i)), \tag{3.1}$$

where π and u are current inflation and unemployment measures, $r^*(t-)$ is the previous target rate and the shocks $\varepsilon_*(\tau_M(1)), \varepsilon_*(\tau_M(2)), \dots$ are, under $\mathcal{P}$, iid standard normal and independent of the other right hand side variables. These may be interpreted as monetary policy shocks in the sense of Christiano, Eichenbaum, and Evans (1998). The coefficients β_1 and β_2 are the policy-response coefficients on inflation and unemployment. The original Taylor rule (as defined in the previous section) is then obtained by setting β_3 equal to zero, while the extended Taylor rule leaves β_3 as a free parameter. If β_3 is larger than zero, the target rate today depends on the lagged target rate in a way that may be interpreted as interest-rate smoothing. Also, if the macroeconomic variables, π and u, are stationary and if β_3 is smaller than one in absolute value, then the target rate reverts to a long-run mean implied by the long-run means of π and u.

The short rate r of the term-structure model is identified with the overnight interbank lending rate. The spread $s = r - r^*$ between the short rate and the target rate is modeled as a stationary process with an unconditional mean of zero. This implies that r^* takes the role of a stochastic mean for the short rate. In order to capture the well-known conditional heteroscedasticity of interest rates in a setup with a stochastic mean that changes only on central-bank meeting dates, the spread s is taken to be the sum of I independent processes $\widetilde{s} = (\widetilde{s}_1, \dots, \widetilde{s}_I)$, called the 'spread factors.' We let

$$s(t) = \sum_{i=1}^{I} (\widetilde{s}_i - \theta_i), \tag{3.2}$$

where θ_i is a positive constant and $\widetilde{s}_i$ solves

$$d\,\widetilde{s}_i(t) = \kappa_i(\theta_i - \widetilde{s}_i(t))\ dt + \sigma_i \sqrt{\widetilde{s}_i(t)}\, dB_i(t)\ , \tag{3.3}$$

where κ_i and σ_i are the mean reversion and volatility parameters of the spread factor $\widetilde{s}_i$. $B = (B_1, ..., B_I)$ is a standard Brownian motion under $\mathcal{P}$, independent of ε_*. The construction of the spread s ensures that the square-root in equation (3.3) is taken of a positive number, while the spread process s reverts to a long-run mean of zero. Because of the square-root terms in (3.3), the spread exhibits conditional heteroscedasticity. The conditional distribution of a spread factor $\widetilde{s}_i(t')$ is noncentral chi-square,[13] as described in Cox, Ingersoll, and Ross (1985).

The specification of macroeconomic variables takes into account the differing observation frequencies of yields and macroeconomic variables. Empirical approaches that use both types of data typically settle on using the lowest frequency.[14] In this case, the determination of conditional densities is not complicated by the use of macro variables. As higher frequency observations of yields are important to distinguish the effects of biweekly monetary policy interventions from day-to-day movements in interest rates, this approach is not taken here. Instead, we make the following informational assumption regarding the macro variables π and u that enter the central bank's reaction function. The policymakers and all investors in the economy know the exact value of the relevant macroeconomic variables as soon as they are officially released. Policymakers do not have superior information regarding the macro variables relative

[13]The noncentral chi-square density of $\widetilde{s}_i(t')$ conditional on $\widetilde{s}_i(t)$ is given by

$$f(\widetilde{s}_i(t')|\ \widetilde{s}_i(t)) = c\exp(-u-v)\left(\frac{v}{u}\right)^{q/2} I_q\left(2(u\ v)^{\frac{1}{2}}\right),$$

where

$$\begin{aligned}
c &= \frac{2\kappa_i}{\sigma_i^2\ (1-\exp(-\kappa_i(v-t)))}\\
u &= c\ \widetilde{s}_i(t')\ \exp(-\kappa_i(v-t))\\
v &= c\ \widetilde{s}_i(t)\\
q &= \frac{2\kappa_i\theta_i}{\sigma_i^2} - 1,
\end{aligned}$$

and where $I_q(\cdot)$ is the Bessel function of the first kind of order q. The author thanks Qiang Dai for the numerical approximation of the Bessel function used in the Gauss estimation code.

[14]See, for example, Friedman and Kuttner (1998), Estrella and Mishkin (1995), Evans and Marshall (1998) for VARs containing macroeconomic variables and yields as well as Berardi (1998) for a Kalman Filter application.

to bond investors. We now need to distinguish between the inflation and unemployment processes that are measured by the Federal Statistical Office (Statistisches Bundesamt) at the end of each month and the released values of these variables. In discrete time, the vector process of measured inflation and unemployment is assumed to be an autoregressive process with Gaussian innovations. The AR order is set to 1. In the continuous-time model, this vector process is a pure-jump process with time-dependent conditional jump distribution. The measurements of inflation and unemployment are subsequently released to the public at some specific release dates known in advance. Only released values enter the policy rule.

Given the AR discrete-time specification of the measured variables, the released values (π, u) can be modeled as follows. Suppose inflation is released before unemployment on days counted by a deterministic counting process N_π. The released inflation process π is piecewise-constant, with a jump at the i-th announcement date $\tau_M(i)$ given by

$$\pi(\tau_\pi(i)) = \alpha_\pi + \alpha_{\pi\pi}\pi(\tau_\pi(i)-) + \alpha_{\pi u}u(\tau_\pi(i)-) + \sigma_{\pi\pi}\varepsilon_\pi(\tau_\pi(i)), \tag{3.4}$$

where $\varepsilon_\pi(\tau_\pi(1))$, $\varepsilon_\pi(\tau_\pi(2)), \ldots$ is, under $\mathcal{P}$, iid standard normal. Thus, the jump size of released inflation depends on the last released value of inflation and unemployment. The last released value $\pi(\tau_\pi-)$ is labeled $\pi_L(\tau_\pi)$. The unemployment rate u is released on days counted by the deterministic counting process N_u, and its jump size on the i-th release is given by

$$\begin{aligned} u(\tau_u(i)) = \alpha_u &+ \alpha_{u\pi}\pi_L(\tau_u(i)-) + \alpha_{uu}u(\tau_u(i)-) \\ &+ \frac{\sigma_{u\pi}}{\sigma_{uu}}(\pi(\tau_u(i)-) - \alpha_\pi - \alpha_{\pi\pi}\pi_L(\tau_u(i)-) - \alpha_{\pi u}u(\tau_u(i)-)) \\ &+ \sigma_{uu}\varepsilon_u(\tau_u(i)), \end{aligned} \tag{3.5}$$

where $\varepsilon_u(\tau_u(1)), \varepsilon_u(\tau_u(2)), \ldots$ is, under $\mathcal{P}$, iid standard normal. All of ε_u, ε_π, ε_*, and B are assumed to be $\mathcal{P}$-independent.

Without a monetary policy shock ε_*, meeting days have no effect on yields, as the Bundesbank is presumed to act like an automat that executes a policy rule. As soon as the macroeconomic variables are released, agents know the target rate that will be in effect until the next meeting day. This means that yields with a maturity after the next meeting day react to macroeconomic releases by anticipating their impacts on the target rate. The magnitude of the impact of releases on long rates depends on, among other parameters, the mean reversion of the spread process. The slower the spread reverts to zero, the less important is the target rate as stochastic mean of the short rate. One notes that the short rate does not react at all to releases, as the new macroeconomic information affects only future target rates. In the presence of a monetary policy shock ε_*, meeting-day releases do affect longer yields as the policy shock is incorporated into the target rate, at least until the next meeting day. The impact of the monetary policy shock depends on whether the Bundesbank uses the original or the extended Taylor rule, because the policy rule determines whether the policy shock lives longer than just the two weeks until the next meeting.[15]

The central bank is assumed to care about the monthly released series of macroeconomic variables, as opposed to inflation and unemployment at any point in time. This is motivated by a view that the central bank does not want to move interest rates too often. Apart from this reasoning, the specification of macroeconomic variables has the advantage of being able to easily accommodate an arbitrary lag structure for the macroeconomic variables, a feature that would be more difficult to achieve in a continuous-time setting if the policy rule depended on a process with continuous sample paths.[16] In addition, the econometrician is assumed to have the same information as the agents in the economy, so that there is no filtering problem involved in the estimation. Given the specification of π and u, the same policy rule would result if, instead of reacting to the last released value, the central bank reacted to its forecast of the future releases of inflation and unemployment. Because (π, u) is a Gaussian Markov process, the best forecast would be a linear function in $(\pi(t), u(t))$, and the coefficients β_1 and β_2 would remain constant between any two releases, as the central

[15]Note that the macroeconomic variables π and u are not affected by policy and policy shocks ε_* are iid. In order for a monetary policy shock to have a persistent effect, the policy rule needs to depend on the lagged target rate.

[16]See the generalized function approach in Hansen and Sargent (1991) to incorporate lags in a continuous-time model.

bank would not learn any additional information that would cause a revision of its forecast. The last property can of course be relaxed, but at the cost of introducing more state variables into the model.

To summarize, the *state vector* X contains the spread factors, the target rate, inflation, unemployment and the last released inflation rate, or

$$X(t) = (\widetilde{s}(t), r^*(t), \pi(t), u(t), \pi_L(t)) .$$

The *parameter vector* α contains the parameters describing the law of motion of the spread factors

$$\{\kappa_i, \theta_i, \sigma_i\}_{i=1}^I,$$

the target rate

$$(\beta_0, \beta_1, \beta_2, \beta_3, \sigma_*),$$

inflation and unemployment

$$(a_\pi, a_u, a_{\pi\pi}, a_{\pi u}, a_{u\pi}, a_{uu}, \sigma_{\pi\pi}, \sigma_{u\pi}, \sigma_{uu}),$$

and the market-prices-of-risk parameters

$$(b_u, b_\pi, b_*, \{b_{B_i}\}_{i=1}^I)$$

that are associated with the sources of risk: ε_u, ε_π, ε_*, and B.

3.1.4 Estimation Procedure and Results

Seasonal Pattern in Measurement Errors and Release Days

The interest-rate data consist of daily Euro-Deutschmark rates from January 1, 1982 to December 31, 1997. The data were obtained from Datastream. Euro-DM rates are actively traded interbank interest rates on DM deposits maintained in London.[17] The maturities are overnight, 7-day, 1, 3, 6 and 12 months. Summary statistics for all yields are reported in Table 1. The sample means of the yields show that the yield curve is on average increasing. This can also be seen from Figure 2 which shows different Euro rates. In light of the upcoming recession, the yield curve was inverted during the shaded area. The standard deviation and skewness of yields decrease with maturity. Yields are extremely persistent and autocorrelation increases with maturity. The kurtosis of changes in yields is much larger than 3, which confirms the well-known fact that yields are not normally distributed. This is also shown by the Jarque-Bera test statistics displayed in Table 2: The null hypothesis that the process is Gaussian can be rejected at the 1% confidence level.

The seasonal pattern that is partly responsible for the non-normality in these interest rates can be seen from Figure 3, which plots the overnight Euro rate. Large apparent spikes are typically associated with the end of reserve maintenance periods or the ends of years. For example, the largest value that the overnight rate takes during the sample is 14.5% on December 31, 1986. Table 3 shows more comprehensive evidence of seasonality by reporting the results of a least-squares regression of the absolute value of changes in yields on a constant and dummies for each day in the business week before the end of a maintenance period, the end of a quarter, and the end of a year. This regression measures the effect of seasonal dummies on the (unconditional) variance of changes in interest rates. The term structure of maintenance-period and end-of-the-year effects on volatility is clearly decreasing. Maintenance-period effects are significant up to 1 month; for longer maturities the reserve maintenance cycle does not lead to a significant increase in volatility. End-of-the-year effects are more pronounced than maintenance-period effects and also start

[17] Daily data on German government bonds for a sample going back to 1982 are difficult to obtain. The only relevant data available from the Bundesbank are computed from a linear regression of longer yields on a short-term money-market rate and some functions of maturities (For further details, see Schich (1997)). This means that these yields are the fitted values of a one-factor model. In addition, there is no analogue to short-term government securities in Germany, as there are no Treasury bills. In an estimation with yields of different maturities this would imply that government bonds would have to be combined with short-term money market rates. Unless collateralized money-market rates are used, such as repo rates, these two rates may have entirely different risk characteristics.

earlier in the business week. The end of quarters does not have an impact in addition to the one of the end of a year. This means that seasonal patterns at the very short end of the yield curve do exist; they add to the fat tails of yield changes and can not, therefore, be ignored in an estimation with daily data.

Unemployment and inflation releases occur on different days and are made by different government agencies. The release calendar is published one year in advance by the Federal Statistical Office (Statistisches Bundesamt), from where it was obtained. Unemployment releases are made by the Federal Labour Office (Bundesanstalt für Arbeit), a federal agency subordinated to the Federal Ministry of Labor, at 10 a.m. on its release date, which is approximately 10 days after the end of the reference period.[18] Consumer prices are released by the Federal Statistical Office typically two weeks after the end of the reference period at 8 a.m. In addition, a tentative inflation number is released a few days prior to the end of the reference period. This estimate is based on calculations using price information from four regions in Western Germany. In order to determine which of these two releases provides more information about inflation, it is important to assess the quality of the preliminary estimate by the Federal Statistical Office. Figure 4 plots these preliminary estimates together with the final inflation release during the time period for which data is freely available: May 1995 to November 1998, a period covering 36 months. Inflation rates are reported as monthly percentage changes (not annualized). It can be seen that the forecasts never went in the wrong direction, but errors were made regarding the extent of the change. Considering the fact that average inflation over one month was 0.11 percent during this period with a standard deviation of 0.23 percent, the mean absolute error of the forecasts of 0.047 percentage points does not seem high (see Table 4).

Thus, it can be concluded that the first release does provide considerable information about actual inflation. Our convention will therefore be that inflation is released at the first release day around the end of each month, and unemployment is released after that. The data on inflation and unemployment were obtained from Datastream.

[18] The unemployment release day coincides with the day the Federal Statistical Office releases employment information. However, employment releases have a different reference period as employment figures are released with a lag of 3 months. This means that January employment is released together with March unemployment on the 10th of April.

Estimation Procedure

Fixing a parameter vector α, the yield $Y(t,T)$ at time t of maturity date T can be written as an affine function[19] of $X(t)$. With a finite-dimensional state space D, measurement errors are needed to break the singularity that otherwise arises in the model. Yields are therefore assumed to be observed with an error ε_M, independent of X, so that we can write

$$Y(t,T) = \frac{-\ln \psi(X(t), c_\alpha(t,T))}{T-t} + \varepsilon_M(t), \tag{3.6}$$

where the coefficients $c_\alpha(t,T)$ are calculated recursively, using Proposition 1, given the parameter vector α.

Even in the case of only one spread factor ($I = 1$), not all components of X are directly observable from (3.3). Because of the monetary policy shock, it is not possible to distinguish between s and r^* by observing the short rate, so that there are two unobservable factors. With more than one spread factor ($I > 1$), it is not feasible to separate the spread into its different driving forces $\widetilde{s}_i$ from short-rate information only, so that there are $I+1$ unobservable factors. Following Chen and Scott (1993) and others, we therefore assume that $I+1$ yields are observed without measurement error, so that otherwise unobservable factors may be recovered by inverting the map from factors to yields given assumed parameters. In the present setting, a complicating aspect arises as one of the unobservable factors, r^*, is a pure-jump process which changes only on meeting days. When $I+1$ yields are observed without error, the model is singular when using yield observations on two consecutive business days. This is resolved by using information on the $(I+1)$-th yield exclusively at meeting days.

The likelihood function of yields and macroeconomic variables can be obtained through a change of variables from the conditional density of X multiplied by the absolute value of the determinant of the Jacobian of the transformation (3.6) with respect to Y. The $\mathcal{F}(t)$-conditional density of $X(t')$ for $t' > t$, is the product of the densities of $\widetilde{s}(t')$ conditional on $\widetilde{s}(t)$ and the joint density of $(r^*(t'), \pi(t'), u(t'), \pi_L(t'))$

[19] In the notation of section (3), the coefficient c_3 is not needed.

conditional on $(r^*(t), \pi(t), u(t), \pi_L(t))$. As already mentioned, the density of each spread factor $\widetilde{s}_i(t')$ conditional on $\widetilde{s}_i(t)$ is a noncentral chi-square, and the $\widetilde{s}_i$ are independent. The second density depends on whether time t' denotes a meeting day, release day of inflation or unemployment, or any combination of these 'special' days.

Bundesbank meeting days and the release days of macroeconomic information introduce additional seasonal patterns that makes yields nonstationary. In particular, the coefficients $c_\alpha(t,T)$ of the transformation (3.6) that maps X into yields depends not only on the time-to-maturity $T-t$, as in most yield curve models in the literature (such as Chen and Scott (1993), Duffie and Singleton (1997), Dai and Singleton (2000)), but also on time t and maturity T separately.[20] This means that, for each observation time t, the yield curve coefficients must be computed recursively, which is computationally costly even though the coefficients have closed-form solutions between any two 'special' days.

The estimation is performed using the overnight, 1-month, 6-month and 12-month yields. The number of spread factors is set equal to 1, referred to as the *1-spread model* ($I = 1$), and equal to 2, the *2-spread model* ($I = 2$). In the case of $I = 1$, the overnight and 1-month rate are assumed to be observed with measurement error. When $I = 2$, only the overnight rate is measured with error. The fat tails of the short-term yields are allowed to be generated at least in part by measurement error. This means that measurement errors are not assumed to be normally distributed. Instead, for each $\varepsilon_t \equiv \varepsilon_M(t)$ and day $\Delta = 1/365$, the Bernoulli-Gaussian mixture distribution

$$f(\varepsilon_t|\varepsilon_{t-\Delta}) = p(t)\phi(\rho_m\varepsilon_{t-\Delta}, \sigma^2_{m,a}) + (1-p(t))\phi(\rho_m\varepsilon_{t-\Delta}, \sigma^2_{m,b}), \tag{3.7}$$

is used. Here, $\phi(m_1, m_2)$ denotes the density of a Gaussian random variable with mean m_1 and variance m_2; $\sigma^2_{m,a}$ and $\sigma^2_{m,b}$ are variance parameters; ρ_m is an autocorrelation coefficient, and $p(t)$ is the probability of mixing the two Gaussian densities. The mixing probability $p(t)$ is allowed to vary according to whether t is a normal day or

[20] Another model with this feature is Balduzzi, Bertola, and Foresi (1996). In their discrete time model, nonstationarity is introduced through dummies for the biweekly U.S. reserve maintenance cycle. However, the estimation of the model only uses short rate information so that the computation of time-dependent coefficients is not a problem in this case.

a ‘special’ day. In order to economize on parameters, the probability $p(t)$ may only take two different values. If t is a normal business day, then $p(t) = p_1$. If t is the day of or before the end of a maintenance period or the day during the business week before the end of a year, then $p(t)$ is set to p_2. In order to capture the seasonality of short term interest rates, a much richer parametrization of this mixture distribution can easily be designed. This might be seen as only a first step towards adjusting for seasonal effects in measurement errors.

For any given set of parameters, the computing time to evaluate the likelihood function of the model with two spread factors takes 2.32 minutes on a Sun UltraSparc machine. With 29 parameters, a gradient based numerical maximization method is costly. The numerical optimization of the likelihood function uses a combination of a simplex method and a gradient-based method.[21]

3.1.5 General Performance of the Model

Yields are clearly driven by more factors than just inflation and unemployment. This means that the performance of the model depends on the number of unobservable spread factors. The approach taken here is to economize on the number of unobservable factors, but admitting as many unobservable factors into the specification that seem to be needed to avoid obvious mispricing. In a first step, a 1-spread model with the original Taylor rule was estimated. The fitting errors, that is, the differences between yields and their fitted counterparts, were larger than 1 percentage point on average over extended periods of time. The fitted values of longer yields were essentially step-functions, whose step occurred at release days of macroeconomic information. Thus, release days tended to be far too important in this setup, and the standard deviation of changes in longer fitted yields on normal business days was far too low.[22] Therefore, the model was generalized in two directions: (i) the 1-spread

[21] The author thanks Bo Honoré and Ekaterini Kyriazidou for their ‘amoeba’ procedure in Gauss.

[22] First estimation attempts matched the overnight rate without measurement error and the spread factor was assumed to jump to zero on Bundesbank meeting days. Seasonal effects for the maintenance period were introduced through time-varying coefficients in the spread factor. The maximum-likelihood estimate of the mean-reversion parameter κ_1 turned out to be extremely high, so that the resulting modeled yield curve was flat and exclusively driven by the macro factors. This feature

model was estimated with the extended Taylor rule, and (ii) the original Taylor rule was combined with two spread factors. These two models are not nested, so simple model comparisons such as likelihood ratio tests do not apply. The estimation results are reported in Tables 5 and 6.

The ML estimates of the 1-spread specification show high persistence. This means that changes in the spread have an almost equal impact on short and long term yields. The plot of the standardized value of $\widetilde{s}_1$ evaluated at the ML estimate $\widehat{\alpha}$ in Figure 6 confirms that this factor is a *level factor*. The correlation between changes in $\widetilde{s}_1(\widehat{\alpha})$ and changes in yields indicate that the spread factor corresponds to the level of the 12-month yield. The implied factors $\widetilde{s}_1(\widehat{\alpha})$ and $\widetilde{s}_2(\widehat{\alpha})$ from the 2-spread model are graphed in Figure 7. From the graph, it is apparent that the first spread variable takes the role of a level factor. Again, this factor is taken from the 12-month yield. The second spread factor essentially represents a *slope factor* since the correlation of changes in $\widetilde{s}_2(\widehat{\alpha})$ with changes in the difference between the 12-month and the 1-month is -0.25, while its correlation with changes in the 1-month yield (12-month yield) is only 0.08 (-0.21).

As the yield curve model imposes cross-equation restrictions on the parameters describing the dynamics of the factors, it is interesting to compare the ML estimates of the parameters of the macroeconomic variables with those obtained from a traditional VAR. Before turning to these estimates, it is worth noting that both inflation and unemployment are very persistent. In fact, for both variables one fails to reject the null of a unit root in an augmented Dickey-Fuller test. The Dickey–Fuller test statistic for inflation is -2.09 with a 10% critical value of -2.58; the respective numbers for unemployment are -1.42 and -2.58. The sample of monthly macro variables consists of 192 observations and, in small samples, the power of unit-root tests is well-known to be low. It is clear, however, that both variables are persistent which is reflected in Table 9 by the estimates of the autoregressive matrix, the largest eigenvalue of which has a modulus of 0.997. The corresponding numbers for the 1-spread and 2-spread

barely changed when the spread was allowed to continue to 'survive' the meeting days. Even with additional (seasonal) spread factors, high mean reversion continued to be a problem. Subsequently, the overnight rate was taken to be observed with seasonal measurement error. This greatly improved the overall fit of the model, even with only one spread factor.

models are only 0.98 and 0.89, implying that the estimated joint model of yields and macro variables is characterized by less persistence.[23] The volatility estimates in all models look qualitatively similar. In particular, the conditional covariance parameter of inflation and unemployment is positive in all cases.

Both models assume that there are no measurement errors regarding the 12-month rate. The 2-spread model also assumes this for the 1-month rate. In other words, the models match the respective yields exactly. It is then interesting to see how the models perform in explaining other yields. Table 7 and 8 show summary statistics of the implied yields by both models. The average fitting errors reported in Panel A regarding the 1, 3 and 6-month yield in the 1-spread model do not exceed 1 basis point. The overnight and 7-day yield are priced less accurately due to the seasonal adjustment in measurement errors.

Both yield-curve models clearly fail to match the high kurtosis of changes in the overnight and the 7-day yield. This may be in part a result implied by the measurement-error distribution. Both of these yields are measured with a seasonal error characterized by a density which is estimated to be the mixture of a low volatility density which has a very large probability on normal business days (over 99% in both models) and a high volatility density that is in effect on 'special' days with a probability (again, over 99%). The kurtosis of longer yields is matched far better.

3.1.6 Results regarding Monetary Policy and Releases

In order to infer the policy rule of a central bank, the VAR literature typically regresses overnight rates on other variables and makes assumptions that identify the error term in this regression with the monetary policy shock. We will compare the policy rule found by estimating the joint model of yields and macro variables with a conventional one. Table 10 reports the results of regressing the overnight rate on inflation and unemployment. This regression can be interpreted in a VAR setting by assuming that the variables included in the Taylor rule do not depend on lagged values of the short rate. The original Taylor rule in this estimation explains 91.35% of

[23]The high persistence also explains the negative long-run mean of -0.82% of inflation implied by the VAR(1) specification in Table 9. The sample mean of inflation is 2.44%.

variations in the short rate. The Bundesbank raises the short rate by 52 basis points for every percentage point increase in inflation and decreases it by 120 basis points for every percentage point increase in unemployment. Thus, inflation and unemployment have large contemporaneous impacts on interest rates, while their effect on the future short rate is entirely determined by the persistence of the macroeconomic variables themselves. The extended Taylor rule regression has an R^2 of over 99%. Here, macroeconomic variables have a much smaller, but still significant contemporaneous impact on the short rate. A one percentage point increase in inflation (unemployment) leads in expectation to a 3.5 (-5.6) basis-point increase in the short rate. This effect is perpetuated not only through the persistence of the macroeconomic variables themselves, but also through the lagged short rate. This means that while the initial effect of macroeconomic variables differs in these two specifications, their effect on short rates in the medium and long run may be equally important.

These conclusions can now be compared with those obtained from our joint model of yields and macroeconomic variables. Relative to the least-squares regression of the extended Taylor rule, the 1-spread model has a smaller estimated autoregressive coefficient β_3 (0.90 versus 0.95) and a comparable response coefficient on unemployment β_2 (-0.061 versus -0.056). Both these coefficients are significant at conventional confidence levels. The estimated coefficient on inflation, β_1, is surprisingly negative and significant. However, its value is tiny (-0.0038). The estimated response coefficients β_2 and β_3 in the 2-spread model are both smaller than in the simple OLS regression. A one-percentage point increase in inflation (unemployment) increases (decreases) the target rate by 14 basis points (21 basis points) in expectation.

