ALGEBRA

TEACHER'S EDITION

Exploring Systems of Inequalities

GAIL F. BURRILL AND PATRICK W. HOPFENSPERGER

DATA-DRIVEN MATHEMATICS

DALE SEYMOUR PUBLICATIONS®

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About *Data-Driven Mathematics*

Historically, the purposes of secondary-school mathematics have been to provide students with opportunities to acquire the mathematical knowledge needed for daily life and effective citizenship, to prepare students for the workforce, and to prepare students for postsecondary education. In order to accomplish these purposes today, students must be able to analyze, interpret, and communicate information from data.

Data-Driven Mathematics is a series of modules meant to complement a mathematics curriculum in the process of reform. The modules offer materials that integrate data analysis with secondary mathematics courses. Using these materials helps teachers motivate, develop, and reinforce concepts taught in current texts. The materials incorporate the major concepts from data analysis to provide realistic situations for the development of mathematical knowledge and realistic opportunities for practice. The extensive use of real data provides opportunities for students to engage in meaningful mathematics. The use of real-world examples increases student motivation and provides opportunities to apply the mathematics taught in secondary school.

The project, funded by the National Science Foundation, included writing and field testing the modules, and holding conferences for teachers to introduce them to the materials and to seek their input on the form and direction of the modules. The modules are the result of a collaboration between statisticians and teachers who have agreed on the statistical concepts most important for students to know and the relationship of these concepts to the secondary mathematics curriculum.

A diagram of the modules and possible relationships to the curriculum is on the back cover of each Teacher's Edition of the modules.

Using This Module

Why the Content Is Important

Many problems in the world require looking for relationships between two events that are changing at the same time. For example, when will the time for women running the 1600 meters be the same as the time for men, or when did the circulation of morning newspapers exceed the circulation of evening papers? Solving systems of equations and inequalities are important tools in answering questions similar to these questions. Solving systems enables us to find the intersection of two lines on a graph and analyze patterns and relationships.

This module will extend the concept of rate of change and graphing linear equations to graphing two equations or inequalities on the same coordinate system. In Unit I, students will investigate algebraic techniques that can be used to find the point of intersection of two lines. These techniques will to used to graph and analyze inequalities in Unit II. People often use inequalities to determine when a quantity is greater than or less than some given standard. Does the food at a fast-food restaurant exceed nutritionists' guidelines? Will increasing the price of a chocolate-chip cookie decrease sales?

Content

Mathematics content: Students will be able to

- Find and interpret slope.
- Graph a system of equations.
- Identify parallel lines.
- Solve a system of equations using the substitution method.
- Graph a system of inequalities.
- Solve a system of inequalities.

Statistics content: Students will be able to

- Make and interpret plots over time.
- Make and interpret scatter plots.
- Find a line of best fit for a set of data.

Instructional Model

The instructional emphasis *Exploring Systems of Equations and Inequalities,* as in all of the modules in *Data-Driven Mathematics,* is on discourse and student involvement. Each lesson is designed around a problem or mathematical situation and begins with a series of introductory questions or scenarios that can prompt discussion and raise issues about that problem. These questions can involve students in thinking about the problem and help them understand why such a problem might be of interest to someone in the world outside the classroom. The questions can be used in whole-class discussion or in student groups. In some cases, the questions are appropriate to assign as homework to be done with input from families or from others not a part of the school environment.

These opening questions are followed by discussion issues that clarify the initial questions and begin to shape the direction of the lesson. Once the stage has been set for the problem, students begin to investigate the situation mathematically. As students work their way through the investigations, it is important that they have the opportunity to share their thinking with others and to discuss their solutions in small groups and with the entire class. Many of the exercises are designed for groups in which each member does one part of the problem and the results are compiled for final analysis and solution. Multiple solutions and solution strategies are also possible, and it is important for students to recognize these situations and to discuss the reasoning behind different approaches. This will provide each student with a wide variety of ways to build his or her own understanding of the mathematics.

In many cases, students are expected to construct their own understanding by thinking about the problem from several perspectives. They do, however, need validation of their thinking and confirmation that they are on the right track, which is why discourse among students, and between students and teacher, is critical. In addition, an important part of the teacher's role is to help students link the ideas within an investigation and to provide an overview of the "big picture" of the mathematics within the investigation. To facilitate this, a review of the mathematics appears in the summary following each investigation.

Each investigation is followed by a Practice and Applications section in which students can revisit ideas presented within the lesson. These exercises may be assigned as homework, given as group work during class, or omitted altogether if students are ready to move ahead.

Student assessments occur after two units in the student book. These can be assigned as long-range take-home tasks, as group assessment activities, or as in-class work. The assessment pages provide a summary of the lessons up to that point and can serve as a vehicle for students to demonstrate what they know and what they can do with the mathematics. Commenting on the strategies students use to solve a problem can encourage students to apply different strategies. Students also learn to recognize those strategies that enable them to find solutions efficiently. Also included are two student projects.

Where to Use the Module in the Curriculum

This module is about solving systems of equations and graphing and working with linear inequalities. It can be used in a first-year algebra or second-year algebra course in a variety of ways. The module can be used as a complete unit, starting with the solving of a system of equations and finishing with the solving of a system of inequalities. The three units can also be used separately or at different times during the school year.