Figures 8 and 9 graph the target rates associated with the 1-spread and 2-spread model together with the discount and lombard rates. As the long-run mean of the first spread factor in both models is imprecisely estimated, the target rate computed at the ML estimates takes on extreme values in both models. The coefficients of the spreads are therefore set equal to their sample averages to recover the target rate for the graphs. The target rate of the 2-spread model looks too flat, while the target rate in the 1-spread model can be seen to commove with the lombard rate on certain meeting days. A more precise statement regarding the plausibility of the target rates can be made by referring to Table 11, which shows the correlations of the two rates with the discount and lombard rate. Panel A calculates these correlations by including only observations for which either the discount or lombard rate moved, while Panel B conditions on meeting days. The correlation between the target rate from the 1-spread model, and the discount and lombard rate is 0.38 and 0.21, respectively. The target rate from the 2-spread model has a negative correlation with lombard rate. Thus, the target rate implied by the 2-spread model does not seem to capture an unobservable target.

Table 12 shows the results from regressing changes in fitted and actual yields on the factors of the model at meeting days, release days of inflation and release days of unemployment. While the impact of a change in the target rate decreases with maturity for the yields in the model, the actual yield curve seem to be shifted by around 22 basis points for every 100 basis point move in the desired rate. The term structure of unemployment release effects is increasing. The effect of inflation releases has the 'wrong' sign: increases in inflation lower yields significantly, both in the data and in the model.

3.2 Conclusion

This chapter has estimated a special case of the linear-quadratic bond model with German interest-rate data. The estimated yield-curve model shows that the behavior of the Bundesbank (before the introduction of the Euro) is characterized by an interest-rate smoothing motive. In addition, the Bundesbank responds to unemployment and less so to inflation. In anticipation of policy actions at future Bundesbank meetings, bond yields show different reactions to releases of unemployment and inflation according to the meeting calendar that underlies their life span. In the tradition of the Bundesbank, the Board of the European Central Bank has started bi-weekly meetings. Future research will determine whether the yield-curve model presented in this paper also fits Euro-bond yields and whether it provides a good description of ECB policy.

Table 1.
Summary Statistics of Yields

Panel A: Levels

Maturity	Mean	small St.Dev.	Skewness	Kurtosis	Autocorrelation
Overnight	4.0735	1.4352	0.4586	2.0173	0.99015
	(0.0596)	(0.0289)	(0.0664)	(0.1195)	(0.00454)
7 Days	4.0985	1.4355	0.4506	1.9677	0.99606
	(0.0597)	(0.0290)	(0.0648)	(0.0967)	(0.00164)
1 Month	4.1257	1.4440	0.4420	1.9480	0.99867
	(0.0602)	(0.0292)	(0.0649)	(0.0955)	(0.00122)
3 Months	4.1550	1.4381	0.4191	1.9390	0.99899
	(0.0600)	(0.0290)	(0.0638)	(0.0931)	(0.00117)
6 Months	4.1738	1.4156	0.3966	1.9481	0.99899
	(0.0591)	(0.0287)	(0.0624)	(0.0911)	(0.00116)
12 Months	4.1727	1.3529	0.3909	1.9852	0.99903
	(0.0564)	(0.0278)	(0.0605)	(0.0925)	(0.00117)

Panel B: Differences

Maturity	Mean	St.Dev.	Skewness	Kurtosis	Autocorrelation
Overnight	-0.0001	0.1971	-0.3577	426.6437	-0.3458
	(0.0017)	(0.0440)	(3.6775)	(63.7765)	(0.0781)
7 Days	-0.0012	0.1219	1.0348	130.1512	-0.0469
	(0.0014)	(0.0139)	(2.2890)	(40.9277)	(0.0364)
1 Month	-0.0012	0.0642	-0.5459	24.6674	-0.1570
	(0.0008)	(0.0034)	(0.4551)	(3.6553)	(0.0350)
3 Months	-0.0012	0.0509	-0.1150	11.1383	-0.1172
	(0.0007)	(0.0018)	(0.2194)	(1.3226)	(0.0295)
6 Months	-0.0011	0.0506	-0.4962	14.8008	-0.1465
	(0.0006)	(0.0020)	(0.2346)	(2.4584)	(0.0319)
12 Months	-0.0011	0.0464	-0.3361	10.8005	0.0795
	(0.0007)	(0.0014)	(0.2429)	(1.2917)	(0.0210)

NOTE: The sample used for this table consists of daily data on continuously compounded Euro-DM rates over the period January 1, 1982 to December 31, 1997. Standard Errors are in parenthesis and are estimated with Generalized Method of Moments using 6 Newey-West Lags. Let m_i denote the i-central moment. Then the Mean is m_1, the standard deviation (Std.Dev.) is $m_2^{1/2}$, Skewness is $m_3/m_2^{3/2}$ and Kurtosis is m_4/m_2^2. For a Gaussian random variable, the skewness is 0 and the kurtosis is 3.

Table 2.

Jarque-Bera Test Statistics

	Overnight	7 Days	1 Month	3 Months	6 Months	12 Months
Jarque-Bera	302.04	313.86	315.86	306.89	292.78	280.30

NOTE: For a description of the data see Table 1. Jarque-Bera is a test statistic for testing whether the process is Gaussian. It is chi-square distributed with 2 degrees of freedom. As the data consists of 4015 observations, the relevant 5% (1%) quantile is 5.99 (9.21).

Table 3. Seasonal Pattern in Yield Changes

	Overnight	7 Days	1 Months	3 Months	6 Months	12 Months
Constant	0.0626*	0.0410*	0.0323*	0.0289*	0.0291*	0.0275*
	(0.0032)	(0.0018)	(0.0013)	(0.0011)	(0.0011)	(0.0009)
Maintenance						
t	0.0448*	0.0109*	0.0189*	0.0001	-0.0007	0.0018
	(0.0159)	(0.0061)	(0.0076)	(0.0040)	(0.0050)	(0.0042)
$t+1$	0.0117	0.0110*	-0.0012	-0.0043	-0.0077*	-0.0020
	(0.0096)	(0.0055)	(0.0044)	(0.0030)	(0.0027)	(0.0040)
$t+2$	0.0018	0.0042	-0.0033	-0.0037	-0.0053	-0.0052
	(0.0097)	(0.0055)	(0.0041)	(0.0036)	(0.0031)	(0.0028)
$t+3$	-0.0082	-0.0057	-0.0009	0.0012	-0.0014	-0.0001
	(0.0077)	(0.0055)	(0.0046)	(0.0042)	(0.0037)	(0.0036)
$t+4$	-0.0073	0.0019	-0.0043	-0.0041	-0.0028	-0.0015
	(0.0073)	(0.0080)	(0.0031)	(0.0029)	(0.0032)	(0.0031)
Quarter						
t	0.0343	0.0143	-0.0118	0.0204*	0.0017	-0.0107
	(0.0300)	(0.0142)	(0.0095)	(0.0092)	(0.0074)	(0.0060)
$t+1$	0.0047	0.0102	0.0043	0.0012	0.0018	-0.0080
	(0.0216)	(0.0154)	(0.0101)	(0.0072)	(0.0062)	(0.0058)
$t+2$	-0.0072	-0.0053	0.0009	-0.0004	0.0008	0.0070
	(0.0148)	(0.0106)	(0.0064)	(0.0063)	(0.0056)	(0.0053)
$t+3$	0.0413	0.0309	0.0092	-0.0062	0.0043	0.0100
	(0.0246)	(0.0208)	(0.0082)	(0.0065)	(0.0066)	(0.0094)
$t+4$	0.0169	0.0196	0.0066	0.0042	0.0064	0.0054
	(0.0162)	(0.0197)	(0.0077)	(0.0061)	(0.0062)	(0.0065)
Year						
t	0.7499*	0.6175*	0.1552*	0.0561*	0.0259*	0.0043
	(0.3823)	(0.1678)	(0.0514)	(0.0187)	(0.0115)	(0.0100)
$t+1$	0.2903*	0.2131*	0.0706*	0.0360*	0.0271	0.0080
	(0.1403)	(0.0530)	(0.0250)	(0.0175)	(0.0147)	(0.0106)
$t+2$	0.0086	0.0713	0.0117*	-0.0079	-0.0124	-0.0056
	(0.0462)	(0.0482)	(0.0167)	(0.0087)	(0.0070)	(0.0082)
$t+3$	0.0477	0.4228*	0.0412	0.0032	-0.0034	-0.0038
	(0.0342)	(0.1822)	(0.0326)	(0.0124)	(0.0076)	(0.0101)
$t+4$	0.1033*	0.2378*	0.0468*	0.0245	0.0064	-0.0040
	(0.0344)	(0.0829)	(0.0194)	(0.0137)	(0.0112)	(0.0078)

NOTE: This Table reports the least squares estimates of regressing the absolute value of the change in different yields on a constant and a number of dummies. The dummies included in the regression are the ones listed in the Table; "Maintenance $t+j$" is a dummy which is equal to 1 at time t if $t+j$ is the end of a reserve maintenance period. Analogue definitions apply to "Quarter" and "Year". "Maintenance" includes the end of a year, while "Quarter" does not. Newey-West standard errors are reported in parethesis and are computed with 9 lags. Stars indicate significance at the 5% level. For a description of the data, see Table 1.

Table 4.

Evaluation of the Federal Statistical Office's Preliminary Inflation Release

RMSE	0.0913
MAE	0.0472
MAPE	0.2484

NOTE: The sample used for this table is May 1995 to November 1998. The data consists of the preliminary inflation releases by the German Federal Statistical Office (Statistisches Bundesamt) at the end of each month and the actual value of monthly inflation released around the middle of the next month. The data was obtained from the Federal Statistical Office (Statistisches Bundesamt). The table reports the Root Mean Squared Error (RMSE), the Mean Absolute Error (MAE) and the Mean Absolute Percentage Error (MAPE) of the preliminary releases.

Table 5.
Maximum Likelihood Estimation of the 1-Spread Factor Model
(Extended Taylor Rule)

Parameter	Estimate	St. Error
κ_1	0.0163	0.0834
θ_1	0.0860	0.4334
σ_1	0.0510	0.0013
a_π	0.0134	0.0018
a_u	0.0010	0.0003
$a_{\pi\pi}$	0.9715	0.0175
$a_{\pi u}$	-0.1288	0.0171
$a_{u\pi}$	0.0167	0.0025
a_{uu}	0.9813	0.0030
$\sigma_{\pi\pi}$	0.0040	0.0021
$\sigma_{u\pi}$	0.00009	0.00003
σ_{uu}	0.0012	0.00007
β_0	0.0149	0.0410
β_1	-0.0038	0.0011
β_2	-0.0610	0.0023
β_3	0.9052	0.0015
σ_*	0.0025	0.00009
b_{B_1}	-2.7916	1.6329
p_1	0.00001	0.0010
p_2	0.9999	0.0007
ρ_{1day}	0.7490	0.0049
$\sigma_{1day,a}$	0.0016	0.00002
$\sigma_{1day,b}$	0.0039	0.0007
ρ_{1mth}	0.9195	0.0068
$\sigma_{1mth,a}$	0.0007	0.000007
$\sigma_{1mth,b}$	0.0010	0.00002

NOTE: This Table reports the estimation results of the model described by equations (3.1), (3.4), (3.5) and (3.7). ρ_{1day}, $\sigma_{1day,a}$and $\sigma_{1day,b}$ denote the parameters describing the dynamics of the measurement error of the overnight rate; ρ_{1mth}, $\sigma_{1mth,a}$ and $\sigma_{1mth,b}$ denote the respective parameters of the 1 month rate.

Table 6.

Maximum Likelihood Estimation of the 2-Spread Factor Model (Original Taylor Rule)

Parameter	Estimate	St.Error
κ_1	0.0028	0.0108
θ_1	0.1292	0.1436
σ_1	0.0265	0.0038
κ_2	2.8112	0.0315
θ_2	0.0930	0.0136
σ_2	0.0848	0.0065
a_π	0.0270	0.0013
a_u	0.0055	0.0007
$a_{\pi\pi}$	0.8513	0.0079
$a_{\pi u}$	-0.2706	0.0140
$a_{u\pi}$	0.0521	0.0060
a_{uu}	0.9310	0.0075
$\sigma_{\pi\pi}$	0.0039	0.0002
$\sigma_{u\pi}$	0.0002	0.00003
σ_{uu}	0.0017	0.0001
β_0	-0.2370	0.1363
β_1	0.1367	0.0112
β_2	-0.2142	0.0234
σ_*	0.0048	0.0002
b_{B_1}	-0.6270	0.2064
b_{B_2}	-0.1145	0.2734
p_1	0.00002	0.0094
p_2	0.9920	0.0027
ρ_{1day}	0.7794	0.0051
$\sigma_{1day,a}$	0.0020	0.00004
$\sigma_{1day,b}$	0.0041	0.0004

NOTE: This Table reports the estimation results of the model described by equations (3.1), (3.5), (3.4) and (3.7). ρ_{1day}, $\sigma_{1day,a}$and $\sigma_{1day,b}$ denote the parameters describing the dynamics of the measurement error of the overnight rate, note that there is no measurement error associated with the 1 month rate in this specification.

Table 7.

Summary Statistics of Fitted Yields from the 1-Spread Factor Model

Panel A: Levels

Maturity	Mean	St.Dev.	Skewness	Kurtosis	Autocorrelation
Overnight	4.1138	1.4973	0.4273	1.8779	0.99862
	(0.0624)	(0.0291)	(0.0661)	(0.0904)	(0.0012)
7 Days	4.1184	1.4935	0.4259	1.8805	0.99868
	(0.0623)	(0.0291)	(0.0659)	(0.0904)	(0.0012)
1 Month	4.1312	1.4785	0.4203	1.8899	0.99876
	(0.0617)	(0.0290)	(0.0654)	(0.0904)	(0.00121)
3 Months	4.1551	1.4472	0.4092	1.9166	0.99895
	(0.0604)	(0.0288)	(0.0640)	(0.0908)	(0.0012)
6 Months	4.1719	1.4111	0.4002	1.9520	0.99907
	(0.0589)	(0.0287)	(0.0624)	(0.0917)	(0.0012)

Panel B: Differences

Maturity	Mean	St.Dev.	Skewness	Kurtosis	Autocorrelation
Overnight	-0.0012	0.0680	0.2720	36.0575	-0.0139
	(0.0010)	(0.0034)	(1.0138)	(8.1843)	(0.0197)
7 Days	-0.0012	0.0656	0.0362	32.5645	-0.00002
	(0.0010)	(0.0031)	(0.9397)	(7.5416)	(0.0202)
1 Month	-0.0012	0.0619	0.0387	27.7948	-0.0178
	(0.0009)	(0.0027)	(0.8035)	(6.4019)	(0.0205)
3 Months	-0.0012	0.0547	-0.1168	18.2627	-0.0321
	(0.0008)	(0.0020)	(0.5064)	(3.8872)	(0.0219)
6 Months	-0.0011	0.0474	-0.2482	11.0222	-0.0550
	(0.0007)	(0.0015)	(0.2206)	(1.2473)	(0.0218)

NOTE: The 12 month yield is omitted as it is fitted without error in the 1-spread model.

Table 8.

Summary Statistics of Fitted Yields from the 2-Spread Model

Panel A: Levels

Maturity	Mean	St.Dev.	Skewness	Kurtosis	Autocorrelation
Overnight	4.0945	1.4590	0.4058	2.1477	0.99403
	(0.0601)	(0.0320)	(0.0618)	(0.0970)	(0.0017)
7 Days	4.1033	1.4493	0.4293	2.0846	0.99672
	(0.0603)	(0.0311)	(0.0624)	(0.0973)	(0.0016)
3 Months	4.1698	1.4116	0.4043	1.9631	0.99910
	(0.0589)	(0.0288)	(0.0623)	(0.0929)	(0.0016)
6 Months	4.1727	1.3529	0.3909	1.9852	0.99903
	(0.0564)	(0.0288)	(0.0623)	(0.0929)	(0.0012)

Panel B: Differences

Maturity	Mean	St.Dev.	Skewness	Kurtosis	Autocorrelation
Overnight	-0.0011	0.1546	-0.1480	38.8912	-0.08396
	(0.0021)	(0.0078)	(0.9132)	(6.333)	(0.0159)
7 Days	-0.0011	0.1106	0.6059	24.6338	0.05418
	(0.0018)	(0.0048)	(0.7358)	(7.6304)	(0.0270)
3 Months	-0.0011	0.0464	-0.4888	9.4176	-0.03561
	(0.0007)	(0.0014)	(0.2054)	(0.9326)	(0.0205)
6 Months	-0.0010	0.0464	-0.3361	10.8005	-0.07945
	(0.0007)	(0.0014)	(0.2429)	(1.2917)	(0.0210)

NOTE: The 1 and 12 month yields are omitted as they are both fitted without error in the 2-spread model.

Table 9.

Panel A.
Macroeconomic Variables in a VAR

Parameter	Estimate	St.Error
a_π	0.0059	0.0019
a_u	-0.0027	0.0007
$a_{\pi\pi}$	0.9338	0.0162
$a_{\pi u}$	-0.0525	0.0197
$a_{u\pi}$	0.0329	0.0050
a_{uu}	1.0242	0.0074
$\sigma_{\pi\pi}$	0.0028	0.00009
$\sigma_{u\pi}$	0.00003	0.00009
σ_{uu}	0.0009	0.00004

Panel B.

Model Selection Criteria for Higher Order VAR

	Order 1	Order 2	Order 3	Order 4	Order 5	Order 6
AIC	-1.5609	-1.6803	-1.7409	-1.8068	-1.8164	-1.7771
SC	-1.4587	-1.5094	-1.5008	-1.4969	-1.4363	-1.3261

NOTE: Panel B reports the Akaike (AIC) and Schwarz (SC) criterion for different orders of VARs in inflation and unemployment. The VAR(1) is the specification used for Panel A.

Table 10.
Estimation using Short Rate Information Only

Panel A: Original Taylor Rule

Parameter	Estimate	St.Error
β_0	0.1482	0.2899
β_1	0.5182	0.0277
β_2	-1.1962	0.0279
R^2_{adj}	0.9135	

Panel B: Extended Taylor Rule

Parameter	Estimate	St.Error
β_0	0.6933	0.2614
β_1	0.0345	0.0124
β_2	-0.0561	0.0213
β_3	0.9452	0.0172
R^2_{adj}	0.9935	

NOTE: In this Table, the extended Taylor rule is obtained by regressing the overnight rate on inflation, unemployment and the lagged overnight rate at meeting days

$$r_t = \beta_0 + \beta_1 \pi_t + \beta_2 u_t + \beta_3 r_{t-1} + \varepsilon_t \ ;$$

in the original Taylor rule estimation, the lagged overnight rate is omitted. Standard Errors are obtained using 5 Newey-West lags.

Table 11.
Correlations of Target Rate with Discount and Lombard Rates

Panel A.
Conditioning on Policy Rate Changes

	r^* in 1SM	r^* in 2SM	Lombard	Discount
r^* in 1SM	1	-0.7158	0.3787	0.2113
r^* in 2SM	-0.7158	1	0.0411	0.0796
Lombard	0.3787	0.0411	1	0.7494
Discount	0.2113	0.0796	0.7494	1

Panel B.
Conditioning on Meeting Days

	r^* in 1SM	r^* in 2SM	Lombard	Discount
r^* in 1SM	1	-0.7191	0.1512	0.0892
r^* in 2SM	-0.7191	1	-0.0015	0.0129
Lombard	0.1512	-0.0015	1	0.7500
Discount	0.0892	0.0129	0.7500	1

Table 12.

Reactions of Yields to Changes in the Target, Inflation and Unemployment implied by the 1-Spread Model

	Meeting Days: r^*		Release Days: π		Release Days: u	
	Model	Data	Model	Data	Model	Data
Overnight	1.0000	0.2240	0	-0.0588	0	-0.1294
	-	(0.0617)	-	(0.0322)	-	(0.0679)
7 Days	0.7989	0.2554	-0.0019	-0.0602	0.0323	-0.1636
	(0.0233)	(0.0586)	(0.0023)	(0.0630)	(0.0307)	(0.0562)
1 Month	0.7366	0.2625	-0.0044	-0.0250	-0.0046	-0.1513
	(0.0240)	(0.0409)	(0.0022)	(0.0256)	(0.0310)	(0.0486)
3 Months	(0.6103)	0.2242	-0.0131	-0.0500	-0.0969	-0.1631
	(0.0216)	(0.0390)	(0.0020)	(0.0187)	(0.0278)	(0.0391)
6 Months	0.4704	0.2370	-0.0260	-0.0686	-0.1945	-0.1903
	(0.0169)	(0.0305)	(0.0018)	(0.0206)	(0.0231)	(0.0381)
1 Year	0.3027	0.3027	-0.0485	-0.0485	-0.2889	-0.2889
	(0.0113)	(0.0113)	(0.0022)	(0.0022)	(0.0180)	(0.0180)

NOTE: This Table reports least squares estimates from regressing changes in model implied yields and actual yields on a constant and changes in all factors of the 1-spread model at (i) meeting days, (ii) release days of inflation and (iii) release days of unemployment. The coefficient reported is the coefficient on changes in the (i) target rate, (ii) released inflation and (iii) released unemployment. Standard Errors are computed using 6 Newey-West Lags and are reported in parenthesis.

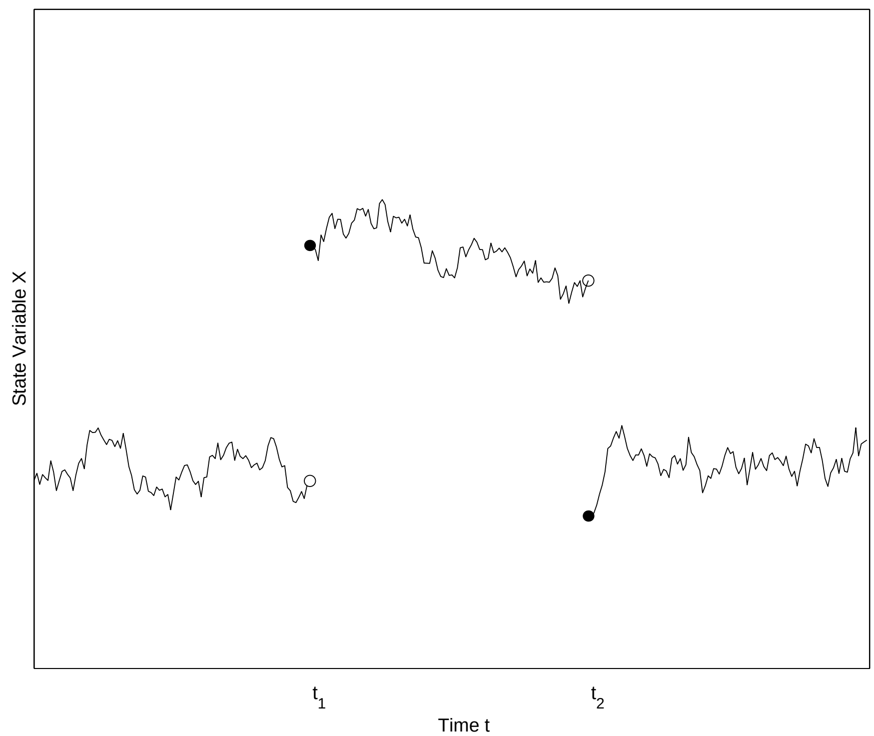


Figure 3.1: Discontinuous Moves in the State Variable X at t_1 and t_2.

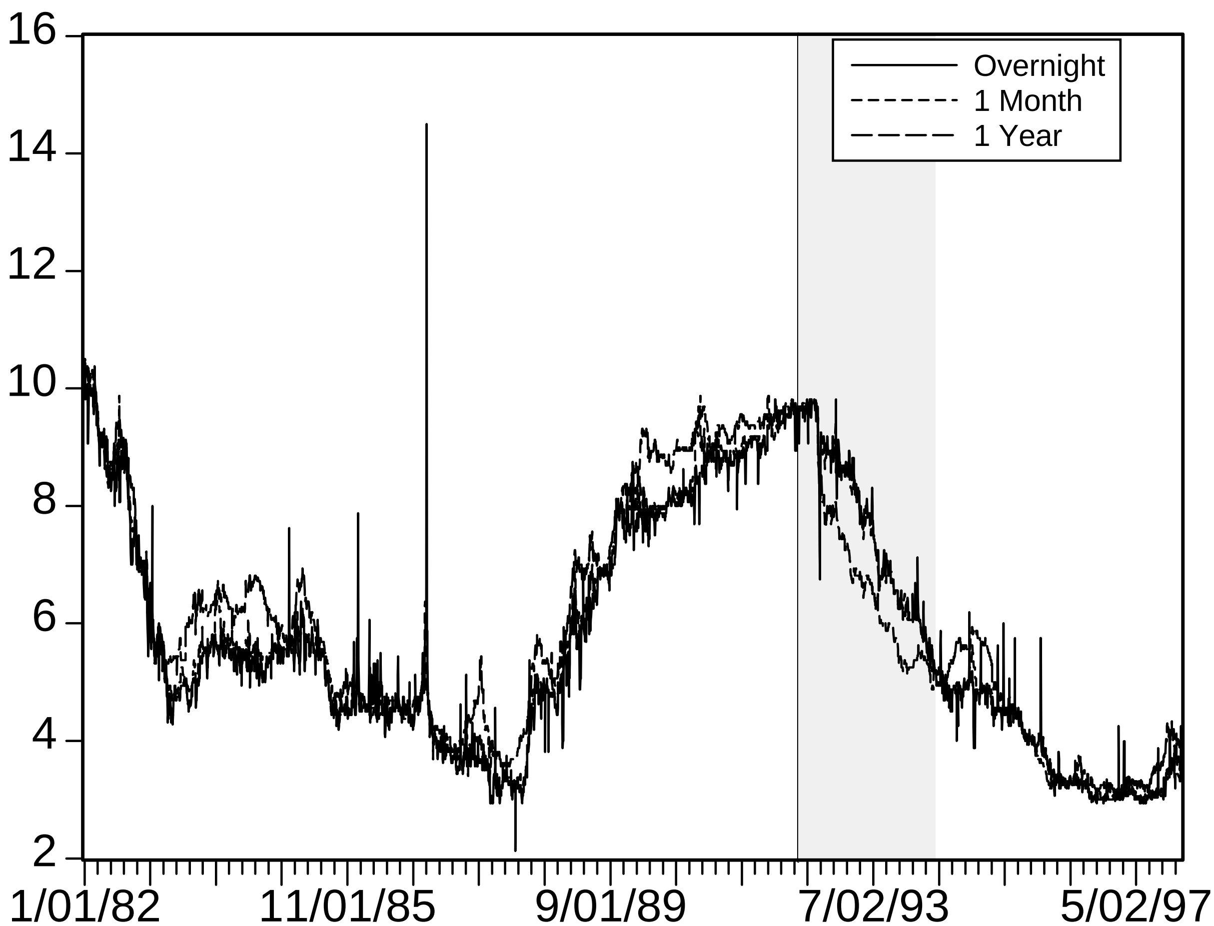


Figure 3.2: Term Structure of Euro Rates from January 1982 to December 1997.

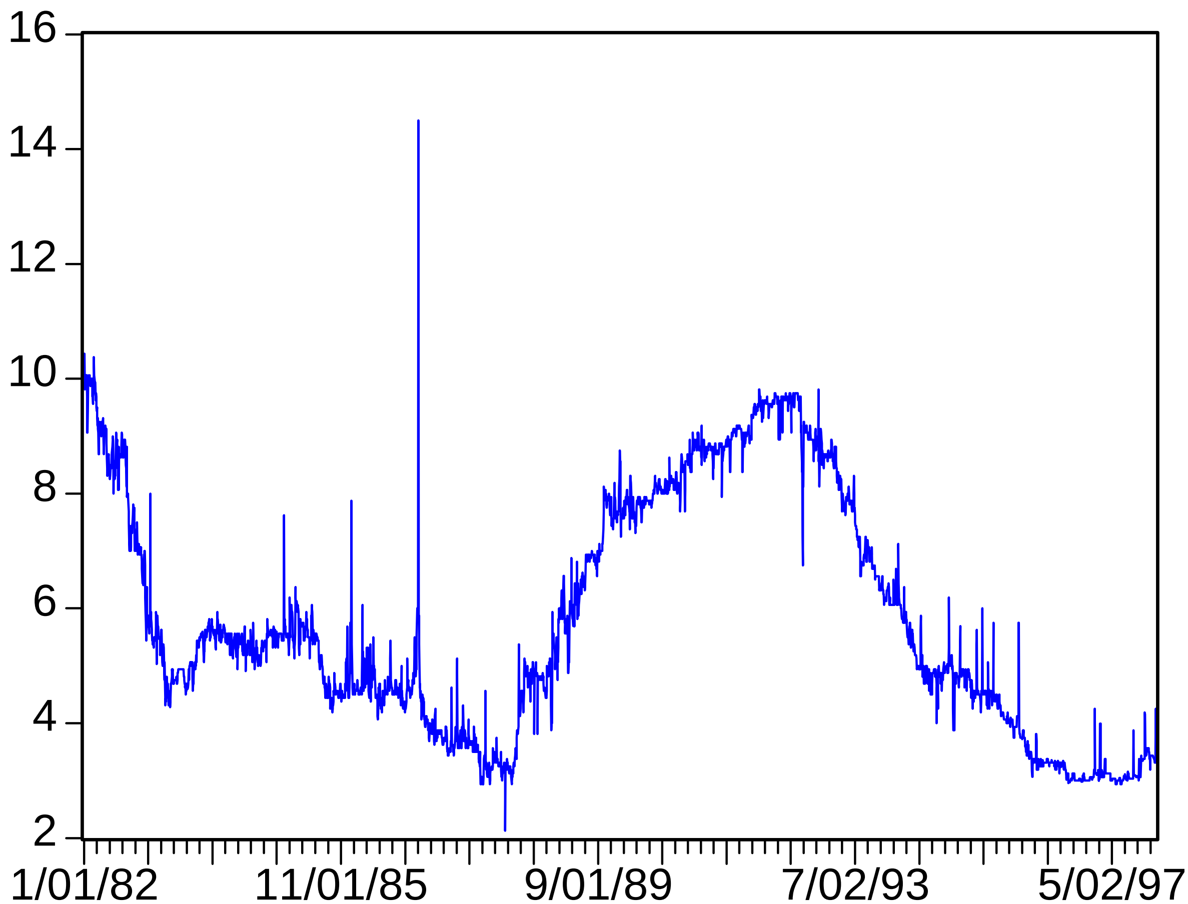


Figure 3.3: Daily data on the Euro-DM Interest Rate from January 1982 to December 1997.

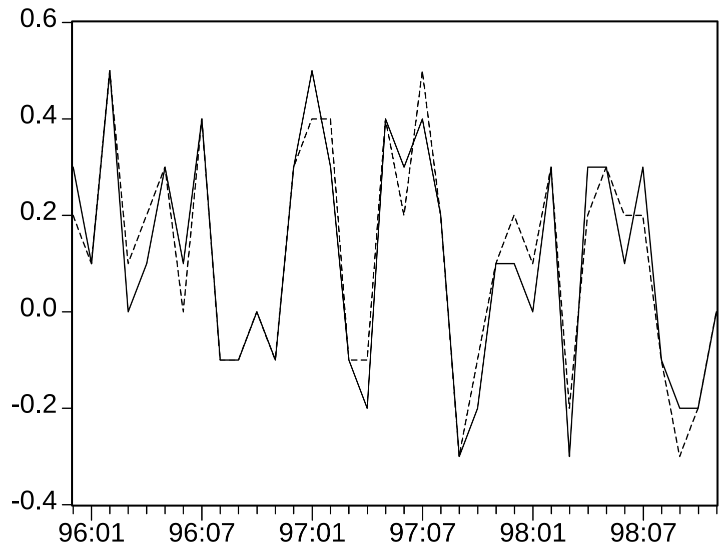


Figure 3.4: Preliminary Inflation Release (dotted line) and Final Inflation Release.

Figure 3.5: Monthly Data on Inflation and Unemployment (dotted line); the vertical line indicates German unification.

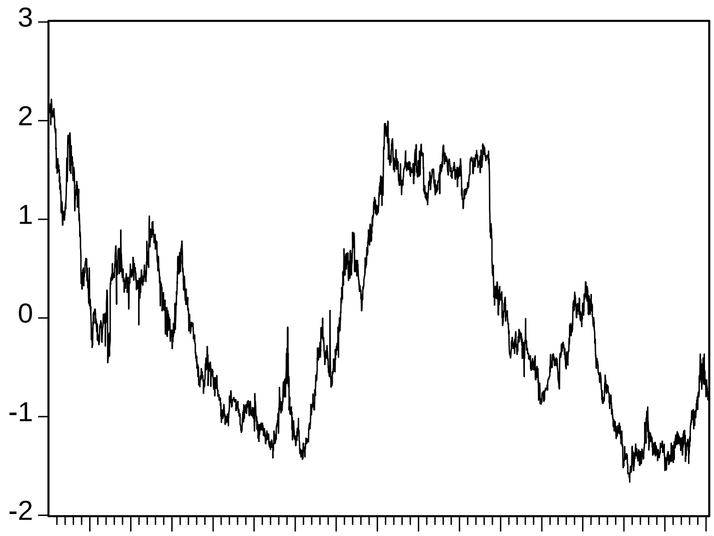


Figure 3.6: Implied $\tilde{s}_1(\hat{\alpha})$ from 1-spread factor model.

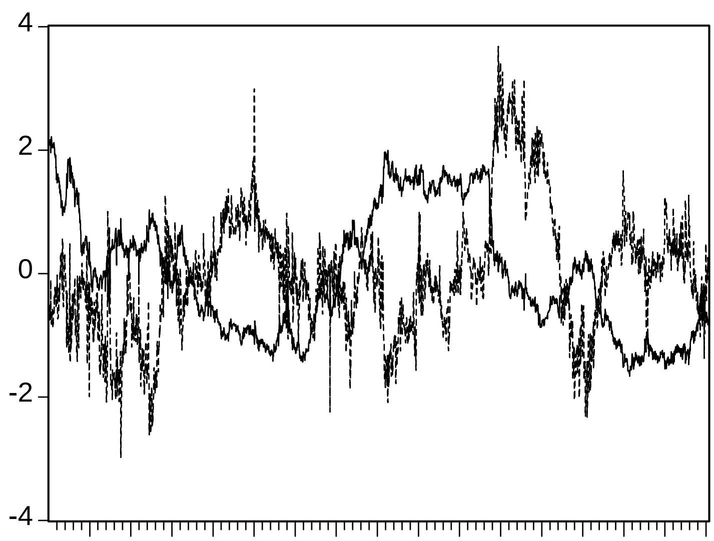


Figure 3.7: Implied $\tilde{s}_1(\hat{\alpha})$ and $\tilde{s}_2(\hat{\alpha})$ (dotted line) from 2-spread factor model.

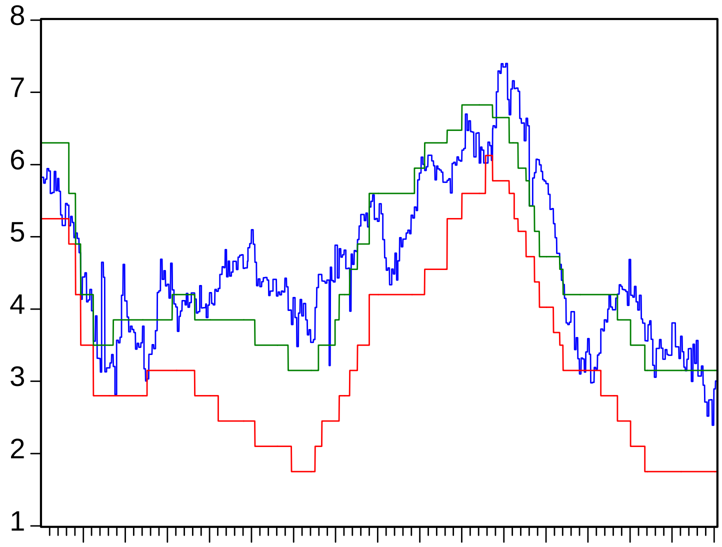


Figure 3.8: Implied Target Rate $r^*(\widehat{\alpha})$ by the 1-Spread Factor Model together with Discount Rate (lower step-function) and Lombard Rate (upper step-function). All rates are continuously compounded.

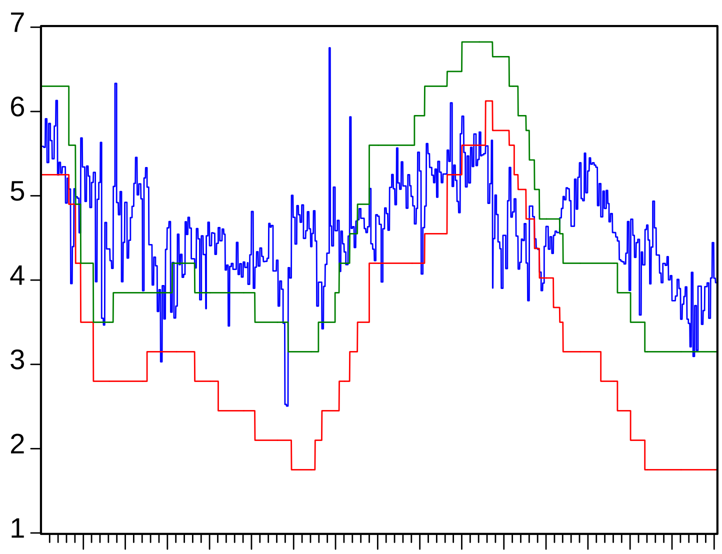


Figure 3.9: Implied Target Rate $r^*(\widehat{\alpha})$ by the 2-Spread Factor Model together with discount (lower step-function) and lombard rate (upper step-function). All rates are continuously compounded.

Chapter 4

Application to U.S. Data

4.1 Institutional Background

Important changes in 1994 to Fed-policy operating procedures underly the choice of sample period in this paper, which focuses on the policy framework in place today. The Fed conducts monetary policy by targeting the overnight rate in the federal funds market.[1] The FOMC fixes a value for the target and communicates it to the Trading Desk of the Federal Reserve Bank of New York, which then implements it through open-market operations (Meulendyke (1998)). Figure 4.1 shows the fed funds market rate and the target rate from 1984 to 1998, illustrating that, on average, the Fed is able to closely target the federal funds rate, except for occasional spikes. (Section 4.3.5 provides a description of the target data that is used in this paper.) These spikes are usually associated with "settlement Wednesdays" and other special calendar effects, such as the end of the year.[2] The target has been changed 114 times over the entire fifteen-year time frame. Figure 4.1 also shows that target-rate changes

[1] While this statement is true for the '70s and for the Fed under Greenspan, it does not apply to the Volcker era. From October 1979 until 1982, the Fed was targeting nonborrowed reserves. Starting in 1983 and at least until the change in chairmanship from Volcker to Greenspan in August 1987, the Fed was targeting borrowed reserves, a practice that has, since then, been increasingly abandoned, especially after the stock-market crash of October 1987 (Meulendyke (1998)).

[2] During bi-weekly reserve-maintenance periods, banks must hold "good funds" in the form of cash or in accounts at the Fed, or be penalized. These reserve-maintenance periods end on "settlement Wednesdays".

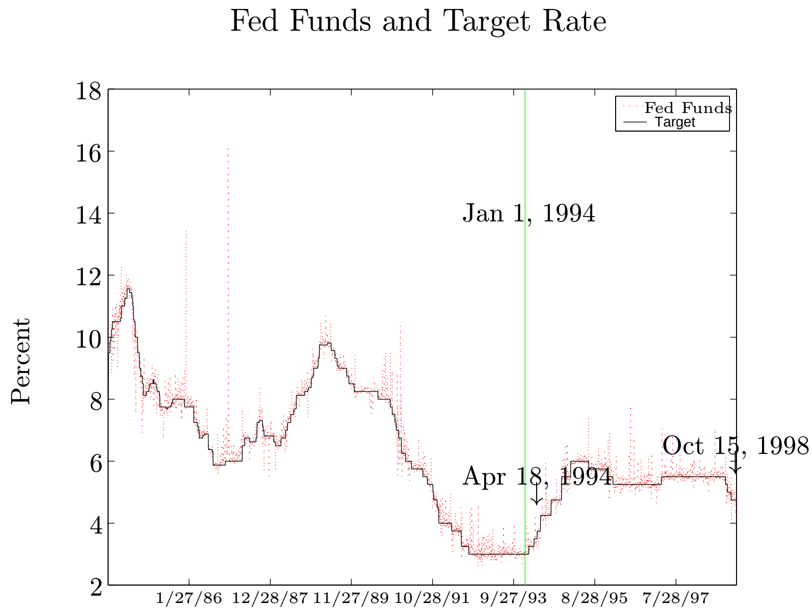


Figure 4.1: Daily federal funds and target rate from 3/1/1984 to 12/31/1998.

are often followed by additional changes in the same direction. This feature will be referred to as "policy inertia".[3]

Starting with the first FOMC meeting of 1994, the Fed made two changes in monetary-policy operating procedures that effectively divide the past fifteen years into two "regimes." First, the Fed increased transparency by publicly announcing target moves at FOMC meetings. From 1983 to 1990, the Fed did not disclose its target rate at all. Since 1994, target moves have been disclosed right after they were made. More recently, the FOMC has even published its carefully worded views about the likelihood of a rate change in the upcoming inter-meeting period.[4]

More importantly, the timing and size of policy moves have changed. Figure 4.2 illustrates this difference in timing by showing histograms, pre-1994 and post-1994, of the number of days between a target-rate change and the preceding FOMC meeting.

[3]This may indicate the Fed's unwillingness to move the target immediately and entirely to its desired rate. Instead, the Fed adjusts the target in small steps to avoid possible policy mistakes, because of political motives, parameter uncertainty (Sack (1998)) or to affect long-maturity yields with minimal changes in short yields (Woodford (1999)).

[4]For details on the art of reading policy directives, see Meulendyke (1998).

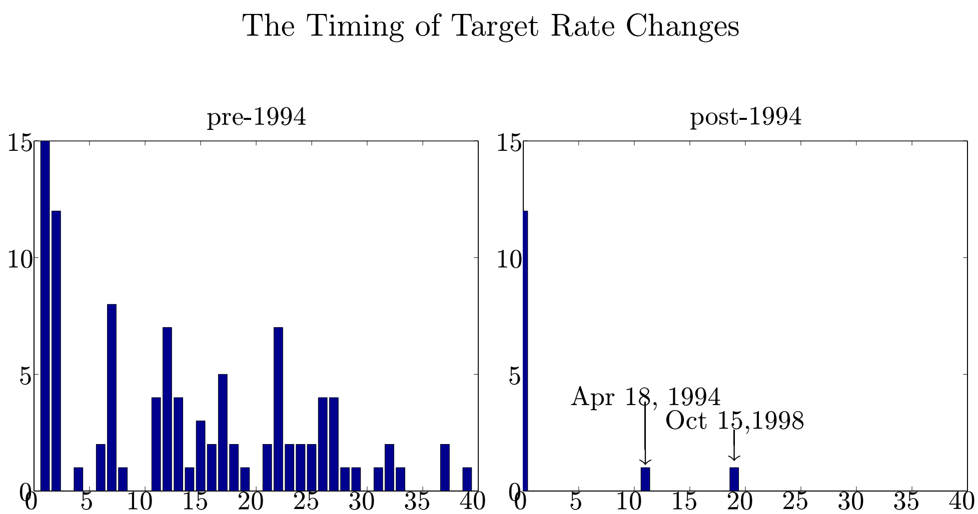


Figure 4.2: For any given target rate change between 1984-1993 and 1994-1998, these graphs show the histogram of days since the last FOMC meeting. In the first subperiod, there have been a total of 100 target moves, while there were 14 in the second subperiod.

If, in a given subperiod, the Fed had moved its target only at FOMC meetings, we would see a single spike at 0 in the corresponding histogram. One sees a definite change in 1994 of re-targeting mainly at FOMC meeting days,[5] with two exceptions highlighted in Figure 4.2. The first exception occurred on April 18, 1994 after high car sales in March, a leading business-cycle indicator. The financial press speculated that the surprise move was intended as a manifestation of authority by Alan Greenspan, as no vote was held on the move.[6] The second exception was decided upon in a

[5]Does a closer look at the timing of target moves pre-1994 reveal any other calendar effects that we could use as an alternative to the FOMC meeting calendar? There is a clear tendency to implement changes in the target at the beginning of a new settlement period, as 37 out of 100 moves pre-1994 happened on the Thursday after a settlement Wednesday. Other possible candidates for calendar effects are release schedules of macro information. In fact, 11 moves occurred on the release days of employment information by the Bureau of Labor Statistics. Together, the releases of consumer and producer price indices account for another 8 changes. These releases, however, are on a monthly basis, and on different days respectively, so that they cannot serve as a calendar for target moves. Other variables such as the Producing Managers' Index, an index often referred to in the "Minutes" of the FOMC meetings, and released on the first business day of each month, do not coincide with target moves. Monetary aggregates might seem relevant in this context, but these are published by the Fed itself.

[6]*The Financial Times*, April 19, 1994, page 3, "Greenspan plays an early hand: US rates rise" by Michael Prowse and *The New York Times*, April 19, 1994, page 1, "Fed again raises short-term

conference call on October 15, 1998, and came in response to the Asian and Russian financial crises.[7]

The Size of Target Rate Changes

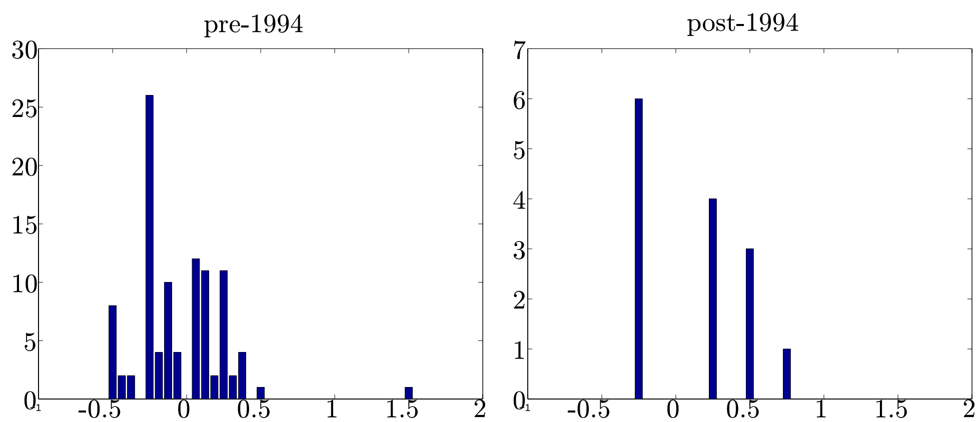


Figure 4.3: These graphs show the histogram of the size of target changes between 1984-1993 and 1994-1998.

Along with the change 1994 in the timing of Fed moves, it can be seen from Figure 4.3 that there was a big change in the size distribution of target-rate changes. While pre-1994 target-rate changes came in multiples of 6.25 basis points,[8] after 1994 the Fed used multiples of quarter-percentage points.

4.2 The Target as an Observable Factor

This section presents the econometric model with Fed targeting.

4.2.1 Target Dynamics

Since the beginning of 1994, the target was usually set at FOMC meetings. Only in emergency cases ('Peso events') has the Fed adjusted the target between FOMC

rate on loans" by Keith Bradsher.

[7] *The New York Times*, October 16, 1998, page 1, "Federal Reserve cuts rates again; Wall St. surges" by Richard W. Stevenson.

[8] One basis point is 0.01%.

meeting dates. The timing of these two types of policy events and the discrete distribution of target changes can be modeled by taking the i-th FOMC meeting to be an interval $[\tilde{t}_M(i), t_M(i)]$. During this interval, the Fed may move the target in steps of 25 basis points (in light of the histogram in Figure 4.2) according to the state of the economy. There may be more than one move during the i-th meeting, but the econometrician will only observe the target announced at $t_M(i)$. Confronted with important macroeconomic "Peso" events, the Fed may also decide to move the target outside of a meeting. Peso events are assumed to be triggered by Poisson processes with small constant arrival rates. More concretely, the target process solves

$$d\theta(t) = 0.0025\,\left(dN^U(t) - dN^D(t)\right), \tag{4.1}$$

where N^U and N^D are counting processes with stochastic intensities given by

$$\lambda^j(t) = \begin{cases} \left(\lambda_0^j + \lambda_X^j \cdot X(t-)\right)^+, & \text{for} \quad t \in [\tilde{t}_M(i), t_M(i)], \\ \lambda_P^j, & \text{otherwise}, \end{cases} \tag{4.2}$$

for $j = U$ ("up") and D ("down"), and where $x^+ = \max\{x, 0\}$.

While the truncated linear intensities (4.2) are outside the LQJD class, this specification has several advantages over a pure LQJD specification. First, it allows for negative correlation among intensities (similar to a quadratic formulation), while the approximating map from factors to yields is invertible. This means that, even in the presence of latent variables, we can use an estimation method that relies on the likelihood function of the factors.[9] Second, the dependence of the intensities on the target θ itself allows for interest-rate "smoothing". Moreover, together with the max-operator, this dependence permits mean reversion in the target.[10]

By defining the martingale $M = M^U - M^D$, where $dM^j(t) = dN^j(t) - \lambda^j(t)\,dt$

[9] Ahn, Dittmar, and Gallant (1999) apply EMM to the Saints Model which similarly generates a non-invertible map from factors to yields. The efficient method of moments by Gallant and Tauchen (1996), or more generally the simulated method of moments (Duffie and Singleton (1993)), simulates the state variables, computes yields as a function of the simulated states, and matches moments of the resulting simulated data to actual data. This makes inversion unnecessary.

[10] Unfortunately, this is impossible with a quadratic specification, as the target rate itself is not a Gaussian process. Assumption 1 requires squared variables to be Gaussian.

for $j = U, D$, the dynamics of the target in (4.1) (for times at which the positivity constraints on λ^U and λ^D are not binding) can be rewritten as

$$\begin{aligned} d\theta(t) &= \kappa_\theta(t) \left(\bar{\theta}(t-) - \theta(t)\right) dt + J_\theta \, dM(t), \\ \bar{\theta}(t) &= c_0(t) + c_X(t) \cdot X(t), \end{aligned}$$

for piecewise constant functions $c_0(t), \kappa_\theta(t) \in \mathbb{R}$ and $c_X(t) \in \mathbb{R}^N$, which can be recovered from the intensity parameters. This representation shows that, during times at which $\kappa_\theta(t) > 0$, the target reverts to $\bar{\theta}(t)$.

There are different ways to think about the Fed's policy rule in this setting. As an illustrative example, the original Taylor (1993) rule may be represented by letting the target revert instantly at FOMC meetings to a linear combination $\bar{\theta}$ of measures of inflation π and output y given by

$$\bar{\theta}(t) = \pi(t) + r_R + 0.5y(t) + 0.5(\pi(t) - \pi^*). \tag{4.3}$$

Here, r_R is the real rate and π^* is the Fed's inflation target.

To economize on parameters, it is assumed that the slope parameters in (4.2) are symmetric, in that $\lambda_X := \lambda_X^U = -\lambda_X^D$. Mean-reversion of the target it is also imposed by assuming $\lambda_0^D = \lambda_0^U + 2\lambda_X \bar{x}$, where $\bar{x}$ denotes the long-run mean of X. The arrival rates of Peso events are fixed to their empirical frequency. There has been one up and one down move outside of FOMC meetings in the 5 years from 1994 to 1998, so that we set $\lambda_P^U = \lambda_P^D = 0.2$. For given long-run mean parameters, we therefore have $N + 1$ free intensity parameters: λ_0^U and λ_X.

4.2.2 Additional Latent Factors of Base Models

A number of alternative approaches to the use of latent state variables are introduced. In all of the setups, the target is included as observable state variable and the spread $s = r - \theta$ between the short rate and the target is among the latent factors, with s mean-reverting to zero. In addition, we consider a traditional stochastic volatility factor as well as a Gaussian *inertia factor* that affects only the target dynamics. This inertia factor proxies for variables (in addition to s, θ, and v) to which the Fed reacts when conducting monetary policy. The stochastic intensity of policy events at FOMC meetings may depend on all of the state variables in X. These alternative specifications form a set of base-case models, in the sense that they are not maximally flexible in the sense of Dai and Singleton (2000): some of the correlation parameters can be freed up without loss of statistical identification. Extensions allowing for additional correlation between state variables will be examined in Section 4.5. These base-case models are summarized next.

An Intensity-State Model (The λ Model)

For this base-case model, the state vector X consists of the target rate θ and the bivariate Gaussian variable (s, z), where $s = r - \theta$, and z is the inertia factor, with

$$\begin{aligned} ds(t) &= -\kappa_s s(t)\,dt + \sigma_s\,dW_s(t), \\ dz(t) &= -\kappa_z z(t)\,dt + dW_z(t), \end{aligned}$$

where W_s and W_z are independent standard Brownian motions.

A Model with Stochastic Volatility (The SV Model)

In this second base-case model, an additional factor v, beyond the spread s, serves both as the stochastic volatility of s and as a factor affecting the stochastic intensity of policy events, in that

$$\begin{aligned} ds(t) &= -\kappa_s s(t)\,dt + \sqrt{v(t)}\,dW_s(t), \\ dv(t) &= \kappa_v(\bar{v} - v(t))\,dt + \sigma_v\sqrt{v(t)}\,dW_v(t), \end{aligned}$$

where W_s and W_v are independent standard Brownian motions.

A Model with Volatility and λ-Factor (The SVλ Model)

The final setup, the SVλ model, combines the previous two by specifying the dynamics of three latent variables (s, v, z) by

$$\begin{aligned} ds(t) &= -\kappa_s s(t)\, dt + \sqrt{v(t)}\, dW_s(t), \\ dv(t) &= \kappa_v(\bar{v} - v(t))\, dt + \sigma_v \sqrt{v(t)}\, dW_v(t), \\ dz(t) &= -\kappa_z z(t)\, dt + dW_z(t), \end{aligned}$$

where W_s, W_v, and W_z are independent standard Brownian motions.

4.2.3 Market Prices of Uncertainty

In the SVλ model, the market prices of uncertainty σ_ξ appearing in (2.9) for the Brownian motions W_s, W_v and W_z are of the form

$$\begin{pmatrix} \sigma_\xi^s(t) \\ \sigma_\xi^v(t) \\ \sigma_\xi^z(t) \end{pmatrix} = \begin{pmatrix} q_s \sqrt{v(t)} \\ q_v\, \sigma_v \sqrt{v(t)} \\ q_z \end{pmatrix}$$

leading to risk premia that are affine in the volatility factor v. For the λ model and the SV model, σ_ξ^v and σ_ξ^z are not needed, respectively. In the λ model, the market price of uncertainty σ_ξ^s for W_s is constant.

The parametrization of σ_ξ also captures aversion against target moves that are driven by s, r and v. This means that even without market prices of uncertainty for N^U and N^D, the intensities under the risk-neutral measure $\mathcal{Q}$ may differ[11] from their values under $\mathcal{P}$ because of the state-dependence in (4.2). With only 5 years of data, we choose to not parametrize the market price of jump uncertainty for target-rate moves (for example, λ_0^U is hard to estimate even without any risk adjustment).

[11] For example, the long-run mean of the short rate r under $\mathcal{Q}$ compared to that under $\mathcal{P}$ is higher if $q_s < 0$. If λ^U positively depends r, then the mean intensity of up-moves is higher under $\mathcal{Q}$ than under $\mathcal{P}$.

4.3 Estimation Technique and Data

This section describes the simulation-based method used to approximate the joint likelihood function of the target, LIBOR and swap rates, which is not available in closed form. Moreover, it presents the approximation of the pricing map used in the estimation.

4.3.1 Estimation Problem

Let $f_X(\,\cdot\,, t|X_{\tilde{t}}, \tilde{t}; \gamma)$ denote the true density of the state vector X_t conditional on the last observation $X_{\tilde{t}}$ at some $\tilde{t} < t$. The parameter vector γ contains parameters describing the true distribution of X and parameters governing the market prices of uncertainty. This density involves the nonlinear stochastic intensities in (4.2). Let $p(\,\cdot\,, \gamma)$ denote the true mapping from factors to observed yields and the target for a given γ, in that $p(X_t, \gamma) = Y_t$, where Y_t is the vector of observables at time t: yields and the target rate θ_t. We assume that, $p(\,\cdot\,, \gamma)$ can be inverted to obtain the factors as function $q(\,\cdot\,, \gamma)$ of the observables Y_t, in that $X_t = q(Y_t, \gamma)$.

Ideally, we would like to estimate by maximizing the likelihood of the observations over γ, which can be obtained by a change of variables from the conditional densities of the state variables. For example, the conditional density $f(\,\cdot\,, t\,|\,Y_{\tilde{t}}, \tilde{t}; \gamma)$ of Y_t given $Y_{\tilde{t}}$ at $\tilde{t} < t$ is given by

$$f\left(Y_t, t\,|\,Y_{\tilde{t}}, \tilde{t}; \gamma\right) = f_X\left(q(Y_t, \gamma), t\,|\,q(Y_{\tilde{t}}, \gamma), \tilde{t}; \gamma\right) \left|\bigtriangledown_Y q(Y_t, \gamma)\right|. \tag{4.4}$$

Three problems arise. First, the true density f_X of the state variables is not available in closed form. We therefore extend the simulated maximum likelihood (SML) method of Pedersen (1995) and Brandt and Santa-Clara (1999) to jump-diffusions (Section 4.3.2). Second, the true maps p and q are not available in closed form. In this high-dimensional setting, we can only recover $p(\,\cdot\,, \gamma)$ by Monte-Carlo integration, which is prohibitively expensive. This roadblock is bypassed by using an approximating LQJD model, for which the Jacobian term in (4.4) can be calculated analytically. A time-consuming hill-climbing procedure, based on analytical derivatives, inverts the map from states to LIBOR and swap yields numerically for each observation (Section 4.3.3). Third, an exact computation of yield coefficients for the approximating LQJD model is computationally intensive, so we employ a time-saving algorithm (Section 4.3.4).

4.3.2 Density Approximation (SML)

The conditional density of the likelihood function of the underlying state vector solves a partial differential-integral equation that has a closed-form solution only for a few special cases, such as Gaussian and square-root diffusions (Lo (1988)). To overcome this problem, simulated maximum likelihood (SML) approach is used, which attains approximate efficiency similar to the efficient method of moments technique by Gallant and Tauchen (1996).[12] The density $f_X(\,\cdot\,, t \,|\, \tilde{x}, \tilde{t})$ of the state $X(t)$ conditional on the last observation $X(\tilde{t}) = \tilde{x}$ can be written, using Bayes' Rule and the Markov property of X, as

$$f_X(x, t \,|\, \tilde{x}, \tilde{t}) = \int_D f_X(x, t \,|\, w, t-h)\, f_X(w, t-h \,|\, \tilde{x}, \tilde{t})\, dw, \qquad (4.5)$$

for any time interval h. (This is sometimes called the Chapman-Kolmogorov equation.) SML computes (4.5) by Monte-Carlo integration, replacing the density $f_X(\,\cdot\,, t \,|\, w, t-h)$ by the density of a discretization of X. The method is extended in Appendix D to

[12]EMM implements a simulated method of moments estimator with moments generated by the scores of an auxiliary semi-nonparametric (SNP) density. The SNP density is a Hermite expansion (with analytical scores) that approaches the true density as the degree of the polynomial increases. In the case of SML, the simulated moments are scores from the discretized model. An alternative approximately efficient estimator for is proposed by Singleton (1999), who computes explicit moments using the conditional characteristic function $\psi(\,\cdot\,)$ of X, defined by $\psi(u) = E_{t-1}[\exp(i\, u^\top X_t)]$. Efficiency is achieved by increasing the number of different values taken on by u with one moment associated with each choice of u. This estimator can also be used in the LQJD setting, as the characteristic function can also be obtained in closed form. While SML is used here as a potentially helpful alternative to EMM, the computational costs of explicit moments as in Singleton (1999) are prohibitive in the present seasonal setting. The same caveat applies to other GMM approaches based on explicit moments, such as those of Liu (1999) and Pan (1999).

allow for jump-diffusions. Particular care needs to be taken to accommodate time-dependent stochastic intensities.[13]

4.3.3 Pricing-Formula Approximation (LQJD Model)

Modeling the target-rate with jump intensities defined by (4.2) introduces a form of nonlinearity that takes the state vector outside of the LQJD class. The quality of an approximating LQJD-pricing formula $Y_t = \tilde{p}(X_t, \gamma)$ that ignores the truncation by the max-operator in (4.2) depends crucially on how severely the positivity constraint on the intensities is binding (and on the average impact of hitting the constraint). The inverse $\tilde{q}(\,\cdot\,, \gamma)$ of the approximating map $\tilde{p}$ is defined in the obvious way. We let

$$D_+^{\gamma_0} := \{x \in D : \lambda^j = g(x, l^j) \geq \gamma_0^j,\ j = U, D\}$$

denote the set of states at which the intensity formula $g(\,\cdot\,, l^i)$ is bounded below by a given constant γ_0^j, for $j = U, D$. The approximation $\tilde{q}(\,\cdot\,, \gamma)$ is likely to be better at states in D_+^0.

Two estimation approaches can be taken. The first approach is simply to replace p in (4.4) by $\tilde{p}$ and obtain an estimator of γ by maximizing the total approximate likelihood

$$\prod_{(\tilde{t},t)\in I} \tilde{f}(Y_t, t \,|\, Y_{\tilde{t}}, \tilde{t}; \gamma) = \prod_{(\tilde{t},t)\in I} f_X\left(\tilde{q}(Y_t, \gamma), t \,|\, \tilde{q}(Y_{\tilde{t}}, \gamma), \tilde{t}; \gamma\right) \left|\bigtriangledown_Y \tilde{q}(Y_t, \gamma)\right|,$$

where I denotes pairs of successive observation times in the data set. Here, the true

[13] As any simulation-based technique, SML is computationally intensive. The application considered in this paper does not exploit the computational advantages that would be allowed by analytical gradients and Hessians of the likelihood function, as discussed in Brandt and Santa-Clara (1999), because the map from the state vector to swap yields involves the numerical computation of ODEs that depend on the parameters. The numerical optimization procedure is therefore based on the Nelder-Mead simplex method, starting a gradient-based parameter search only after the simplex algorithm has collapsed.

factor dynamics (captured by f_X) are combined with the inverse $\tilde{q}$ of the approximating map from factors to yields. Since there is no restriction here on the parameter space, the approach is labeled *unconstrained estimation*. The accuracy of this approximation of the likelihood can be assessed *ex post* by checking whether the functions p and $\tilde{p}$ are close at the estimated parameter $\hat{\gamma}$.

One might be concerned that, without *a-priori* restrictions on the parameter space, it is unlikely that a good approximation of p would obtain at the estimated parameter value. Even though this turned out not to be a problem in the application below, an alternative approach was also tried. Specifically, consider performing the *constrained estimation*

$$\begin{array}{rl} \max_{\{\gamma,\gamma_0\}} \prod_{(\tilde{t},t)\in I} \tilde{f}\left(Y_t, t \,|\, Y_{\tilde{t}}, \tilde{t}; \gamma\right) & \\ \text{subject to} \qquad \tilde{q}(Y_t,\gamma) \in D_+^{\gamma_0}, \text{ for all } t \in I. & \end{array}$$

In words, the parameter space is restricted to contain only those parameters at which the observations are explained by a factor realization $\tilde{q}(Y_t,\gamma)$ for which the γ_0-constraint never binds. The special case that was tried in this paper is $\gamma_0 = 0$. Naturally these two problems typically deliver distinct estimators. Any such differences will be further discussed when the estimates are presented.

4.3.4 Coefficient Approximation

Time dependencies introduced by scheduled announcements, such as FOMC meetings and macro releases, immensely increase the computational burden associated with the solution of the approximating LQJD model for yields, and render almost impossible an estimation using data for long-maturity yields. For example, in order to evaluate the likelihood function, we need to compute the 5-year swap rate for each observation in the sample. In the setup of Section 4.6, this takes 16 minutes on a SUN workstation.[14] In setups with only one type of scheduled announcement (at

[14]The computation simulates 9 coefficients (for each of the 8 state variables plus a constant) for 10 different bond maturities $(0.5, 1, \ldots, 4.5, 5$ years) for each of the 261 observations, using Runge-Kutta solutions of the ODEs for the coefficients.

FOMC meetings), the coefficients are therefore computed using the following approximation: the time until the next FOMC meetings is matched exactly only for the next-to-occur meeting. The subsequent meetings are assumed to be equally spaced over the year. For the maturities of the yields used in the estimation (6 months and above), the errors due to this approximation are virtually undetectable. In setups with more than one type of scheduled announcement (FOMC meetings and macro announcements), such an approximation is no longer accurate, and is not pursued in this paper. Instead, only yields with a maturity of up to one year are used in the estimation, economizing on computation time.

4.3.5 Data

With knowledge of an equivalent martingale measure $\mathcal{Q}$, any claim to future payoffs can in principle be priced. This means that the term-structure model may be estimated with data on a broad range of assets including swaps, Treasuries, futures (for example, Fed Funds Futures), and options (swaptions, Eurodollar options, and Treasury options, for example). In the model of this paper, the Fed is targeting the federal funds rate, an interbank rate that reflects default risk, implying that the target rate itself is on average higher than a short Treasury rate. Moreover, Treasury rates are further reduced relative to interbank rates by liquidity, tax effects, and specials in the market for repurchase agreements (Duffie (1996)). For example, the average daily target rate from 1994 to 1998 is 5.22%, while the 3-month T-bill and the 3-month LIBOR-rate averaged 5.06% and 5.44%, respectively. Target data cannot, therefore, be combined with treasury rates without modeling the spread. The empirical results reported here are therefore based on LIBOR and swap rates. LIBOR-quality swap rates are minimally affected by credit risk because of their special contractual netting features (Duffie and Huang (1996)), although they do trade at spreads to Treasuries that have, to this point, resisted a convincing explanation (see, for example, Collin-Dufresne and Solnik (1997)). For example, the 2, 5 and 10-year average swap rates were 6.08%, 6.48% and 6.79% during the post-1994 period, respectively, while the same maturities had average percentage yields in the Treasury market of 5.81, 6.13, and 6.34.

The sample period considered in the estimation is January 1, 1994 to December 31, 1998. The dates of FOMC meetings were obtained from the Board of Governors of the Federal Reserve. The FOMC meets eight times a year. Two of these meetings, the first and the fourth, extend over two days. In the past, if the Fed changed its target during one of these two-day meetings, the announcement was always made on the second of the two meeting days.[15] The target-rate series used in this paper differs from the series in Datastream with respect to the timing of the target change during the two-day meeting of February 1994. Datastream assigns the change to the first meeting day (February 3), while the change was announced on the second meeting day (*The New York Times*, February 5, 1994, page 1, "Federal Reserve, Changing Course, Raises a Key Rate" by Keith Bradsher).

LIBOR data are from the British Bankers' Association, while swap rates are from Intercapital Brokers Limited. Both series are obtained through Datastream. LIBOR rates are recorded at 11 a.m. London time, while swap rates are recorded at the end of the UK business day. Target-rate changes are typically announced from 10 a.m. to 3 p.m. Eastern time.[16] This means that a move in the target on Tuesday, March 1, affects recorded LIBOR rates on Wednesday, March 2. The effect on recorded swap yields is not so precisely separable. The data sample is therefore constructed by using Thursday (London time) observations of LIBOR and swap yields, together with Wednesday (Eastern time) observations of the target rate. Figure 4.6 shows a plot of the data. The asynchronous nature of the observations is ignored in the estimation. Whenever the respective day was a holiday, the observation of the previous business day was used.

The bond-pricing formula (2.2) extends as written to the case of LIBOR bonds, treating r as a default-adjusted discount rate (Duffie and Singleton (1997)). The

[15] This is also true for the target rate increase from 4.75 to 5 that was decided upon during the 2-day meeting on June 29/30 and that was announced only on Wednesday, June 30.

[16] For example: Sep 29, 1998 at 2:15 p.m., Oct 15, 1998 at 3:15 p.m., Nov 17, 1998 at 2:15 p.m.

6-month LIBOR rate $r_L(t)$ at time t is in that case defined by

$$P(t, t + 1/2) = \frac{1}{1 + r_L(t)/2}. \tag{4.6}$$

An interest-rate swap is a contract between two parties to exchange fixed and floating coupons for a stipulated time, say τ years. One party receives a semi-annual floating payment in form of the 6-month LIBOR rate, and pays in exchange a fixed coupon rate, the swap rate, denoted $Y(t, t+\tau)$. At the initiation of the swap contract, the swap rate is set so that the value of the swap contract is zero. For simplicity, we treat swap rates as par-coupon rates on LIBOR-quality bonds of the same maturity, putting aside the distinct institutional features and differences in default risk of LIBOR and LIBOR-swap markets, so that

$$Y(t, t + \tau) = \frac{2\,(1 - P(t, t + \tau))}{\sum_{j=1}^{2\tau} P(t, t + j/2)}. \tag{4.7}$$

4.4 Estimation Results for Base Models

The three base-case models have different numbers of state variables. The λ and SV models have three, while the SVλ model has four. The same set of yields (6-month LIBOR, 2 and 5-year swap) and the target are used for all estimations, creating the need to break the stochastic singularity arising from the exact map of three factors in four observed variables in the lower-dimensional systems. This is achieved by assuming that the 2-year swap rate is observed with measurement error. As this section investigates the properties of the estimated models, the concept of *model-implied factors* will be needed. These are obtained by inverting the map from factors to the target rate, and to those yields that are assumed to be observed without error at the SML estimates. The map from factors to yields is given by the pricing formulas ((4.6) and (4.7)) from the approximating model.

4.4.1 Accuracy of the Approximating LQ Model

As the true state process X is not actually of the LQJD class, because of (4.2), it is important to study the accuracy of the LQ-approximating model. From the swap-yield formula (4.7), one can see that it is sufficient to investigate the approximation accuracy for zero-coupon yields.

Zero-coupon yields $Y_0(t,T)$ for each observation t in the sample implied by the true (nonlinear) model can be computed with Monte Carlo integration. Consider simulating S paths of the short rate for times $i = t+h, t+2h, \dots, T-h$ starting with the model-implied state x at time t. The time t yield of a zero-coupon bond maturing at time T is then

$$Y_0(t,T) \hat{=} -\frac{\ln \hat{P}(t,T)}{T-t},$$

where

$$\hat{P}(t,T) = \frac{1}{S}\sum_{s=1}^{S} \exp\left(-\sum_{i=t}^{T-h} \hat{r}_i^x[s]h\right).$$

These calculations are performed using $S = 10,000$ and $h = 1/365$. FOMC meeting days are additionally subdivided into 30 intervals. Given these choices, the standard errors of the Monte-Carlo approximation of the true yields for even the 5-year yield are sufficiently small, from 0.93 to 1.79 basis points.

These zero-coupon yields implied by the true model can be compared to the yields from the approximating LQ model that are based on the same model-implied state vector. Table 4.1 shows that the mean absolute approximation error made by the LQ model is around 1-3 basis points, with a standard deviation of 1-2 basis points for the constrained SV model and an even smaller error for all SVλ models. The approximation error for these specifications is thus of similar magnitude as the bid-ask spread of swaps. The unconstrained estimates of the SV model and all λ models produce about 5 times the approximation error, which seems too large to be acceptable.

In the following, "SV model" and "λ model" will therefore denote the constrained version of those models, while the unconstrained 4-factor model will be called "SVλ model".

Table 4.1: Approximation Error made by LQ Model (in Basis Points)

Maturity		λ Model		SV Model		SVλ Model	
		Con.	Unc.	Con.	Unc.	Con.	Unc.
6-mth	mean abs.AE	4.08	8.56	2.85	11.32	2.60	2.11
	std of abs.AE	2.73	9.26	2.20	10.69	1.91	1.11
	average SE	0.53	0.38	0.46	0.29	0.59	0.67
2-yr	mean abs.AE	11.43	20.99	2.62	19.43	2.17	1.73
	std of abs.AE	8.78	18.59	1.50	18.18	1.36	0.85
	average SE	0.93	0.66	0.85	0.59	1.07	1.13
5-yr	mean abs.AE	37.74	28.92	1.76	9.37	1.54	1.86
	std of abs.AE	20.16	20.74	0.72	9.30	0.81	0.73
	average SE	1.19	0.93	1.28	1.07	1.64	1.79

NOTE: This table presents summary statistics about the approximation errors in basis points made by the approximating LQ models over the sample January 1, 1994 to December 31, 1998. Due to the seasonality introduced by FOMC meetings, the approximation errors in this setup depend on time t even for a given value of the state vector. The table therefore reports the mean average absolute approximation error $|Y(t,T) - Y_0(t,T)|$ and its standard deviation over the sample (first and second row). The table also reports the average standard errors of the Monte Carlo approximation of true yields $Y_0(t,T)$ (third row). These are obtained using the Delta method by viewing the simulated bond price $\hat{P}(t,T)$ at time t as the estimated mean of an i.i.d. population of random variables $\exp(\sum_{i=t}^{T-h} \hat{r}_i[s]h)$. The table reports the average standard errors over the sample.

4.4.2 Parameter Estimates

Table 4.5 reports the estimated parameters and t-ratios for all base models. In all models, the short rate quickly reverts to the target, which in turn reverts to a parameter $\bar{\theta}$ that is fixed to the sample mean, 5.22%, of the target rate. The speed κ_s of the mean-reversion is highest in the SVλ model, implying a weekly autoregressive coefficient of $\exp(-\kappa_s/52) = 0.83$ and a half life of shocks to the spread of less than 1 month. The mean-reversion of the volatility process v and inertia process z are roughly equally slow in the λ and SV models, while the weekly autoregressive coefficients of z and v in the SVλ model are quite different, 0.82 and 0.99 respectively, implying half lives of 1 and 17 years.

The fitted intensity parameters of the SVλ model imply that a 25-basis-point increase in the spread at the beginning of an FOMC meeting raises the conditional probability of an upward move in the target for that meeting by about 5%, indicating that the Fed does not react much to the short spread s. A 25-basis-point decrease in the target lowers the probability of an upward move during the FOMC meeting by about 11%, reflecting the slow mean-reversion of θ. A one-standard-deviation shock to v and z increases the conditional probability of a target increase by about 14 and 30%, respectively. This shows that the inertia factor z has a larger effect on the stochastic intensity of target moves than v. While the magnitude of these effects varies across the base-case models, the directions of the effects are the same.

The "measurement error" of the 2-year swap rate in the three-factor λ and SV models is persistent, with a weekly autocorrelation coefficient that varies across models from 0.95 to 0.98, and a standard deviation that varies from 12 to 21 basis points.

4.4.3 Interpretation of Model-Implied Factors

Across all base-case models, estimates of the correlation between the factors, LIBOR, swap and target rates are reported in Table 4.2. These correlation estimates are useful for characterizing the factors as 'level,' 'slope,' and 'curvature' (in the language of Litterman and Scheinkman (1993)), and in comparing their respective roles in explaining yields across base models. In addition, these correlations can be used to detect misspecification in the model.

The two model-implied latent factors, for both the λ and the SV model, are almost uncorrelated. For both models, the model-implied short rate r is correlated most highly with the shortest yield in the estimation, the 6-month LIBOR rate, while the second latent factor (z for the λ model and v for the SV model) behaves much like the longest yield. Table 4.3 shows that, for both models, the second latent variables are the main driving force behind the conditional probability of target-rate changes. The short rate is pulled towards these during FOMC meeting days. (This is also

Table 4.2: Correlations of Model-Implied in Factors, Yields and Target

		λ Model		SV Model		SVλ Model			LIBOR & Swaps			Target
Model		r	z	r	v	r	z	v	6-mth	2-yr	5-yr	θ
λ	r	1							.56	.26	.14	.66
	z	.01	1						.44	.89	.97	.07
SV	r	.85	−.18	1					.57	.13	−.01	.75
	v	.07	.99	−.09	1				.55	.93	.99	.12
SVλ	r	.67	−.24	.77	−.16	1			.54	−.03	−.07	−.12
	z	24	.63	.12	.64	−.18	1		.44	.81	.65	.13
	v	-.08	.78	−.19	.78	−.01	.01	1	.37	.55	.76	−.03
DS	r	.07	−.57	.21	−.54	.57	-.90	.05	−.05	−.62	−.50	−.26
	θ	.23	.82	.10	.85	−.16	.93	.34	.63	.96	.86	.29
	v	−.15	−.10	−.26	.62	.02	−.19	.97	.19	.33	.59	−.21

NOTE: This table computes the correlation of the first differences of model-implied factors (r, z and v) from the unconstrained estimations, the model-implied factors (r, θ, v) of the DS model (at their estimated parameter vector), the 6-month LIBOR rate, the 2 and 5-year swap rates, and the target rate θ over the sample January 1, 1994 to December 31, 1998: All correlations with the target rate are computed using the subsample of FOMC meetings.

true under the risk-neutral measures $\mathcal{Q}$). The high correlation of v and z across λ and SV models seems to indicate that the role of v as conditional second moment of shocks to r is dominated by its importance in setting the stochastic intensities of policy events. In the SV model, r is less related to longer yields than in the λ model. This can also be seen from the estimated mean-reversion parameter κ_s, higher for the SV model than for the λ model, and its correlation with target-rate changes on FOMC meetings, which is also higher in the SV model. This may be explained by the fact that, for a nonzero market price q_s of uncertainty, the SV model has the additional flexibility of allowing r to revert under $\mathcal{Q}$ to the continuous variable v. In the λ model, the conditional mean between FOMC meetings is constant.

The SVλ model is characterized by latent variables z and v that roughly correspond to the stochastic mean θ and volatility v factors of the "$\mathrm{A}_1(3)_{DS}$ model" by Dai and Singleton (2000, DS), as can be seen from their correlation of 0.93 and 0.97, respectively. The comovements with yields indicate that z behaves much like the 2-year swap rate, and that v is related to the 5-year swap rate. The two models imply,

Table 4.3: Which Factors drive the Probability of Target Moves ?

	λ Model		SV Model		SVλ Model	
	Constr.	Unconstr.	Constr.	Unconstr.	Constr.	Unconstr.
s	.44	.53	.60	.55	.40	.48
θ	$-.33$	$-.33$	$-.48$	$-.48$	$-.33$	$-.36$
z	.85	.83	–	–	.86	.71
v	–	–	.65	.67	$-.09$	.26

NOTE: To obtain a measure of importance of a factor for the stochastic intensities of policy events, this table shows the correlation between changes in the model-implied factors (s, θ, v, z) and changes in the function $\lambda_0 + \lambda_s s(t) + \lambda_\theta \theta(t) + \lambda_v v(t) + \lambda_z z(t)$ for the weekly sample from January 1, 1994 to December 31, 1998.

however, very different short rates. The sample mean of the model-implied r in the DS model is -0.46%, while the average r in the SVλ model is 5.02%. While the SVλ model implies that r is closely related to the short end of the yield curve, the DS model produces a short rate that behaves like the slope of the short end of the yield curve. This can be seen from the correlations between the model-implied short rates r from the two models and the LIBOR rate (0.54 for the SVλ and -0.05 for the DS model), and the difference between the 2-year swap and the LIBOR rate (0.58 for the SVλ and 0.86 for the DS model). From Table 4.3, the arrival intensity of Fed moves in the SVλ model is mostly driven by z.

More insights can be obtained from Figure 4.4.3, which shows, for the SVλ model, the linear dependence of zero-coupon yields on all factors, as a function of maturity. As shocks to the factors are uncorrelated in the base models, we can interpret these yield coefficients as instantaneous impulse-responses of yields to the various shocks. We see that the response of yields to changes in the target θ is monotonically decreasing with maturity, as are the responses to shocks to the spread s. Both θ and s are "slope factors", but act on different parts of the yield curve, as the impact of target-rate increases dies off more slowly with maturity than do the impacts of shocks to s. Changes in the inertia factor z cause a hump-shaped reaction in yields, with a peak at 2 years, making z a "curvature factor". One can interpret this as a *policy-inertia effect*: A positive shock to z increases the conditional probability of moves up in the target not only at the next FOMC meeting, but also at subsequent

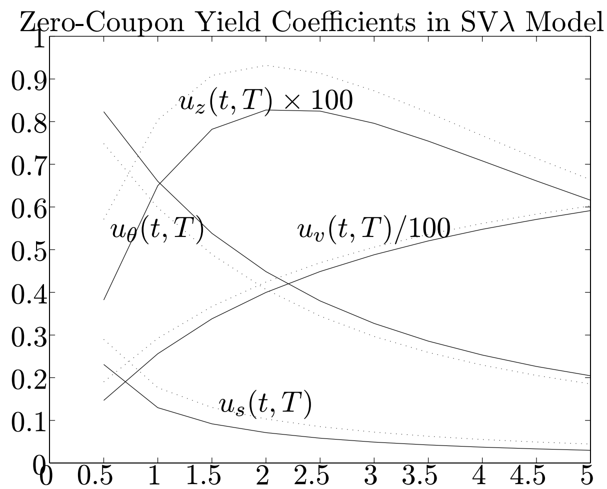


Figure 4.4: Zero-Coupon Yield Coefficients as a function of time to maturity ($T - t = 1, 2, \ldots$ years) taken from the (unconstrained) SVλ Model: $Y_0(t) = u_0(t,T) + u_s(t,T)s(t) + u_\theta(t,T)\theta(t) + u_v(t,T)v(t) + u_z(t,T)z(t)$ at two typical dates t. The solid line is at an FOMC meeting. The dotted line is at the day after the meeting.

meetings, as shocks to z have a half-life of 1 year. Finally, shocks to v affect yields at all maturities, reflecting the strong persistence of volatility which has a half life to shocks of roughly 17 years. In this sense, v is a "level factor".

4.4.4 Results about Yield Dynamics

A measure of goodness of fit of the approximating LQ model is the error with which it determines yields that are not used in the estimation. Pricing errors are defined as the difference between the actual yield and the model-implied yield, which is computed by inserting the model-implied factors into the LIBOR and swap formulas ((4.6) and (4.7)) based on the approximating LQ model, respectively. The pricing error is thus composed of both model misspecification and approximation error.

Table 4.4 reports the average absolute pricing errors and their sample standard deviations for the base-case models and for the DS model as a point of reference. The parameters of the DS are estimated with weekly data on the same LIBOR and swap rates with the exception of using the 10-year instead of the 5-year swap, but over a sample period that only partially overlaps with our sample (April 3, 1987 to August

Table 4.4: Pricing Errors for Yields not used for Fitting (in Basis Points)

		λ Model		SV Model		SVλ Model		DS Model
		Con.	Unc.	Con.	Unc.	Con.	Unc.	
1 mth	mean	13.5	12.53	18.48	25.41	33.57	25.72	237.34
	std	11.07	11.29	11.59	13.96	16.69	16.36	115.48
3 mth	mean	7.90	7.48	11.85	15.48	16.74	11.00	66.17
	std	6.22	6.33	6.65	7.71	7.41	6.75	30.72
12 mth	mean	6.67	6.84	9.90	12.44	4.72	3.22	9.93
	std	6.48	6.58	7.17	8.77	3.53	2.92	4.83
3 year	mean	10.68	9.47	14.96	16.94	8.46	1.76	6.41
	std	5.89	5.66	8.79	9.03	2.35	1.13	1.54
4 year	mean	6.40	5.76	7.98	8.81	10.14	1.78	5.70
	std	3.17	3.08	4.55	4.51	2.45	1.02	1.51

NOTE: This table presents the mean and the standard deviation of the absolute value of the pricing error in basis points over the weekly data sample from January 1, 1994 to December 31, 1998 made by the LQ approximating model and, as a reference, for the DS model (at their parameter estimates). Using their sample period, Dai and Singleton (1999) report mean pricing errors for weekly 3, 5 and 7 year swaps rates of -11.3, 16.9 and -12.7 basis points with standard deviations of 9.6, 16.5 and 10.1 basis points.

23, 1996). They have not been reestimated. The pricing errors of the unconstrained λ and $SV\lambda$ models are lowest among these. The λ model matches the short end of the yield curve extremely well with average absolute errors of around 6 to 13 basis points. In addition to matching the short end well, with 11-26 basis point errors, the $SV\lambda$ model produces low errors, of around 2 basis points, at the long end of the curve. As can be seen from the table, the latter model outperforms the DS model, especially at the short end. The incorporation of the target as a fourth factor appears to provide a manageable way of fixing the short end of the yield curve. The SV model has overall higher pricing errors (ranging from 8 to 18 basis points) when the model is evaluated at the constrained parameter vector, and about 1-7 basis points higher than that for the unconstrained parameter.[17]

[17]While pricing errors are a measure of first moments that depends on the realized path of yields through the use of model-implied factors, we can obtain a history-independent check by simulating $20,000$ samples of weekly yields. The average 6-month LIBOR, 2 and 5-year swap rates in these simulated samples are 5.59, 5.76 and 5.89%, respectively, which shows better how the high persistence of yields makes it difficult to match first moments.

Figure 4.7 shows the standard deviation of yield changes as a function of maturity, the so-called term structure of volatility (or 'vol curve'). The vol curve is "snake-shaped," in that volatility is high at the very short end, declines until maturities about 3-6 months, after which it has a "hump" at a maturity of 2 years. The hump has already been documented by Litterman, Scheinkman, and Weiss (1988). The statistical key to generate a hump in a term structure model is negative correlation between the state variables, which can be attained, for example, by a stochastic mean model (Dai and Singleton (2000)).

In informal accounts, monetary policy has been conjectured to be responsible for the hump (Fleming and Remolona (1999)). This claim is here validated in the sense that the source of the hump is precisely the factor that is responsible for the stochastic intensity of target-rate moves. In other words, the hump in the coefficient on the inertia factor translates into a hump in the vol curve. This can be seen from Figure 4.7, which shows the volatility curve in simulated data from the SVλ model, which reproduces the overall "snake-shaped" pattern quite well. That is, bond investors may view the Fed's policy to be one of slow adjustment of the target rate, so that certain shocks to the economy are allowed to fully affect short term interest rates only with some delay. Rates at medium maturities such as 2 years would respond immediately to the anticipated cumulative effects of short rates over a two-year period, and thus have greater volatility then do the Fed-dampened short rates. At sufficiently long maturities, beyond 2 years, mean reversion in short-term market conditions causes short term shocks to have smaller and smaller impacts on longer and longer rates. The net effect is the hump-shaped vol pattern.

Another feature of the simulated curve is the high volatility of the short rate, the 'head of the snake', especially at FOMC meetings, which was also found in the data. When attention is restricted to the subsample of FOMC meeting days, the base model somewhat overstates the volatility of maturities around 6 months. This point will be taken up further when model extensions are discussed in Section 4.5.

4.4.5 Model Implications Regarding Target Dynamics

From each of the yield-curve setups, it is possible to derive a discrete-choice model in which, at each FOMC meeting, the Fed is viewed as randomizing over three possible choices: moving the target up, down, or not at all. The conditional probability of a particular choice at the FOMC meeting at t depends on the state "right before" t and is obtained from its empirical frequency in a simulated sample of size $S = 10,000$ that is generated by simulating forward in steps of one day's length starting at the actual value of the implied state at the last observation. Outside of FOMC meetings, the discrete-choice model assigns a small and constant probability to Peso events. The conditional probability of target moves up and down for each FOMC meeting since January 1994 is plotted in Figure 4.9. The conditional likelihood of moves up is very high at the end of 1994, when in fact the Fed increased the target in several steps, and again quite large around the target increase in March 1997. The conditional probability of moves down is high in 1995/96 and 1998, both years in which the Fed lowered the rate on several occasions.

Do these conditional probabilities provide a good description of target dynamics? Table 4.6 compares forecasts by the model-implied discrete choice to forecasts based on alternative ways to describe target-rate moves. As there have been only 7 increases and 5 decreases in the target at FOMC meetings over the sample period 1994-1998, these results suffer from small-sample noise, and are not intended to offer a serious forecasting comparison. They do provide, however, a device that might help one characterize the implications of the model regarding Fed moves.

Following the discrete-choice literature, a forecast is taken to be the alternative with the highest conditional probability. The standard reference setup, usually labeled 'constant probability model,' is a version of the discrete-choice model in which the conditional probabilities are set equal to their empirical frequencies. For target moves, these frequencies are small (7/40 for "up" and 5/40 for "down"), so that this version always forecasts that the Fed is not going to change the target. In other words, this specification generates the same forecasts as a random walk for the target, and are reported under 'No Change.' A second reference model that seems useful is a random walk for the first differenced target; its forecasts are reported under 'Same Change.'

For example, the first column of Table 4.6 indicates that of the 7 target-rate increases that occurred at FOMC meetings, the 'Same-Change' model would have predicted 2 correctly, while it would have forecasted no move in the remaining 5 cases.

The forecasts made by the unconstrained base-case models vastly outperform those of the constrained models in terms of the overall percentage of correctly forecasted Fed moves (62.5% and 75%, compared to 20% and 30%, respectively). The reason is that the constrained models are characterized by large probabilities of nonzero moves. For example, none of the constrained models is able to predict a zero target-rate move correctly. It is worth noting, however, that the constrained SV and SVλ models never miss the direction of the move, leading to a perfect score in forecasting nonzero moves. In other words, conditional on a Fed move, these models always predict the right sign of a move. The constrained λ model is special in this regard, as it implies high probabilities of target-rate increases, and therefore performs badly with respect to conditional forecasts as well.

The unconstrained SV and SVλ models produce forecasts that are also more accurate than those of the reference models in terms of the overall correct forecasting percentage: both the SV and the SVλ model predict 75% of the target moves on FOMC meetings correctly, compared to only 60% using the 'Same-Change' model and 70% using the 'No-Change' model. It is also interesting to see that the unconstrained λ model forecasts target-rate increases better than do all other unconstrained models, including the reference models.

4.4.6 Model Implications for Policy Rule

Policy rules are structural equations that specify the map from a set of variables to the policy instrument of the central bank. Recursively identified VARs, for example, typically contain one equation, describing the data-generating process of the federal funds rate, that can be interpreted as a policy rule plus some orthogonal monetary policy shock (Christiano, Eichenbaum and Evans (1998)). This identifying assumption also underlies regressions that one finds in the Taylor-rule literature of the funds rate on current inflation and the output gap, with quarterly data. (See, for example, John Taylor's article in Taylor (1999)). The use of contemporaneous right-hand-side variables has been criticized on practical grounds: policy-rule plots shown by the Fed staff to the FOMC at the meetings cannot be based on variables that are yet to be released (Orphanides (1998)).

In the yield model, we can analogously identify a policy rule by calculating the conditional expected value of the target as a function of model-implied factors, using the estimated SML parameters. Under this identifying assumption, the Fed reacts to the value of the state 'just before' the meeting. This is a *real-time high-frequency policy rule of the Fed*. For a given parameter value, the rule can be backed out by the staff from daily and even higher-frequency data. With weekly observations, the model-implied policy rule at t is

$$\bar{\theta}(t) = 0.36 + 0.10\, s(\tilde{t}) + 0.87\, \theta(\tilde{t}) + 7.51\, v(\tilde{t}) + 0.0033\, z(\tilde{t}),$$

where $\tilde{t} = t - 1/52$. This shows that the spread is not an important determinant of the rule and there is a strong interest-rate smoothing term. While the volatility and inertia factors might seem unfamiliar as arguments in a policy rule, they simply represent yield-based information that the Fed considers when setting a target, which might act as a sufficient statistic for macroeconomic variables the Fed cares about. As the inertia factor mostly drives the conditional probability of target moves, it is also the most important determinant of the policy rule, in addition to interest-rate smoothing.

Figure 4.9 compares the model-implied rule to three rules. The first rule is the original Taylor rule recommended by Taylor (1993). The rule is given by equation (4.3), with the quarterly averaged fed funds rate on the left-hand side. On the right-hand side, π is taken to be the four-quarter average inflation rate, computed using the GDP deflator, while the output gap y is the percentage deviation of real GDP from its trend (based on a Hodrick-Prescott filter). The two other policy rules are a Taylor rule with estimated coefficients and an estimated extended Taylor rule that additionally includes the lagged federal funds rate as explanatory variable in (4.3).[18]

[18] The regressions use quarterly data from 1994:1 to 1998:12. In the Taylor rule, the estimated

To mimic the decision process of the Fed, we plot for each FOMC meeting the policy rule that corresponds to the quarter in which the meeting took place, leaving us with 40 data points.

By eyeballing, the model-implied rule seems to be a better description of the actual target. This is confirmed by the mean absolute difference between actual target and the value of the target prescribed by the policy rule. For the original Taylor, the estimated Taylor, the extended Taylor and the model-implied rule, the mean absolute difference is 162, 41, 22 and 10 basis points, respectively. The reason for the better fit of the model-implied rule is that the model-implied state variables, especially the inertia factor, anticipate many of the target rate changes, while the Taylor-based rules only catch up slowly with the target.

4.5 Extensions

The base-case models restrict many correlation parameters to zero. Freeing up certain parameters of the SVλ model, we have estimated special cases of:

$$\begin{aligned}
ds(t) &= -\kappa_s s(t)\,dt + \sqrt{v(t)}\,dB_s(t) + \sigma_{sv}\sqrt{v(t)}\,dB_s(t) + \sigma_{sz}\,dB_z(t) \\
&\quad + J_s\,(dN^U(t) - dN^D(t)), \\
dv(t) &= \kappa_v(\bar{v} - v(t))\,dt + \sigma_v\,\sqrt{v(t)}dB_v(t) + J_v\,(dN^U(t) - dN^D(t)), \\
dz(t) &= -\kappa_z z(t)\,dt + \sigma_{zs}\sqrt{v(t)}\,dB_s(t) + \sigma_{zv}\sqrt{v(t)}\,dB_v(t) + dB_z(t) \\
&\quad + J_z\,(dN^U(t) - dN^D(t)).
\end{aligned}$$

Especially interesting extensions are those that allow the spread and the inertia factor to jump at FOMC meetings, as they would introduce a seasonal correlation between factors that may help in producing a hump in yield reactions to target-rate changes and in the *vol curve in weeks of FOMC meetings*. As the coefficients determining the

constant is 6.1031, the coefficients on π and y are -0.4871 and -0.8058 with standard errors 0.5485, 0.3764 and 0.3068, respectively, calculated with 2 Newey-West lags. The R^2 is 23%. In the extended Taylor rule, the estimated constant is -0.3602, the coefficients on π, y and the lagged fed funds rate are 0.4968, 0.3411 and 0.9105 with standard errors 0.7473, 0.1907, 0.2251 and 0.0861, respectively. The R^2 is 91%. The data was obtained from the Federal Reserve Database.

dependence of yields on the inertia factor z are hump-shaped, z has the potential to generate just such hump-shaped patterns. The estimate in Table 4.7 of J_z is positive, meaning that an increase in the target is estimated to increase z as well, triggering yet more future θ-increases. The plot of the resulting yield reactions in Figure 4.10 shows that, for $J_z = 0.3$, there would be hump, but the estimated $J_z = 0.1$ does not suffice to generate it nor does it generate a hump in the vol curve at FOMC meetings.

Freeing up J_s allows for negative correlation between the target and the spread, opening another channel for a hump. Table 4.7 shows an estimate for J_s that is negative, -25.29 basis points, and significant. Figure 4.10 shows that this negative correlation produces humps, but that the hump in the yield reaction to θ-changes leads to a low impact of θ-changes on the short end of the yield curve, which is counterfactual (at least from a comparison with Figure 4.7).[19]

4.6 FOMC Meetings and Macro Variables

As the Fed reacts to macroeconomic variables (taken to be nonfarm payroll employment[20] and CPI inflation) when fixing its target, expectations about future macro variables matter for current yields. In this section, time-series models are explored as possible candidate descriptions of macro dynamics, but a state-space system for the joint data-generating process of analyst forecasts and actual releases is eventually preferred due to its more accurate measure of 'macro news.' The state-space system is set up after establishing two facts: One cannot reject the unbiasedness of analyst forecasts at conventional p-values (at least post-1994), and the correlation between employment and inflation is weak. The system is then build into the model, respecting the exact timing of analyst forecasts and macro releases.

[19] Allowing for jumps in the volatility process v turned out to be unnecessary, as the estimate for J_v was close to zero and not significant. Table 4.10 also shows an estimate of σ_{sv} that turns out to be positive, an old stylized fact that goes back to Cox, Ingersoll, and Ross (1985): Conditional volatility and the short rate are positively correlated.

[20] The relevance of this variable can, for example, be seen from the Minutes of past FOMC meetings. In six out of eight FOMC meetings in 1996, the Board's discussion of the "economic and financial outlook" started with a general overview of the state of the economy and then immediately turned to the value of nonfarm payroll employment.

4.6.1 Data on Analyst Forecasts and Actuals

Employment and CPI releases are made by the Bureau of Labor Statistics. Employment releases are at 8:30 a.m. on the first Friday of each month, while CPI figures are released about two weeks after the end of the reference month, also at 8:30 a.m. This means that the LIBOR recorded at 11 a.m. London time is affected by a macro release on the preceding day, while swap rates recorded at the end of the London business day react on the same day as the macro release.[21]

The actual and released CPI and nonfarm payroll employment (NPE) series are from Money Market Services (MMS). The raw series obtained from MMS are the monthly percentage change in the CPI and changes in nonfarm payroll employment in thousands. The CPI series is multiplied by 1200 to obtain the annualized inflation rate, and changes in employment are divided by 100 (to obtain a series that is similar in magnitude to CPI inflation). MMS collects data on analyst forecasts each Friday prior to the actual release from about 40 money market managers and reports their median forecast. These analyst-forecast data have been used in most studies of release surprises (for example, Balduzzi, Elton, and Green (1998), Fleming and Remolona (1998), and Li and Engle (1998)).

4.6.2 Dynamics of Macro Variables

The monthly time series of changes in nonfarm payroll employment (NPE) and CPI inflation (CPI) for the sample period considered in Section 4.2 contains only 60 monthly observations. Evidence about the macro dynamics will therefore be collected using all available data from MMS, which started surveying NPE forecasts[22] in January 1985.

Are actual investors' forecast errors well approximated by the time-series model? Over the sample period 1985:6 to 1998:12, it is not possible to outperform analyst

[21] This asynchronicity does not matter for estimation results, as they are obtained with LIBOR rate only. Swap rates are only used to generate stylized yield facts around macro releases.

[22] The extension of the sample period to pre-1994 is justified if the precise timing of policy events (target moves at FOMC meetings versus moves at random business days) does not matter for how policy impacts monthly macro series, an assumption which seems to be reasonable.

forecasts in the mean-squared-error sense with one-step-ahead forecasts of univariate or bivariate ARMA specifications (even conditioning on past target values). The errors of analyst forecasts are positively correlated with those of time-series models, but this correlation not perfect. For instance, the sample correlation coefficient is at most 0.65 for the CPI and 0.85 for NPE. A reason for the relatively low correlation between analyst errors and model errors is the oversimplified informational structure assumed by the low-dimensional time-series model. When forecasting, actual investors are able to condition on a wealth of state variables. The approach to be taken here is therefore to explore a state-space model of macro variables and analyst forecasts that introduces latent variables summarizing the conditioning information.

The first step in setting up this state-space system is to check for the unbiasedness of analyst forecast $m_F = (m_F^{CPI}, m_F^{NPE})$ of the vector $m = (m^{CPI}, m^{NPE})$ of CPI and NPE. We test for each variable whether $c_0^i = 0$ and $c_1^i = 1$ when fitting

$$m^i(t) = c_0^i + c_1^i\, m_F^i(t) + \epsilon^i(t), \qquad i = CPI,\ NPE, \tag{4.8}$$

where ϵ^i is white noise. Unbiasedness cannot be rejected at the 1% level for NPE, but is strongly rejected for the CPI series for the period 1985-1998. Concentrating on the post-1994 subsample, that used for the yield-curve model, CPI forecasts also "pass" the unbiasedness test at the 1% confidence level.[23] Finally, three lagged values of CPI, NPE and the target rate were included on the right-hand side of (4.8), but none had a significant coefficient, except perhaps for a weak effect of the first lagged CPI on NPE. To conclude, analyst forecasts of CPI and NPE provide a reasonably good description of the conditional expected values of these variables, at least in the post-1994 period, so that (4.8) will be used with $c_0^i = 0$ and $c_1^i = 1$.

The second step in setting up the state-space system for NPE and CPI is an examination of their correlation. The contemporaneous correlation between the two variables is small (0.017). When lagging one of these variables, the correlation estimates rarely exit approximate 95% confidence bounds around zero, as can be seen from Figure 4.5. In fact, NPE does not help much in predicting future CPI, as shown

[23] Balduzzi, Elton, and Green (1997) and Li and Engle (1998) conduct this test with a short history as well, and fail to reject the null.

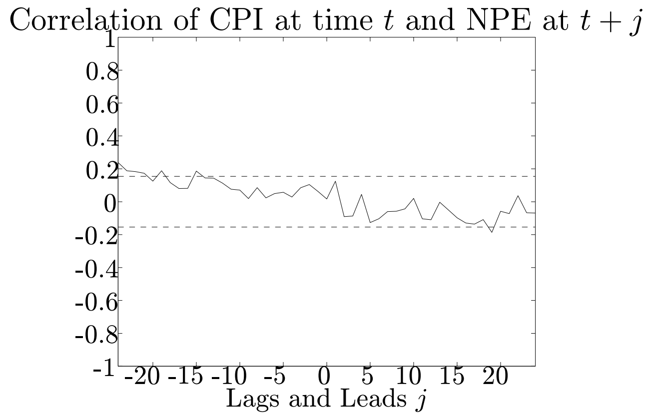


Figure 4.5: The cross correlation between CPI at time t and NPE at time $t+j$ together with approximate 95% confidence bounds ($\pm 2/\sqrt{T}$).

by Table 4.8. The CPI might, however, help in forecasting NPE. Including lagged values of the CPI in an AR(3) specification, for example, of NPE leads to a small gain in adjusted R^2 (9.5% to 12.1%). For each of CPI and NPE, we therefore specify conditionally independent subsystems, using the target rate as an exogenous variable.

The final requirements of the state-space system are that the macro variables are part of the state, and that the state is a four-dimensional autoregressive process of order 1 (this last choice will be further discussed below).

In the continuous-time economy, one deterministic counting process N^i records macroeconomic releases and another, N^i_F, counts the times at which analyst forecasts are made. Selecting a release time τ^i and the succeeding analyst forecast time τ^i_F, we can summarize the specification as:

$$\begin{aligned} m^i(\tau^i) &= m^i_F(\tau^i) + \epsilon^i(\tau^i) \\ m^i_F(\tau^i_F) &= a^i_0 + a^i_1 m^i_F(\tau^i_F-) + a^i_2 m^i(\tau^i_F-) + a^i_3\theta(\tau^i_F-) + \epsilon^j_F(\tau^i_F), \end{aligned} \tag{4.9}$$

where $\epsilon^i(\tau^i)$ and $\epsilon^i_F(\tau^i_F)$ are jointly Gaussian independent across time with mean zero, respective variances σ^i and σ^i_F, and covariance σ^i_{mF}.[24]

[24]Fore more intuition, consider the problem of modeling m_F and m in discrete time. Let the state be denoted by $z = (z^{CPI}, z^{NPE})$ with $z^i \in \mathbb{R}^2$, $i = CPI, NPE$. Leaving out the dependence on the target for the moment, the observation equations in the state space system are (1) $m^i_F(t+1) = \alpha_0 + \alpha_1\, z^i_1(t) + \alpha_2\, z^i_2(t)$ and (2) $m^i(t) = \alpha_0 + z^i_1(t)$. This shows that z_2 is the latent state that

Maximum-likelihood parameter estimates for (4.9) are reported in Table 4.9, except for the covariance parameter σ^i_{mF}, which was estimated to be essentially zero for both CPI and NPE. The forecast m^i_F of m^i does not depend on the past value of m^i (both a_2^{CPI} and a_2^{NPE} are small and insignificant), and depend only slightly on the past target (a_3^{CPI} is somewhat larger than a_3^{NPE}, but neither is significant). Based on this, m^{CPI} and m^{NPE} are both treated as the sum of an AR(1), the forecast m_F, and Gaussian white noise. This means that with this specification, we find an ARMA(1,1)-structure. This is also the specification selected by comparing Akaike and Schwarz criteria of ARMA(p,q) models that include p lagged values of the target rate. The restrictiveness of a four-dimensional state can be checked by including additional lagged values of m_F and m in the equation determining analyst forecasts. We do not find additional significant terms for the CPI forecasts, but an additional moving average for NPE forecasts improves the model-selection criteria. For the term-structure model, we adopt the specification (4.9), and re-estimate the parameters after setting a_2 to zero, but leaving a_3 in the specification so as to leave a free channel for both an effect of monetary policy on macro variables and some correlation between the CPI and NPE.

4.6.3 Estimation

The state vector X is now augmented by $(m(t), m_F(t))$, of which only $m(t)$ enters the stochastic intensities (4.2). If an FOMC meeting and a macro jump event happen on the same day, the Fed's target decisions are able to condition on the newly released information, as CPI and NPE releases (at 8:30 a.m.) precede FOMC meetings. The approximation of the likelihood function is derived in Appendix F. The estimation uses 1, 3, 6, and 12-month LIBOR rates, the target rate, CPI, NPE, and analyst forecasts. In addition to the weekly observations from Section 4.2, we include the day

summarizes the information used by investors to form the forecast m_F. The state equation is $z^i(t) = A^i\, z^i(t-1) + u(t)$ with $u(t) \sim \mathrm{N}(0, \Omega^i)$, $A^i = \begin{pmatrix} 0 & 1 \\ \alpha_1^i & \alpha_2^i \end{pmatrix}$. This system is the maximally flexible system that imposes that (i) $m^i_F(t)$ is the conditional mean of $m^i(t)$, (ii) independence between CPI and NPE, (iii) m^i is part of the state z^i, and (iv) z^i is an AR(1). This is equivalent to (4.9).

of and preceding CPI and NPE releases into the sample.

4.6.4 Results with Macroeconomic Variables

The SML parameter estimates are not reported here. A major difference to the estimation with swap yields is the magnitude of the mean-reversion parameter of the interia factor z. This already hints that the hump in yield-coefficients at this parameter vector peaks before maturities around 2 years; in fact, the peak is at 6 months.

The impact of the surprise component $m^i(\tau^i) - m^i_F(\tau^i)$ of a macroeconomic release on yields is determined by how it affects the stochastic intensity of policy events. There are two possible channels for the impulse response of a release surprise. First, the surprise can directly affect λ^U and λ^D at FOMC meetings that are scheduled before the next macro release. The effect is propagated to future intensities only through the dependence of the intensities on the past target. Second, the surprise can impact the future path of macro variables that enter the intensities at much later FOMC meetings, in addition to its direct effect. Given the dynamics (4.9), the surprise is a *temporary component of the macro variable*: it does not affect the path of future macro variables. Release suprises therefore have a 'short life' by being propagated only through the first channel. The estimated parameters λ_{CPI} and λ_{NPE} indicate that this effect is small. In the base-case model, we thus get the result that *release surprises are not inertia factors themselves.* By introducing, for example, a jump in the inertia factor z at release days that is correlated with the release surprise, we can make the release surprise 'live longer,' as a shock to z affects intensities in the farer future.

Figure 4.11 shows the cross-sectional contemporanous impulse response of yields to a one-standard deviation release surprise to NPE and the CPI. We can compare these model-implied impulse responses to a least-squares regression of yield changes on release surprises (also in Figure 4.11). We can see that a NPE release surprise has a larger impact than a CPI surprise, which translates into a stronger seasonal effect on the term structure of volatilities of yields. The response of yields monotonically descreases with maturity because of the shock's propagation through only the first channel.

4.7 Conclusion

The estimated yield-curve model explains the "snake-shaped" term structure of volatility in yields, based on interest-rate smoothing and policy inertia. Macroeconomic surprises are only temporary components of macro variables. This means that the impact of these surprises on longer yields needs to occur over time through a "policy-inertia factor." The model improves the fit of bond prices over a 3-latent-factor model, especially for short maturities. A policy rule is identified from weekly yield data and is found to provide a good description of the target. In fact, model-based forecasts of future target rates outperform several benchmarks.

Table 4.5: Estimation Results for Base Models

Parameter	λ Model		SV Model		SVλ Model	
	Constr.	Unconstr.	Constr.	Unconstr.	Constr.	Unconstr.
κ_s	1.277306	1.556912	2.861167	3.332016	6.437270	9.748661
	(2.528269)	(4.102793)	(3.895496)	(4.202992)	(3.832683)	(4.693933)
κ_v	–	–	0.1659727	0.129618	0.124326	0.040925
	–	–	(3.738001)	(3.796853)	(0.411320)	(0.415643)
κ_z	0.276054	0.288299	–	–	0.466113	0.724400
	(2.690688)	(3.028778)	–	–	(4.730788)	(4.336113)
$\bar{\theta}$	0.052213	0.052213	0.052213	0.052213	0.052213	0.052213
	–	–	–	–	–	–
$\bar{v}$	–	–	4.480144e-05	5.899065e-05	2.281740e-04	4.150809e-04
	–	–	(5.694430)	(4.528638)	(0.506240)	(1.067214)
σ_s	7.122488e-05	7.989389e-05	–	–	–	–
	(51.385560)	(12.499803)	–	–	–	–
σ_v	–	–	5.756028e-03	6.525899e-03	0.054158	0.089933
	–	–	(2.040981)	(1.367602)	(0.721646)	(0.775665)
λ_0^u	758.158092	338.590888	341.664950	-323.321781	1.160316e+03	273.665764
	(0.106748)	(0.395815)	(0.033362)	(-0.020226)	(0.153202)	(0.548656)
λ_s	6.769235e+03	7.582229e+03	2.013169e+04	1.868451e+04	6.117227e+03	7.267293e+03
	(1.477492)	(2.246950)	(2.304910)	(2.135644)	(2.324139)	(1.856327)
λ_θ	$-5.269678e+03$	$-4.876673e+03$	$-1.263833e+04$	$-1.479579e+04$	$-8.738577e+03$	$-9.408883e+03$
	(-3.895510)	(-4.302838)	(-7.475722)	(-9.228838)	(-6.527465)	(-4.175201)
λ_v	–	–	1.783100e+07	1.861178e+07	$-8.919179e+04$	5.483153e+05
	–	–	(2.549629)	(2.256711)	(-0.601905)	(1.629651)
λ_z	123.034098	119.4507	–	–	234.823691	237.629397
	(18.737795)	(18.630551)	–	–	(21.788983)	(17.870117)
q_s	70.120506	70.285118	-257.530126	-241.323541	-89.933014	-47.618971
	(3.274451)	(8.673452)	(-5.289520)	(-4.785031)	(-3.222643)	(-2.904623)
q_v	–	–	499.896256	1051.160708	$-1.018602e+04$	$-2.537544e+03$
	–	–	(0.047090)	(0.105902)	(-1.192666)	(-0.810859)
q_z	-0.209059	-0.213215	–	–	0.2553252	0.112632
	(-132.732571)	(-33.360167)	–	–	(27.818100)	(0.181483)
σ_M	1.364800e-03	2.132152e-03	1.155277e-03	1.145397e-03	–	–
	(15.119999)	(14.766891)	(17.539221)	(18.142008)	–	–
ρ_M	0.961300	0.955110	0.982425	0.983664	–	–
	(58.641087)	(56.126410)	(78.932304)	(88.109031)	–	–

NOTE: This table reports the SML parameter estimates and t-ratios (in brackets) obtained with $S = 2500$, $h = \frac{1}{M}\frac{1}{365}$, $M = 1$, $M_s = 30$ and weekly observations of the 6-month LIBOR, 2 and 5-year swap rate from January 1, 1994 to December 31, 1998. σ_M is the standard deviation of the measurement error contaminating observations of the 2-year swap rate and ρ_M is its autocorrelation. For the SV Model, $\lambda_0^d = 619.6040$ and $\bar{\lambda}^u = \bar{\lambda}^d = 480.6345$ for the constrained model; $\lambda_0^d = 327.3511$ and $\bar{\lambda}^u = \bar{\lambda}^d = 2.0146$ for the unconstrained model. For the SVλ Model, $\lambda_0^d = 207.0786$ and $\bar{\lambda}^u = \bar{\lambda}^d = 683.6972$ for the constrained model; $\lambda_0^d = 273.6658$ and $\bar{\lambda}^u = \bar{\lambda}^d = 9.9949$ for the unconstrained model.

Table 4.6: Forecasting Evaluation of Target Model

	Same Change				No Change			
Predicted Actual	up	no	down	total	up	no	down	total
up	2	5	0	7	0	0	0	0
no	5	20	3	28	7	28	5	40
down	0	3	2	5	0	0	0	0
correct	2	20	2	24	0	28	0	28
total	7	28	5	40	7	28	5	40
% correct	28.57	71.42	40	60	0	100	0	70

	Unconstr. λ Model				Constr. λ Model			
Predicted Actual	up	no	down	total	up	no	down	total
up	7	10	0	17	7	28	4	39
no	0	18	5	23	0	0	0	0
down	0	0	0	0	0	0	1	1
correct	7	18	0	25	7	0	1	8
total	7	28	5	40	7	28	5	40
% correct	100	64.29	0	62.50	100	0	20	20

	Unconstr. SV Model				Constr. SV Model			
Predicted Actual	up	no	down	total	up	no	down	total
up	5	3	0	8	7	17	0	24
no	2	25	5	32	0	0	0	0
down	0	0	0	0	0	11	5	16
correct	5	25	0	30	7	0	5	12
total	7	28	5	40	7	28	5	40
% correct	71.43	89.29	0	75	100	0	100	30

	Unconstr. SVλ Model				Constr. SVλ Model			
Predicted Actual	up	no	down	total	up	no	down	total
up	4	2	0	6	7	23	0	30
no	3	26	5	34	0	0	0	0
down	0	0	0	0	0	5	5	10
correct	4	26	0	30	7	0	5	12
total	7	28	5	40	7	28	5	40
% correct	57.14	92.86	0	75	100	0	100	30

NOTE: The sample used in this table is January 1, 1994 to December 31, 1998. During this time, there have been 40 FOMC meetings, 8 moves up in the target (1 outside of an FOMC meeting) and 6 moves down (1 outside of an FOMC meeting). This means that in a constant probability model, the estimated probability of a move up and down is 7/40 and 5/40, respectively. Forecasting a particular choice (up, down, no) is defined as the alternative with the highest probability. As to the 2 changes outside of FOMC meetings, all models would have missed them, so they are not part of the table.

Table 4.7: Estimation Results for Some Extensions of the SVλ-Model

Parameter	Jumps in z	Jumps in s	Corr. σ_{sv}
κ_s	14.967792	7.820412	9.933076
	(6.398363)	(5.214696)	(4.473048)
κ_v	0.0115368	0.054233	0.042497
	(0.215357)	(1.584036)	(0.454876)
κ_z	0.641243	0.679306	0.748360
	(3.648210)	(7.190619)	(4.523098)
$\bar{\theta}$	0.052213	0.052213	0.052213
	–	–	–
$\bar{v}$	1.056887e-03	2.804654e-04	3.659218e-04
	(2.137352)	(1.342108)	(1.059371)
σ_v	0.017776	0.066227	0.085224
	(0.455049)	(1.390437)	(0.843764)
λ_0^u	62.358567	49.883635	271.189584
	(0.111839)	(0.073574)	(0.0520345)
λ_s	6.998324e+03	5.582408e+03	7.324409e+03
	(3.115227)	(1.587551)	(1.860092)
λ_θ	-1.560489e+04	-1.526898e+04	-9.240203e+03
	(-4.552850)	(-3.920868)	(-4.836343)
λ_v	7.271786e+05	1.073800e+06	6.046136e+05
	(2.160996)	(1.299366)	(1.659181)
λ_z	187.103829	312.683229	240.092789
	(9.144435)	(9.095217)	(22.691191)
q_s	-27.058453	-101.979831	-46.167988
	(-3.770240)	(-3.427597)	(-2.961026)
q_v	-1.349147e+03	-3.588000e+03	-2.322817
	(-1.526325)	(-1.152997)	(-0.741050)
q_z	0.090891	0.067808	0.165448
	(0.151579)	(-0.069894)	(0.281479)
J_z	0.103886	–	–
	(1.985465)		
J_s	–	-0.002529	–
		(-6.145468)	
σ_{sv}	–	–	20.150802
			(1.221437)

NOTE: This table resports SML parameter estimates and t-ratios (in brackets) obtained with $S = 2500$, $h = \frac{1}{M}\frac{1}{365}$, $M = 1$, $M_s = 30$ and weekly observations of the 6-month LIBOR, 2 and 5-year swap rate from January 1, 1994 to December 31, 1998.

Table 4.8: Granger Causality

	n=1	n=3	n=6
CPI predictable by NPE	0.4650	0.7415	0.9608
NPE predictable by CPI	0.1144	0.0592	0.1260
CPI predictable by target	0.0001	0.0197	0.0300
NPE predictable by target	0.0862	0.5028	0.8110

NOTE: This table reports the p-values corresponding to the usual F-test that the coefficients on all lags of the indicated regressor are zero. More precisely, the dependent variable is regressed on n lags of itself and n lags of the regressor. The sample used for this table is 1985:6 to 1998:12.

Table 4.9: Joint Dynamics of Analyst Forecasts and Actual Releases

	$j = CPI$	$j = NPE$
a_0^j	0.006095	0.005817
	(0.603442)	(1.077137)
a_1^j	0.556357	0.549290
	(3.078901)	(2.874358)
a_2^j	-0.027615	-0.011684
	(-0.187600)	(-0.106604)
a_3^j	0.164741	0.037299
	(1.006975)	(0.507469)
σ^j	0.021181	0.016771
	(7.303732)	(8.385483)
σ_F^j	0.018219	0.009146
	(12.146177)	(10.161710)

NOTE: This table reports maximum likelihood estimates of (4.9) using the sample 1985:2 to 1998:12. The target rate $\theta(\tau_F^j)$ in (4.9) is the value of the target on the day before the analyst forecast survey τ_F^j, as releases occur at 8:30 am and FOMC meetings after that.

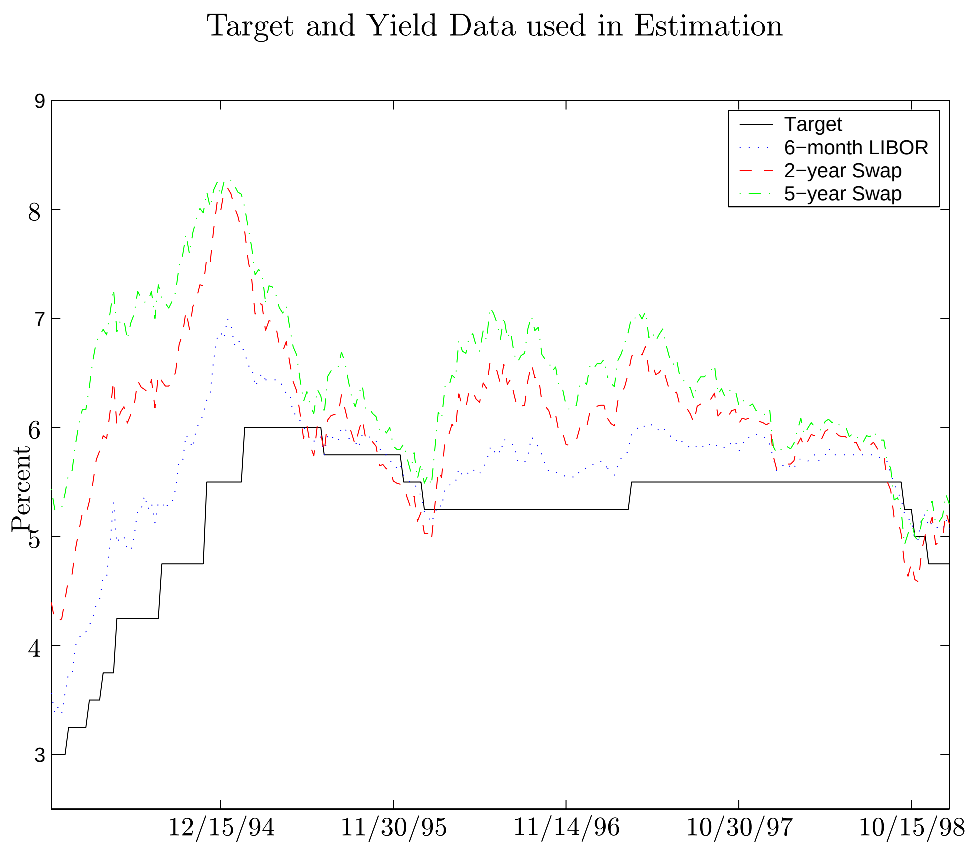


Figure 4.6: This graph shows weekly data on the 6-month LIBOR, 2 and 5-year swap rate together with the Fed's target from January 1, 1994 to December 31, 1998.

The Effects of FOMC Meetings

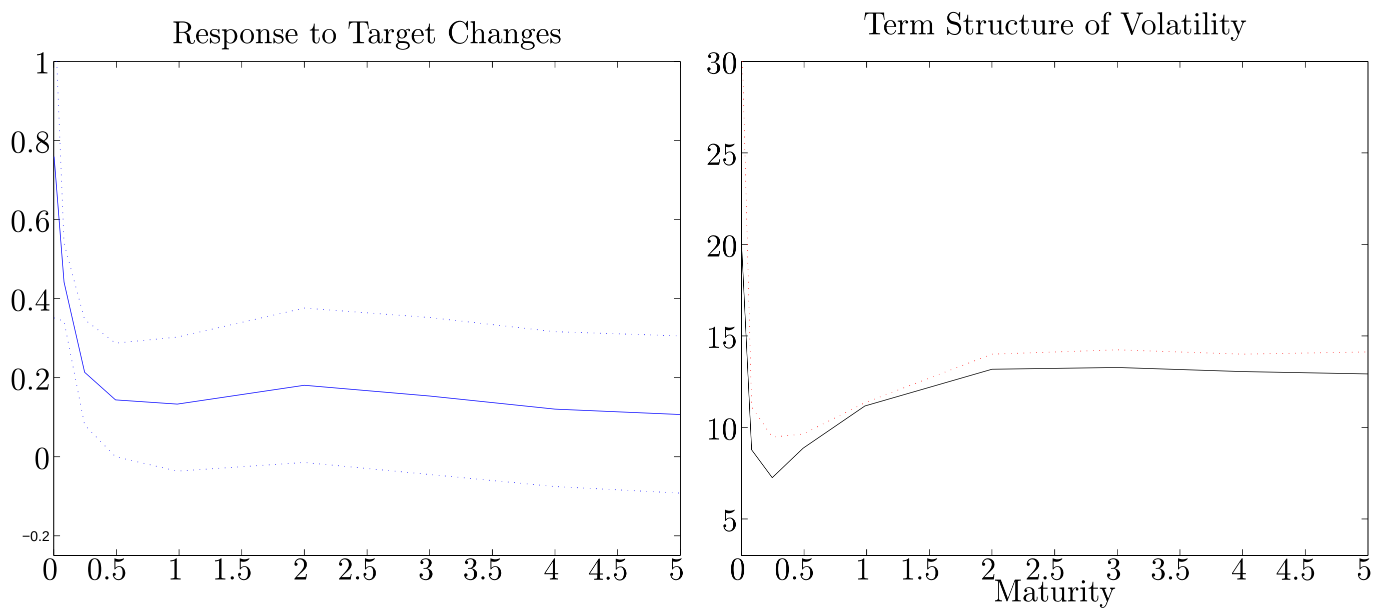


Figure 4.7: These graphs show the response of yields to target changes and the term structure of volatility in the data. The response of yields to target changes is measured by the slope parameter of weekly yield changes regressed on target rate changes (and an intercept) in weeks of FOMC meetings, including the weeks of April 24, 1994 and October 15, 1998. Dotted lines are standard-error bounds computed using a SUR specification. The term structure of volatility during weeks of FOMC meetings (dotted line) and the remaining weeks is measured as standard deviations of yield changes. The weekly data are Wednesday observations from 1994 to 1998 on the target rate, overnight repo rate, and Thursday observations on the 1, 3, 6, 12-month LIBOR and 2, 3, 4, 5-year Swap Rates. Standard-error bounds around the volatility estimates are computed in the following table with GMM using 5 Newey-West lags.

Standard Errors around Volatility Estimates (in Basis Points)

	Repo	LIBOR Rates				Swap Rates			
	Overnight	1 mth	3 mth	6 mth	12 mth	2 yr	3 yr	4 yr	5 yr
Vol	19.90	8.78	7.26	8.87	11.17	13.18	13.27	13.05	12.92
'Normal'	(2.01)	(1.53)	(0.86)	(0.84)	(0.79)	(0.75)	(0.76)	(0.70)	(0.67)
Vol	31.72	11.13	9.47	9.64	11.35	14.01	14.24	14.00	14.12
FOMC	(2.21)	(1.69)	(1.37)	(1.88)	(2.12)	(1.90)	(1.77)	(1.75)	(1.83)

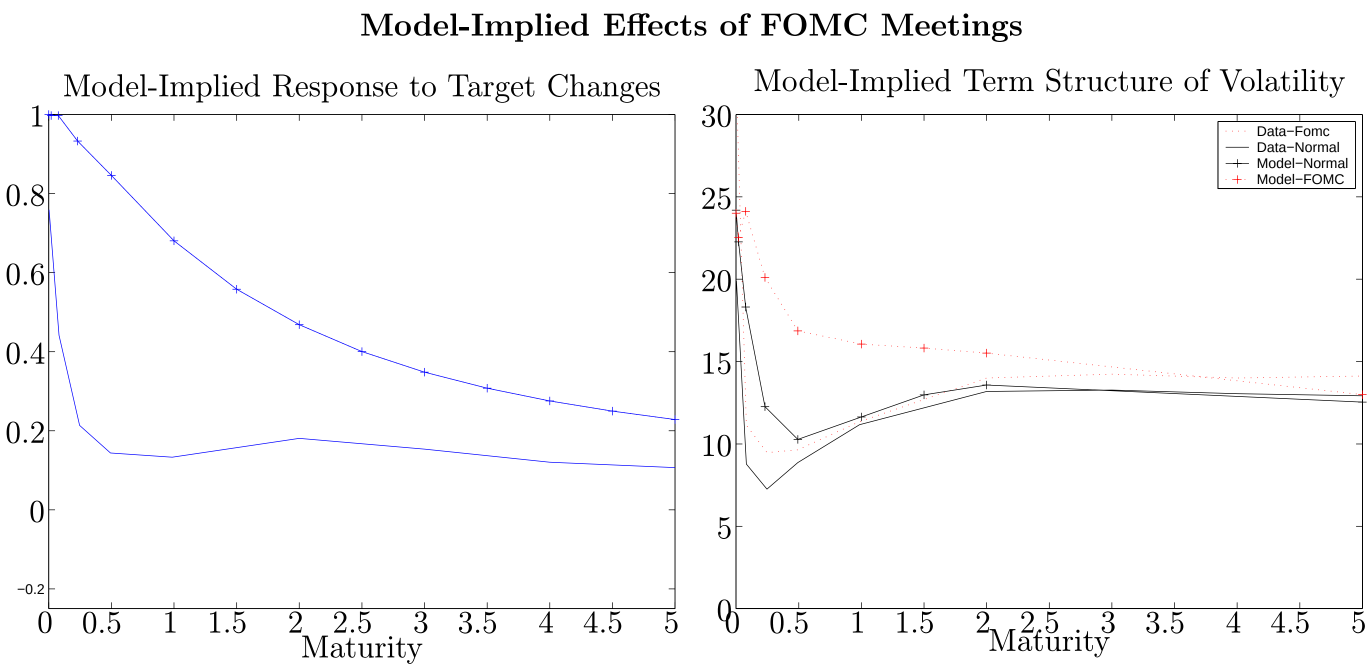
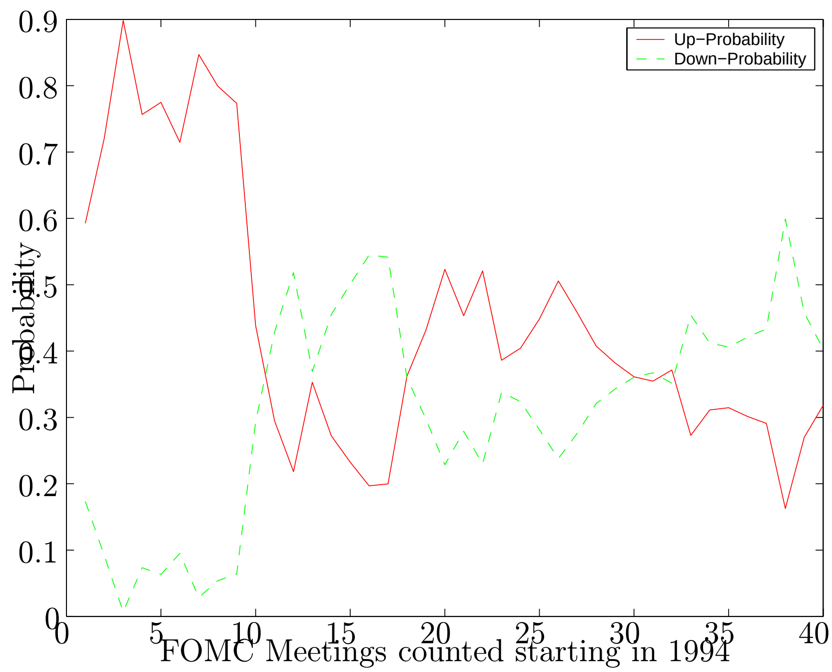


Figure 4.8: The left graph plots the response of yields to target changes estimated in an unrestricted estimation (solid line, same as in Figure 4.7) together with the model-implied response measured by calculating the analytical derivative dy/dx and multiplying it by $400J_U$ (dotted line with +). The right figure is the term structure of volatility during weeks of FOMC meetings (dotted line) and the remaining weeks (black line) in the data (without +) and in a simulation of the model (with +). The simulations start with $S = 20,000$ initial states $\hat{X}_0$ that are obtained by simulating the state dynamics for 10 years, starting at the unconditional mean. Each day is subdivided into 2 intervals and each FOMC meeting is subdivided into 30 intervals. Given $\hat{X}_0$, S different samples of yields are simulated, each sample uses the actual FOMC calendar.



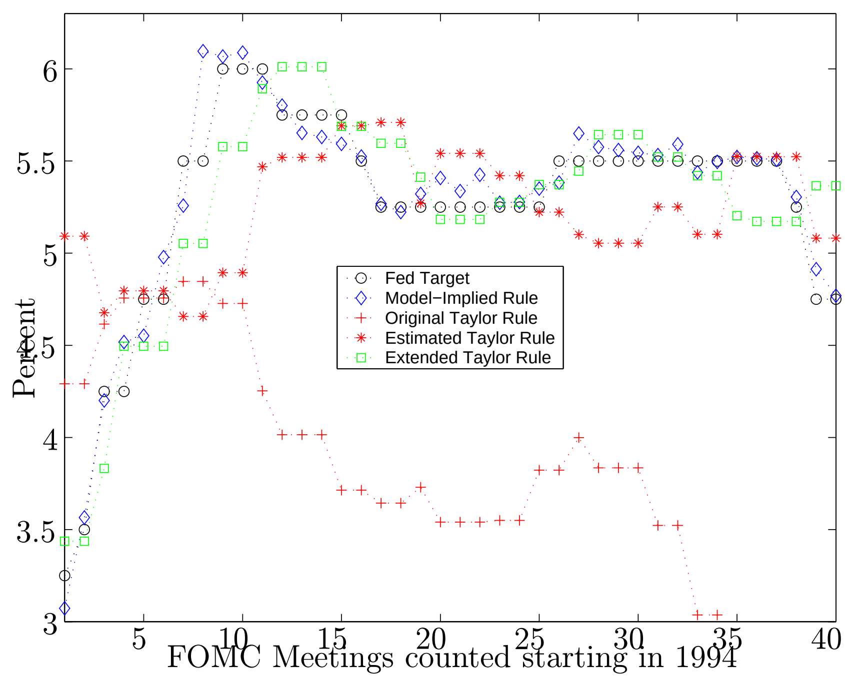


Figure 4.9: For each FOMC meeting since January 1994, the upper graph plots the conditional probability of target rate moves up (straight line) and down (dotted line) from the unconstrained SVλ model. The lower graph shows the model-implied policy rule together with versions of the Taylor rule (see Section 4.4.6).

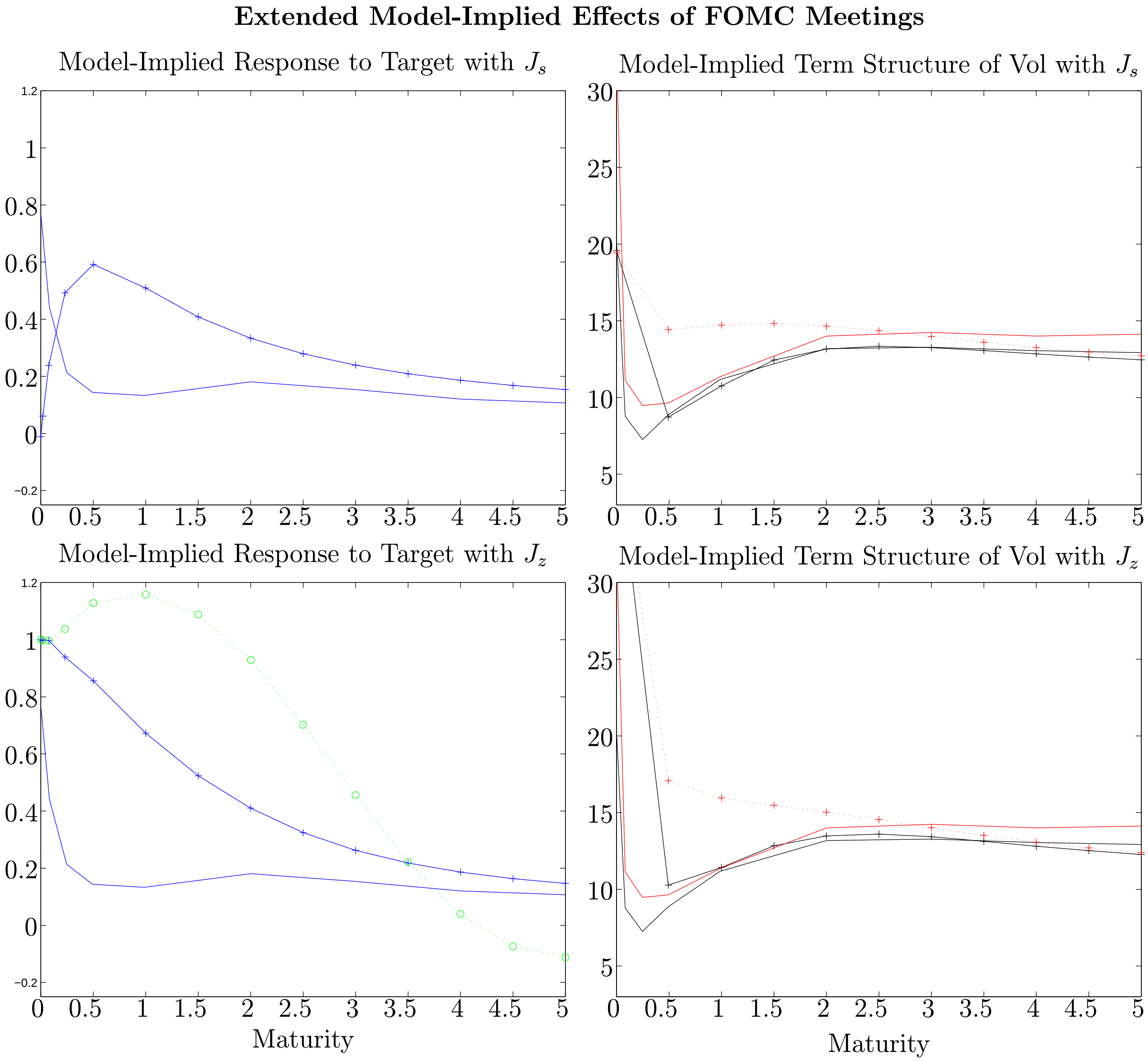


Figure 4.10: Analogues of Figure 4.8 for extended SVλ models using parameters from Table 4.7: J_s (first row) and J_z (second row). The line marked with circles in the lower left corner shows sets $J_z = 0.3$.

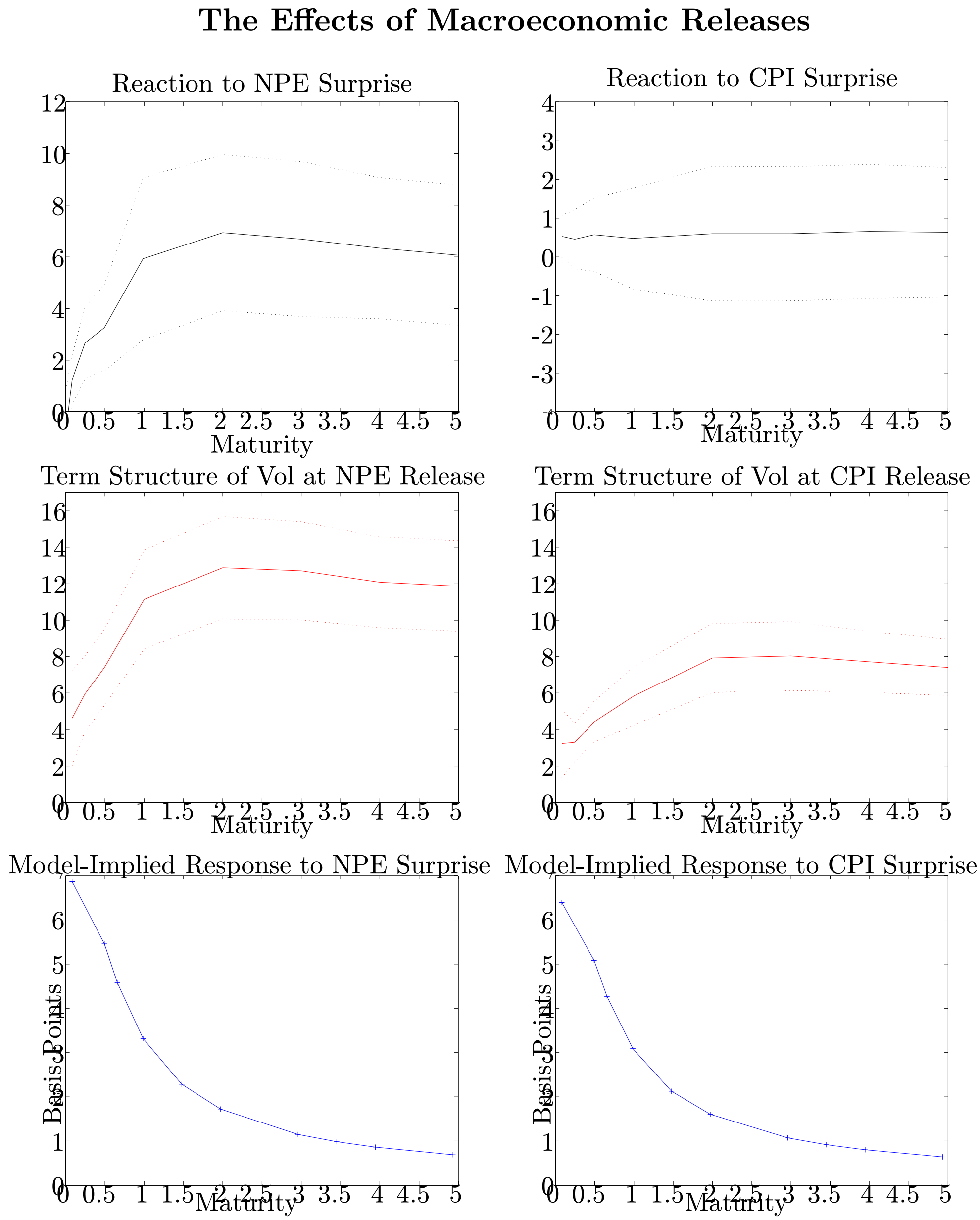


Figure 4.11: The first row of graphs shows the slope parameter of daily yield changes regressed on NPE and CPI standardized surprises (and an intercept) using the subsample of the respective release days. Dotted lines are standard-error bounds computed with 5 Newey-West lags. Standardized surprises are defined as the analyst forecast error $m(t) - m_F(t)$, normalized by its standard deviation, so that the regression coefficients can be interpreted as reactions to a one-standard deviation analyst forecast error. The second row of graphs is the term structure of volatility at the respective release days. Standard errors are computed using 5 Newey-West lags. The third row of graphs shows the model-implied cross-sectional (contemporanous) impulse response of yields to NPE and CPI release surprises. Due to time-zone differences, the data used for this estimation are same-day observations from 1994 to 1998 on 2, 3, 4, 5-year swap rates, and next-day observations of 1, 3, 6 and 12-month LIBOR rates.

Appendix A

Details for Change of Measure

Suppose that ξ solves the SDE (2.9). We first state Assumption 2, under which ξ is a square-integrable $\mathcal{P}$-martingale (Proposition 1).

Assumption 2 (Assumptions on Market Prices of Uncertainty): The processes σ_ξ, J_ξ^d, and J_ξ^p are progressively measurable and satisfy:

(a) $J_\xi^p(t) > -1$ and $J_\xi^d(t) > -1$.

(b) $E_{t-}^{\mathcal{P}}\left[J_\xi^d(t)\right] = 0$.

(c) Novikov's Condition: $E^{\mathcal{P}}\left[\exp\left(\int_0^T \sigma_\xi(t)\sigma_\xi^\top(t)\,dt\right)\right] < \infty$.

(d) $E^{\mathcal{P}}\left[\exp\left(2\int_0^T \ln(1+J_\xi^p(t))\,dM_p(t)\right)\right] < \infty$
$E^{\mathcal{P}}\left[\exp\left(2\int_0^T \ln(1+J_\xi^d(t))\,dN_d(t)\right)\right] < \infty$.

(e) Given a Poisson jump time τ, the $\mathcal{F}(\tau-)$-conditional distribution of $J_\xi^p(\tau)$ is $\mathcal{F}(0)$-measurable.

The integrability conditions A2(c) and A2(d) are easily satisfied, for example, for constant J_ξ^d, J_ξ^p and σ_ξ, but in this case we would need to set $J_\xi^d = 0$ from A2(b), meaning that investors do not demand an uncertainty premium for jump risk at deterministic jump times. Appendix B provides an example in which J_ξ^d is stochastic.

Proposition 1 (Martingale Property for ξ): Under A2, the solution ξ to (2.9) is a positive, square-integrable martingale.

Proof: Applying Ito's Lemma (for semimartingales, Theorem 33 in Protter (1990), p. 74) to (2.9), we get

$$\begin{aligned} d\ln\xi(t) &= -\sigma_\xi(t)\,dW(t) - \frac{1}{2}\sigma_\xi(t)\sigma_\xi(t)^\top dt \\ &\quad + \ln\left(1+J_\xi^d(t)\right)\,dN_d(t) + \ln\left(1+J_\xi^p(t)\right)\,dM_p(t). \end{aligned}$$

The solution for ξ is well defined, due to A2(a). Moreover, it is exponential and therefore positive. The proof that ξ is a martingale is by induction over the deterministic jump times $\tau_1^d, \ldots, \tau_N^d$. We derive now two intermediate results.

(R1) At the i-th deterministic jump time τ^d, from A2(b),

$$\begin{aligned} E^{\mathcal{P}}_{\tau_i^d-}\left[\xi(\tau_i^d)\right] &= E^{\mathcal{P}}_{\tau_i^d-}\left[\xi(\tau_i^d-) + \xi(\tau_i^d-)J_\xi^d(\tau_i^d)\right] \\ &= \xi(\tau_i^d-)E^{\mathcal{P}}_{\tau_i^d-}\left[1+J_\xi^d(\tau_i^d)\right] \\ &= \xi(\tau_i^d-). \end{aligned} \tag{A.1}$$

(R2) The SDE in (2.9) has no drift term (W and M_p are martingales), so that, for t and s in $[\tau_{i-1}^d, \tau_i^d)$ with $t \le s$, we have $E_t^{\mathcal{P}}[\xi(s)] = \xi(t)$ because of the integrability conditions A2(c) and A2(d).

The process ξ is a martingale for $t \in [\tau_N^d, T]$ because of (R2). Suppose that for any τ_i^d, (R1) holds at τ_{i+1}^d. We can apply (R2) to get the desired property for $t \in [\tau_i^d, \tau_{i+1}^d)$, and then apply (R1) to obtain this property for τ_i^d as well. By induction, ξ is a martingale during $[0,T]$ ((R2) can also be applied to $[0, \tau_1^d)$).

Due to the exponential form of ξ, square integability is implied from Novikov's condition A2(c), the corresponding integrability condition for jump sizes A2(d), and the fact that W, M_p, and N_d are independent. ∎

We can now fix T and let $d\mathcal{Q}/d\mathcal{P} = \xi(T)$ be the Radon-Nikodym derivative of $\mathcal{Q}$ with respect to $\mathcal{P}$. The next proposition provides a representation of the dynamics of the state process X under $\mathcal{Q}$. The proposition is stated for one-dimensional versions of N_p and N_d, but is easily extended to the multidimensional case by attaching subscripts i to all jump sizes and intensities in the statements (b) and (c), so that they hold for each component of N_p and N_d.

Proposition 2 ("Generalized Girsanov Theorem"): Suppose that A2 holds and fix T. Under the probability measure $\mathcal{Q}$ with density process ξ, we have:

(a) X solves an SDE with drift and diffusion coefficients

$$\begin{aligned} \mu^{\mathcal{Q}}(x,t) &= K(t)(\bar{x}(t)-x) - \sigma(x,t)\sigma_\xi(t)^\top, \\ \sigma^{\mathcal{Q}}(x,t) &= \sigma(x,t). \end{aligned}$$

(b) The counting process N_p has a stochastic intensity

$$\lambda^{\mathcal{Q}}(t) = \lambda(t)\, E^{\mathcal{P}}\left[1 + J^p_\xi(t)\right].$$

(c) For any bounded measurable function $h : \mathbb{R}^N \to \mathbb{R}$,

$$\begin{aligned} E^{\mathcal{Q}}\left[h(J^p(t))\right] &= E^{\mathcal{P}}\left[h(J^p(t))\frac{(1+J^p_\xi(t))}{E^{\mathcal{P}}[1+J^p_\xi(t)]}\right], \\ E^{\mathcal{Q}}_{t-}\left[h(J^d(t))\right] &= E^{\mathcal{P}}_{t-}\left[h(J^d(t))\frac{\left(1+J^d_\xi(t)\right)}{E^{\mathcal{P}}_{t-}\left[1+J^d_\xi(t)\right]}\right]. \end{aligned}$$

Moreover, $W^{\mathcal{Q}}(t) = W(t) + \int_0^t \sigma_\xi(u)\, du$ defines a standard Brownian motion under $\mathcal{Q}$, and $M^{\mathcal{Q}}_p(t) = N_p(t) - \int_0^t \lambda^{\mathcal{Q}}(u)\, du$ is a compensated Poisson process under $\mathcal{Q}$.

Proof: First, we need to construct a Brownian motion and the intensities of the Poisson processes under $\mathcal{Q}$. For this part of the proof, it is possible to refer to the relevant results in Duffie, Pan, and Singleton (1998, Proposition 4).

Second, we need to show how to obtain the density of jump sizes at deterministic jump events under $\mathcal{Q}$. For $h(\cdot)$ bounded and measurable, we have

$$E_{t-}^{\mathcal{Q}}\left[h(J^d(t))\right] = E_{t-}^{\mathcal{P}}\left[h(J^d(t))\frac{\xi(t)}{\xi(t-)}\right] = E_{t-}^{\mathcal{P}}\left[h(J^d(t))\frac{\xi(t-)\left(1+J_\xi^d(t)\right)}{\xi(t-)}\right].$$

Because of A2(b), we get the result.■

The last proposition shows how to specify market prices of uncertainty, so that X is a LQJD under both measures, $\mathcal{P}$ and $\mathcal{Q}$. For example, from (a) we see that $\sigma(X(t),t)\sigma_\xi(t)^\top$ must be affine in X if the affine drift structure is to be preserved.

Appendix B

Example for J_ξ^d

This Appendix provides an example in which the market price of jump uncertainty at scheduled announcements is non-trivial. For a deterministic jump time τ^d, let $\epsilon \sim \mathrm{N}(0,1)$ be an $\mathcal{F}(\tau^d)$-measurable random variable, and let $z = \mu + \epsilon\,\sigma$ for constants[1] μ and σ. Suppose that at time τ^d the price of an asset is $F(\tau^d) = \exp\left(u_0(\tau^d) + u_1(\tau^d)X(\tau^d)\right)$, for time-dependent coefficients u_0 and u_1, so that $J_F^d(\tau^d) = \exp\left(u_1(\tau^d)\Delta X(\tau^d)\right) - 1$. For simplicity, let X be one-dimensional and let its jump size at τ^d be $J^d(\tau^d) = z$. Now let $J_\xi^d(\tau^d) = \tilde{\xi} - 1$, where $\tilde{\xi} = \exp\left(-\sigma_\xi^d\,\epsilon - \frac{1}{2}(\sigma_\xi^d)^2\right)$.

Assumption A2(a) holds because $\tilde{\xi} > 0$, while A2(b) is satisfied because, for the deterministic jump time τ^d,

$$E_{\tau^d-}^{\mathcal{P}}\left(J_\xi^d(\tau^d)\right) = E_{\tau^d-}^{\mathcal{P}}\left(\tilde{\xi} - 1\right) = 0.$$

Condition A2(d) is satisfied since N_d does not explode. All other parts of A3 are not needed here. Finally, we need to verify that

$$\begin{aligned} E_{\tau^d-}^{\mathcal{Q}}\left[J_F^d(\tau^d)\right] &= \frac{E_{\tau^d-}^{\mathcal{P}}\left[\xi(\tau^d)J_F^d(\tau^d)\right]}{\xi(\tau^d-)} = \frac{E_{\tau^d-}^{\mathcal{P}}\left[\xi(\tau^d-)(1+J_\xi^d(\tau^d))J_F^d(\tau^d)\right]}{\xi(\tau^d-)} \\ &= E_{\tau^d-}^{\mathcal{P}}\left[\tilde{\xi}J_F^d(\tau^d)\right] = E_{\tau^d-}^{\mathcal{P}}\left[\tilde{\xi}\exp(u_1(\tau^d)z)\right] - 1 = 0. \end{aligned}$$

[1] For ease of notation, μ and σ are assumed constant. Everything goes through if they are bounded functions of time.

This is equivalent to

$$\mu + \frac{1}{2}\sigma^2 u_1 = \sigma_\xi^d \sigma,$$

which shows that any σ_ξ^d solving this last equation can be used to adjust for uncertainty at deterministic jump times.[2]

[2]Underlying this example is the following basic result about static changes of measure. Suppose that $\epsilon \sim \mathrm{N}(0,1)$ under the original measure $\mathcal{P}$. Define $\tilde{\xi} = \exp(-\sigma_\xi^d \epsilon - \frac{1}{2}(\sigma_\xi^d)^2)$ and the (equivalent) probability measure $d\mathcal{Q}/d\mathcal{P} = \tilde{\xi}$. Under $\mathcal{Q}$, $\epsilon \sim \mathrm{N}(-\sigma_\xi^d, 1)$.

Appendix C

Statement of Lemma 1

Let the times at which deterministic jumps occur between t and T be denoted $\tau_1^d, \ldots, \tau_n^d$.

Lemma 1: Suppose that Assumptions 1 and 3 hold under $\mathcal{Q}$. Additionally, for $t = \tau_i^d$, for some i, suppose that $P(t,T) = \exp\left(g\left(X(t), \bar{c}\,\right)\right)$ and for some $\bar{c} = (\bar{c}_0, \bar{c}_1, \bar{c}_2) \in C$. Then there exist coefficients $c \in C$ such that $P(t-,T) = \lim_{s\uparrow t} P(s,T)$ is given by

$$P(t-,T) = \exp\left(g\left(X(t-), c\,\right)\right). \tag{C.1}$$

Proof: From equation (2.2),

$$\begin{aligned}
P(t-,T) &= E_{t-}^{\mathcal{Q}}\left[P\left(t,T\right)\right] \\
&= E_{t-}^{\mathcal{Q}}\left[\exp\left(g\left(X(t), \bar{c}\,\right)\right)\right] \\
&= \exp\left(g\left(X(t-), \bar{c}\,\right)\right) E_{t-}^{\mathcal{Q}}\left[\exp\left(\bar{c}_1 \cdot \Delta X(t)\right)\right] \\
&= \exp\left(g\left(X(t-), \bar{c}\,\right)\right)\, \exp\left(g\left(X(t-), a(t;\bar{c}_1)\right)\right) \\
&= \exp\left(g\left(X(t-), \bar{c} + a(t;\bar{c}_1)\right)\right),
\end{aligned}$$

where $E_{t-}^{\mathcal{Q}}$ denotes $\mathcal{F}(t-)$-conditional expectation under $\mathcal{Q}$, and the fourth equality holds for some $a(t;\bar{c}_1) \in C$ because of Definition (A1.c.). ■

Appendix D

Statement of Lemma 2

Lemma 2: Suppose that, for some i such that $s = \tau_{i+1}^d$, $P(s-,T) = \lim_{t \uparrow s} P(t,T)$ can be represented as $P(s-,T) = \exp\left(g\left(X(s-), \bar{c}\,\right)\right)$ for some $\bar{c} \in C$. Let Assumptions 1, 3 be satisfied under $\mathcal{Q}$. Also suppose that Assumption 4 below is satisfied at $(s, \bar{c})$. Then for each $t \in [\tau_i^d, s)$, there exist coefficients $c(t,s) \in C$ such that

$$P(t,T) = \exp\left(g\left(X(t), c(t,s)\,\right)\right), \tag{D.1}$$

where $c(\,\cdot\,, s) : [\tau_i^d, s] \to C$ solves the system of ordinary differential equations (ODE's) in (D.5)-(D.7) stated below, with terminal condition $c(s,s) = \bar{c}$.

Proof:

Lemma 2 applies the standard Feynman-Kac approach to equation (2.2) between deterministic jump times. The approach proceeds in two steps. In a first step, the relevant Cauchy problem is stated and solved. In a second step, integrability conditions are imposed so that the bond price at time $t \in [\tau_i^d, s)$ can indeed be viewed as the Feynman-Kac solution to the Cauchy problem of Step 1.

Step 1: Set up and solve the relevant Cauchy Problem.
Consider the following Cauchy problem. For all $t \in [\tau_i^d, s)$ and $x \in D$, let $F(t,s,x)$

solve the partial differential-integral equation (PDIE)

$$
\begin{aligned}
0 &= F_t(t,s,x) + F_x(t,s,x)\cdot\mu(x,t) \\
&+ \frac{1}{2}\, tr\,\left[F_{x\,x}(t,s,x)\sigma(x,t)\sigma(x,t)^\top\right] \\
&+ \sum_{i=1}^{p} g(x, l_i^{\mathcal{Q}}(t)) E^{\mathcal{Q}}\left[F(t,s,x+J_i^p(t)) - F(t,s,x)\right] - R(x,t)F(t,s,x),
\end{aligned} \tag{D.2}
$$

with terminal condition $F(s,s,x) = \exp\left(g\left(x,\bar{c}\right)\right)$.

We guess a solution of the form

$$
F(t,s,x) = \exp\left(g\left(x, c(t,s)\right)\right), \tag{D.3}
$$

where the coefficients $c(t,s) = (c_0(t,s), c_1(t,s), c_2(t,s))$ satisfy terminal conditions at s given by $\bar{c} = (\bar{c}_0, \bar{c}_1, \bar{c}_2)$. Now we verify that the guess in (D.3) solves the PDIE (D.2) for all $t \in [t_i, s)$. By applying Ito's Lemma to (D.3) and using the fact that $F(t,s,x)$ is strictly positive, we have

$$
\begin{aligned}
0 &= \frac{dc_0(t,s)}{dt} + \frac{dc_1(t,s)}{dt}\cdot x + x^\top\,\frac{dc_2(t,s)}{dt}\,x \\
&+ \left(c_1(t,s) + 2\,c_2(t,s)\,x\right)\cdot K^{\mathcal{Q}}(t)(\bar{x}^{\mathcal{Q}}(t) - x) \\
&+ \frac{1}{2} tr\big[\left((c_1(t,s) + 2c_2(t,s)x)(c_1(t,s) + 2c_2(t,s)x)^\top + 2c_2(t,s)\right) \\
&\left(\Sigma(t)\,S(x,t)S(x,t)^\top\Sigma(t)^\top\right)\big] \\
&+ \sum_{i=1}^{p} g\left(x, l_i^{\mathcal{Q}}(t)\right) E^{\mathcal{Q}}\left[\exp(c_1(t,s)\cdot J_p^i(t)) - 1\right] - g\left(x, \delta(t)\right),
\end{aligned} \tag{D.4}
$$

where the coefficients with subscripts are subvectors and submatrices of the coefficients in equations (2.4), (2.5), (2.6), and (2.7). This equation must hold for all $x \in D$, which is assumed to contain an open set, so that we can apply the usual method of undetermined coefficients which equates the coefficients of x and the quadratic forms in x to zero. This shows that $c(t,s)$ solves the ODE's:

$$
\begin{aligned}
\frac{dc_0(t,s)}{dt} &= \delta_0(t) - c_1(t,s)^\top K^{\mathcal{Q}}(t)\,\bar{x}^{\mathcal{Q}}(t) \\
&- \frac{1}{2}\sum_{i=1}^{N}[c_1(t,s)^\top \Sigma(t)]_i^2\, s_{i0} \\
&- \frac{1}{2} tr[2\, c_3(t,s)\Sigma(t)S(x,t)S(x,t)^\top \Sigma(t)^\top] \\
&- \sum_{i=1}^{p} l^{\mathcal{Q}}_{0,i}(t)\ E^{\mathcal{Q}}\left[\exp(c_1(t,s)\cdot J_p^i(t)) - 1\right]
\end{aligned} \tag{D.5}
$$

$$
\begin{aligned}
\frac{dc_1(t,s)}{dt} &= \delta_1(t) + K^{\mathcal{Q}}(t)^\top c_1(t,s) - 2\, c_2(t,s)K^{\mathcal{Q}}(t)\,\bar{x}^{\mathcal{Q}}(t) \\
&- \frac{1}{2}\sum_{i=1}^{N}[c_1(t,s)^\top \Sigma(t)]_i^2 s_{i1}(t) \\
&- 2\, c_2(t,s)\,\Sigma(t)S(x,t)S(x,t)^\top \Sigma(t)^\top c_1(t,T) \\
&- \sum_{i=1}^{p} l^{\mathcal{Q}}_{1,i}(t)\ E^{\mathcal{Q}}\left[\exp(c_1(t,s)\cdot J_p^i(t)) - 1\right]
\end{aligned} \tag{D.6}
$$

$$
\begin{aligned}
\frac{dc_2(t,s)}{dt} &= \delta_2(t) - c_2(t,s)K^{\mathcal{Q}}(t) - K^{\mathcal{Q}}(t)^\top c_2(t,s) \\
&- 2\, c_2(t,s)\,\Sigma(t)S(x,t)S(x,t)^\top \Sigma(t)^\top\, c_2(t,s) \\
&- \sum_{i=1}^{p} l^{\mathcal{Q}}_{2,i}(t) E^{\mathcal{Q}}\left[\exp(c_1(t,s)\cdot J_p^i(t)) - 1\right],
\end{aligned} \tag{D.7}
$$

with terminal conditions given by $\bar{c}_0, \bar{c}_1$, and $\bar{c}_2$, respectively. ■

Step 2: Here, we impose sufficient integrability conditions so that, for $t \in [\tau_i^d, s)$,

$$P(t,T) = E_t^{\mathcal{Q}}\left[\exp\left(-\int_t^s R(X(u),u)du\right) \ \exp\left(g\left(X(s-),\bar{c}\right)\right)\right]$$

can be viewed as the Feynmac-Kac solution to the Cauchy problem (D.2).

Assumption 4. (Integrability Conditions):
We say that the *integrability conditions hold at* $(s, \bar{c})$ if

1. $c(\cdot, s) : [\tau_i^d, s] \to C$ uniquely solve equations (D.5)-(D.7) with terminal conditions $\bar{c}$ at time s.

2. $E^{\mathcal{Q}} \int_0^s |\gamma_1^i| \, dt < \infty$, for all $i = 1, \ldots, p$, where
   $\gamma_1^i(t) = \Psi\,(t-)\, E^{\mathcal{Q}}\left[\exp(c_1(t,s) \cdot J_p^i(t)) - 1\right] g\left(X(t-), l_i^{\mathcal{Q}}(t)\right).$

3. $E^{Q}\left(\int_0^s |\gamma_2(t) \cdot \gamma_2(t)| \, dt\right)^{1/2} < \infty$, where
   $\gamma_2(t) = \Psi(t-)\left[c_1(t,s) + 2\,X(t-)^{\top} c_2(t,s)\right] \sigma(X(t-),t).$

4. $E^{\mathcal{Q}}\left(|\Psi(s)|\right) < \infty$,

where $\Psi(t)$ is defined for $t \in [\tau_i^d, s]$ by

$$\Psi(t) = \begin{cases} \exp\left(-\int_0^t R(X(u),u)du\right) \exp\left(g\left(X(t), c(t,s)\right)\right) & \text{for } t \in [\tau_i^d, s) \\ \exp\left(-\int_0^s R(X(u),u)du\right) \exp\left(g\left(X(s-), \bar{c}\right)\right) & \text{for } t = s. \end{cases}$$

Lemma 3: If the integrability conditions hold at $(s, \bar{c})$, then $\Psi(t)$ given by (D) is a martingale for $t \in [\tau_i^d, s]$.

Proof: Applying Ito's Lemma[1] to equation (D) for $t \in [\tau_i^d, s]$ and using the

[1]See Protter (1990), p. 74.

coefficient calculation (D.5)-(D.7) gives

$$d\Psi(t) = \Psi(t-)\left[\widehat{c}_1(t,s) + 2\,X(t-)^{\top}\widehat{c}_2(t,s)\right]\sigma(X(t-),t)\;dW(t) +$$
$$+\sum_{i=0}^{p}\Psi(t-)\;\left[\exp\left(\widehat{c}_1(t,s)\cdot J_p^i(t)\right)-1\right]\;dM_P^i(t),$$

where M_P^i denotes the i-th compensated Poisson process. Duffie, Pan, and Singleton (2000), p. 26, show that with assumptions 4.1 and 4.2, $\int \eta_2(t)dW$ and $\int\;\Psi(t-)\;\left[\exp\left(\widehat{c}_1(t,s)\cdot J_p^i(t)\right)\right.$ for $i = 1,\ldots,p$, are martingales during the interval $[\tau_i^d, s]$.

Appendix E

Recursive Calculation of $c(t, T)$

Here, we provide an algorithm for computing $c(t, T)$.

Step 0 (Initialization):

The terminal condition for $c(t, T)$ at T consists of a collection of zeros denoted by $\bar{c}_{n+1}$ in C. Let $\tilde{c}_n(t, T)$ solve the ODE's in (D.5)-(D.7) during the interval $[\tau_n^d, T]$ with terminal condition $\bar{c}_{n+1}$, and define $\tau_{n+1}^d = T$.
Go to Step 1.

Step $\boldsymbol{i}$, for $\boldsymbol{i = 1, \ldots, n}$:

- Calculate the new terminal condition for time τ_{n+1-i}^d as

$$\bar{c}_{n+1-i} = \tilde{c}_{n+1-i}\left(\tau_{n+1-i}^d, \tau_{n+2-i}^d\right) + c_{n+1-i}\left(\tau_{n+1-i}^d, \tilde{c}_{n+1-i}(\tau_{n+1-i}^d, \tau_{n+2-i}^d)\right),$$

  where $c_{n+1-i}\left(\tau_{n+1-i}^d, \tilde{c}_{n+1-i}(\tau_{n+1-i}^d, \tau_{n+2-i}^d)\right) \in C$ is taken from equation (2.7) evaluated at $t = \tau_{n+1-i}^d$.

- For a given terminal condition $\bar{c}_{n+1-i} \in C$, let $\tilde{c}_{n-i}(t, T)$ solve the ODE's in (D.5)-(D.7) during the interval $[\tau_{n-i}^d, \tau_{n+1-i}^d]$, with terminal condition $\bar{c}_{n+1-i}$.

Stop if $i = n$. Go to Step $i + 1$.

Coefficient Collection:

The coefficients $c(t, T)$ are then equal to $\tilde{c}_i(t, T)$ for any $t \in (\tau_i^d, \tau_{i+1}^d)$ and equal to $\bar{c}_i$ at any $t = \tau_i^d$.

Appendix F

Proof of Proposition 3

Under condition A2, the solution ξ of (2.9) is a square-integrable martingale by Proposition 1, stated in Appendix A. As ξ is the density process for $\mathcal{Q}$, the state-price density (or pricing kernel) process π is defined by $\pi(t) = \xi(t)/F^0(t)$. For the equivalent measure defined by ξ to be a martingale measure, it suffices that, for each i, πF^i is a $\mathcal{P}$-martingale. The SDE solved by π is given by

$$\frac{d\pi(t)}{\pi(t-)} = -r(t)dt - \sigma_\xi(t)\, dW(t) + J_\xi^d(t)dN_d(t) + J_\xi^p(t)dM_p(t).$$

Using integration by parts (Corollary 2 in Protter (1990), page 60), we get

$$\begin{aligned}\frac{d\left(F^i(t)\pi(t)\right)}{\left(F^i(t-)\pi(t-)\right)} &= \mu_{F^i}(t)dt + \sigma_{F^i}(t)\, dW(t) + J_{F^i}^d(t)\, dN_d(t) + J_{F^i}^p(t)\, dN_p(t) \\ &\quad -r(t)dt - \sigma_\xi(t)\, dW(t) + J_\xi^d(t)\, dN_d(t) + J_\xi^p(t)\ \left(dN_p(t) - \lambda(t)dt\right) \\ &\quad -\sigma_{F^i}(t)\sigma_\xi^\top(t)dt + J_{F^i}^d(t)J_\xi^d(t)\, dN_d(t) + J_{F^i}^p(t)J_\xi^p(t)\, dN_p(t).\end{aligned}$$

The process πF_i is a $\mathcal{P}$-local martingale if and only if (i) this SDE has a zero drift and (ii) the $\mathcal{F}(t-)$-conditional expected value of $\Delta(\pi(t)\xi(t))$ at a deterministic jump time t is zero. Collecting "dt"-terms for condition (i) and "dN_d"-terms for condition (ii) results in the two equations stated in Proposition 3 (because of A2(b)). As F^i/F^0 and ξ are both assumed to be square-integrable, we get that πF_i is in fact a $\mathcal{P}$-martingale. ■

Appendix G

Simulated Maximum Likelihood with Jumps

Suppose X contains the target rate θ, modeled as the (observable) difference of the "up" and "down" counting processes, with state-and time-dependent intensities as in (4.1). We abstract for the moment from the time dependence of stochastic intensities introduced by FOMC meetings, assuming that these intensities are always "active." Starting from $\tilde{x}$ at time $\tilde{t}$, we can simulate X with the scheme

$$\begin{aligned} \Delta \hat{X}^{\tilde{x}}_t &= \mu(\hat{X}^{\tilde{x}}_{t-h}, t-h)\, h + \sqrt{h}\, \sigma(\hat{X}^{\tilde{x}}_{t-h}, t-h)\, \epsilon_t + J^X_t z_t \\ \hat{X}^{\tilde{x}}_{\tilde{t}} &= \tilde{x}, \end{aligned} \tag{G.1}$$

where ϵ_t is *i.i.d.* standard normal and J^X is the deterministic jump[1] in X at random times, determined by a 2-dimensional vector of Bernoulli variables z_t that determine jumps, up and down. Using the conditional independence of the counting processes N^U and N^D, and assuming that the econometrician observes only the difference of the two, the simulation rolls a "three-sided die" to determine z_t. The three sides are "up" ("U", meaning $\theta_t - \theta_{t-h} = J_\theta$), "down" ("D", meaning $\theta_t - \theta_{t-h} = -J_\theta$), and "no change" ("0", meaning $\theta_t = \theta_{t-h}$). Their conditional probabilities at time t are

[1] The notation goes through with Gaussian jumps J^X, but needs to be adjusted in the case of other jumps size distributions.

approximately

$$p^j_{h,t} = \begin{cases} \lambda^U_{t-h} h\,(1-\lambda^D_{t-h}h), & \text{for} \quad j = \mathrm{U}, \\ \lambda^D_{t-h} h\,(1-\lambda^U_{t-h}h), & \text{for} \quad j = \mathrm{D}, \end{cases}$$

and $p^0_{h,t} = p^U_{h,t}p^D_{h,t} + (1-p^U_{h,t})(1-p^D_{h,t})$.

We write X^θ for all variables in X other than the target θ. The Monte-Carlo approximation of the conditional density is

$$f_X(X(t), t \,|\, \tilde{x}, \tilde{t}) \approx \frac{1}{S}\sum_{s=1}^{S} \sum_{i=\{U,D,0\}} \phi(X^\theta_t, t \,|\, \theta_t, \hat{X}^{\tilde{x}}_{t-h}[s], t-h)\, \hat{p}^i_{h,t}[s] 1_{i,t}[s], \tag{G.2}$$

where $\phi(\,\cdot\,, t \,|\, \hat{X}^{\tilde{x}}_{t-h}[s], t-h)$ is the Gaussian density of X_t at time t conditional on the value $\hat{X}^{\tilde{x}}_{t-h}[s]$ at time $t-h$, $\hat{X}^{\tilde{x}}_{t-h}[s]$ denotes the s-th simulated path from the scheme (G.1), $1_{i,t}[s]$ is the indicator for the i-th side of the die at time t in the s-th simulation, and $\hat{p}^i_{h,t}[s]$ is constructed using $\hat{X}^{\tilde{x}}_{t-h}[s]$. Let $\hat{\theta}^{\tilde{x}}_{t-h}$ be the target component of $\hat{X}^{\tilde{x}}_{t-h}$. If the simulated target $\hat{\theta}^{\tilde{x}}_{t-h}$ at time $t-h$ cannot reach the observed time t-value of target in at most one jump, that simulation is assigned zero likelihood.

We now turn to a case with *time-dependent* intensities that is relevant in Section 4.2. In that application, policy interventions on meeting days are modeled by activating state-dependent Poisson intensities only during FOMC meeting days. More precisely, suppose the i-th meeting day is during the interval $[\tilde{t}_M(i), t_M(i)]$. It is straightforward to modify the simulation scheme (G.1) to allow jumps only during such meeting-day intervals. We refer to the s-th path drawn from this modified scheme in what follows as $\hat{X}^{\tilde{x}}_t[s]$. We now construct analogues of (G.2) for this time-dependent case. As long as the observation time t lies within a meeting-day interval, in that $\tilde{t}_M(i) \le t < t_M(i)$, the approximation (G.2) itself still applies. If the observation time t is made outside an FOMC meeting, however, then one might want to replace the Bernoulli-density terms with an indicator function for sample paths leading up to the actual value of the target at t,

$$f_X(X_t, t \,|\, \tilde{x}, \tilde{t}) \approx \frac{1}{S}\sum_{s=1}^{S} \phi\left(X_t^{\theta}, t|\hat{X}_{t-h}^{\tilde{x}}[s], t-h\right) 1_{\theta_t = \hat{\theta}_{t-h}^{\tilde{x}}[s]}. \tag{G.3}$$

In (G.3), jumps in the target enter the SML objective function only through the indicator function and the simulated values $\hat{X}_{t-h}^{\tilde{x}}$. This creates a serious problem when maximizing the objective: For a given (finite) number S of simulations, a small change in the parameter vector does not necessarily affect the average number of jumps across simulations and may thus leave the value of the likelihood function unchanged. Only changes in a parameter that are large enough to affect the number of simulated jumps change the objective function, but possibly by a large amount.[2] In order to overcome this discontinuity, an alternative to (G.3) is constructed as follows. The joint conditional density of factors can be written in the form

$$f_X\left(X_t, t \,|\, \tilde{x}, \tilde{t}\right) = f_\theta\left(\theta_t, t \,|\, \tilde{x}, \tilde{t}\right)\, f_{X^\theta|\theta}\left(X_t^\theta, t \,|\, \theta_t, \tilde{x}, \tilde{t}\right). \tag{G.4}$$

The first term of equation (G.4) can be approximated by

$$\begin{aligned} f_\theta\left(\theta_t, t \,|\, \tilde{x}, \tilde{t}\right) &\approx \frac{1}{S}\sum_{s=1}^{S} f_\theta\left(\theta_t, t \,|\, \hat{X}_{t_M(i)-h}^{\tilde{x}}[s], t_M(i)-h\right) \\ &\approx \frac{1}{S}\sum_{s=1}^{S}\sum_{i=U,D,0} \hat{p}_{h,t_M(i)}^{i}[s] 1_{i,t}[s] \\ &\approx \bar{S}/S, \end{aligned}$$

where $\bar{S}$ denotes the total number of simulated paths that resulted in the observed value $\theta(t)$. In words, $\bar{S}/S$ is the frequency of "correctly simulated" target rates in the

[2] A similar issue is encountered by Anderson, Benzoni, and Lund (1999) who estimate a jump-diffusion model for equity returns with EMM. In their specification, the size of the jump is Gaussian and the occurrence of jumps is not observed, so that they smooth the mapping from parameters to the estimation's objective function by allowing for partial jumps. In the setup considered in this paper, the target only moves in observable 25 basis points increments, so that a simulation of partial jumps is not feasible. A conjecture is that the method proposed in the following can also be applied with EMM, since the efficiency results of Gallant and Tauchen (1996) do not rely on a particular leading density.

simulations (starting with $\tilde{x} = X_{\tilde{t}}$), while the expression in the first row weights the simulated paths by their likelihoods. In practice, with time-dependent intensities h must be chosen carefully, as intensities can become large during an FOMC meeting. Details about the choice of h can be found in Appendix H.

The second term in (G.4) can be approximated by

$$\begin{aligned} f_{X^\theta|\theta}(X_t^\theta, t \,|\, \theta_t, \tilde{x}, \tilde{t}) &= \frac{f_X\left(X_t, t \,|\, \tilde{x}, \tilde{t}\right)}{f_\theta\left(\theta_t, t \,|\, \tilde{x}, \tilde{t}\right)} \\ &\approx \frac{1}{\bar{S}} \sum_{s=1}^{S} \phi\left(X_t^\theta, t \,|\, \hat{X}_{t-h}^{\tilde{x}}[s], t-h\right) \, 1_{\theta_t = \hat{\theta}_{t-h}^{\tilde{x}}[s]}. \end{aligned}$$

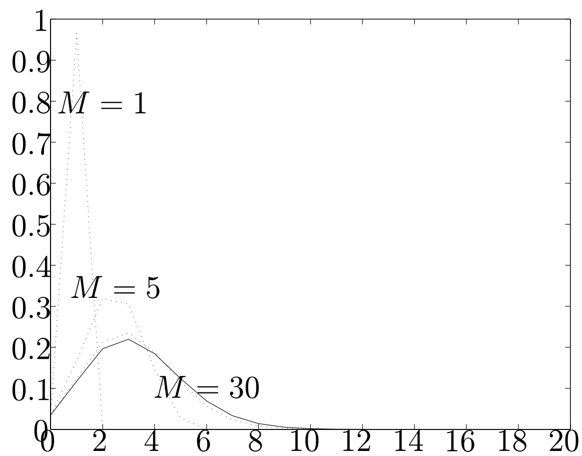
Variance-reduction techniques can improve the efficiency of the Monte Carlo integration (see Geweke (1996)). Here, antithetic variates are used in simulating the paths of the state vector. That is, with each new pseudo-random Gaussian $\epsilon[s]$ and uniform $u[s]$, the antithetic variates $-\epsilon[s]$ and $1 - u[s]$ are used as a subsequent scenario.

Appendix H

Simulation of the Target

The highest value that is reached by the intensities λ^U and λ^D in a typical estimated model[1] is 1225. At this value, the next Figure shows that a Bernoulli approximation that allows for only one jump during an FOMC meeting is not accurate. We can see that the Bernoulli density for $h = \frac{1}{365}$ overstates the true probability of one jump. If we increase the number of Bernoulli trials during an FOMC meeting so that $h \geq \frac{1}{30}\frac{1}{365}$, the Bernoulli approximation becomes accurate. To economize on the number of simulated steps (and thereby the computation time for the likelihood evaluation), the FOMC meeting day is divided into $M_s + 1$ intervals, where M_s is a number divisable by 5. During 5 subintervals $[t_i, t_{i+1}]$ of length $h = \frac{M_s}{5}\frac{1}{M_s+1}\frac{1}{365}$, jumps are drawn from a Poisson distribution with constant parameter $\lambda^j(t-h)h$ by truncating the distribution at $\frac{M_s}{5}$ jumps. In the last subinterval of length $h = \frac{1}{M_s+1}$, a Bernoulli discretization is applied. This approximation procedure is equivalent to 31 Bernoulli trials (with appropriately chosen success probability). In the body of the paper, a choice of $M_s = 30$ is called 'subdividing the FOMC meeting into 30 intervals.'

[1]The value is taken from λ^U in the unconstrained SVλ specification introduced in Section 4.2.



Approximation of a Poisson density (solid line) with $\lambda = 1225$ for daily data ($t - \tilde{t} = 1/365$) with Bernoulli trials with success probability $p = 1 - \exp(-\lambda h)$, with $h = \frac{1}{M}\frac{1}{365}$, for different choices of M.

Appendix I

SML with Macro Variables

The state vector X is now augmented with the macro-related information $M(t) = (m(t), m_F(t))$, and we write $X^{(\theta,M)}$ for the vector consisting of all coordinates of the state X except θ and M. Analogous to the decomposition (G.4), we can write the density of X conditional on the last observation $X_{\tilde{t}} = x$ in the form

$$f_X(X_t, t \,|\, x, \tilde{t}) = f_{\theta,M}(\theta_t, M_t, t \,|\, x, \tilde{t}) f_{X^{(\theta,M)}|\theta,M}(X_t^{(\theta,M)}, t \,|\, \theta_t, M_t, x, \tilde{t}). \qquad \text{(I.1)}$$

The first term in (I.1) can be written as

$$f_{\theta,M}(\theta_t, M_t, t \,|\, x, \tilde{t}) = f_M(M_t, t \,|\, x, \tilde{t}) f_\theta(\theta_t, t|x, M_t, \tilde{t}).$$

If an FOMC meeting and a macro jump event happen on the same day, the Fed's target decisions are able to condition on the newly released information, as CPI and NPE releases (at 8:30 a.m.) precede FOMC meetings. Since each observation interval $[\tilde{t}, t]$ is chosen so that it does not contain more than one FOMC meeting, the density of M conditional on $X(\tilde{t})$ depends only on $\theta(\tilde{t})$. Moreover, $[\tilde{t}, t]$ is short enough so as to not contain the release of the current month's macro variable together with the analyst survey of forecasts for the next month. For the i-th macroeconomic release, we therefore have

$$f_M(M_t, t \,|\, x, \tilde{t}) = \begin{cases} f_{m_F}(m_F(t), t \,|\, m_F(\tilde{t}), \theta_{\tilde{t}}, \tilde{t}), & \text{if } \tau_F^i \in [\tilde{t}, t], \\ f_m(m(t), t \,|\, m_F(\tilde{t}), \tilde{t}), & \text{if } \tau^i \in [\tilde{t}, t], \\ f_{m_F}(m_F(t) \,|\, m_F(\tilde{t}), \theta_{\tilde{t}}, \tilde{t}) f_m(m(t), t \,|\, m_F(\tilde{t}), \tilde{t}), & \text{if } \tau^i, \tau_F^i \in [\tilde{t}, t]. \end{cases}$$

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