Following are suggestions for using the module:

- Use with the standard chapter on solving systems of equations to provide real-world applications.
- Replace the standard chapter on graphing linear inequalities in any traditional mathematics text.
- Use after students have completed a section on graphing linear inequalities to illustrate how to apply those concepts in real-world contexts.
- Use in a second-year algebra course as a review for students solving a system of equations and inequalities.
- Use as the third unit in a course, following *Exploring Symbols: An Introduction to Expressions and Functions* and *Exploring Linear Relations*, from the *Data-Driven Mathematics* series.

Prerequisites

Students should have had experience in using variables to describe relationships, plotting points, simplifying expressions, solving equations, drawing a line that best fits a set of ordered pairs, finding the slope of a line, finding the intercepts of a line, and writing the equation of a line.

Pacing/Planning Guide

The table below provides a possible sequence and pacing for the lessons.

LESSON	OBJECTIVE	PACING
Unit I: Solving Systems of Equations		
Introductory Activity: Men's and Women's Olympic Times	Observe trends in data sets.	$\frac{1}{2}$ class period
Lesson 1: Systems of Equations	Find the point of intersection of two lines on a graph; solve a system of equations.	2–3 class periods
Lesson 2: Lines with the Same Slope	Recognize that parallel lines have the same slope or rate of change; recognize systems that do not have a solution.	1 class period
Assessment for Unit I	Apply knowledge of systems of equations.	1 class period or homework
Unit II: Graphing Inequalities		
Introductory Activity: Estimating the Number of Raisins	Compare an estimate to an actual count.	$\frac{1}{2}$ class period
Lesson 3: Shading a Region	Find the solution to an inequality using the line $y = x$.	1 class period
Lesson 4: Graphing Inequalities	Graph and interpret a linear inequality.	1 class period
Assessment for Unit II	Solve problems by graphing inequalities.	1 class period or homework
Unit III: Solving Systems of Inequalities		
Introductory Activity: Fast Foods	Investigate ordered pairs that satisfy a constraint.	$\frac{1}{2}$ class period
Lesson 5: Graphing Conditions	Graph and interpret systems of inequalities in the form $x < a$ and $y > b$.	1 class period
Lesson 6: Systems of Inequalities	Graph and interpret systems of inequalities in the form $y < mx$.	2 class periods
Lesson 7: Applying Systems of Inequalities	Graph and interpret systems of inequalities in the form $ax + by < c$.	2 class periods
Assessment for Unit III	Apply knowledge of systems of inequalities.	1 class period or homework
		Approximately 3 weeks total time

Use of Teacher Resources

At the back of this Teacher's Edition are the following:

- Quizzes for selected lessons
- End-of-Module Test
- Solution Key for quizzes and test
- Activity Sheets

These items are referenced in the *Materials* section at the beginning of the lesson commentary.

LESSONS	RESOURCE MATERIALS
Unit I: Solving Systems of Equations	
Introductory Activity: Men's and Women's Olympic Times	*Activity Sheet 1* (Problems 3–6)
Lesson 1: Systems of Equations	*Activity Sheet 2* (Problem 3) *Activity Sheet 3* (Problem 9) *Lesson 1 Quiz*
Lesson 2: Lines with the Same Slope	*Activity Sheet 4* (Problems 2 and 7)
Assessment for Unit I	*Activity Sheet 5* (Problem 2)
Unit II: Graphing Inequalities	
Introductory Activity: Estimating the Number of Raisins	
Lesson 3: Shading a Region	*Activity Sheet 6* (Problem 6)
Lesson 4: Graphing Inequalities	*Activity Sheet 7* (Problems 1 and 2) *Lesson 4 Quiz*
Assessment for Unit II	
Unit III: Solving Systems of Inequalities	
Introductory Activity: Fast Foods	
Lesson 5: Graphing Conditions	*Activity Sheet 8* (Problems 1–7, 11, and 12) *Activity Sheet 9* (Problem 15)
Lesson 6: Systems of Inequalities	*Activity Sheet 10* (Problems 2–6 and 8) *Activity Sheet 11* (Problems 11–13 and 18 *Lesson 6 Quiz*
Lesson 7: Applying Systems of Inequalities	*Activity Sheet 12* (Problems 2, 3, and 7) *Activity Sheet 13* (Problems 9, 14, and 18) *Activity Sheet 14* (Problems 27 and 28) *Lesson 7 Quiz*
Assessment for Unit III	*Activity Sheet 15* (Problems 1 and 2) *End-of-Module Test*

UNIT I

Solving Systems of Equations

INTRODUCTORY ACTIVITY

Men's and Women's Olympic Times

Materials: rulers, *Activity Sheet 1*
Technology: graphing calculators (optional)
Pacing: $\frac{1}{2}$ class period

Overview

This opening activity uses the men's and women's 800-meter Olympic times to focus on the concept of slope. Students are asked to draw a line that best fits the data and compare the slopes. The slopes that they find are then compared to the information regarding the speeds of men and women racers.

Teaching Notes

This activity can be used as homework or completed during class. This activity can also be used as a review of fitting a line to data and writing the equation of a line. These skills are used throughout the module, and this activity serves as an introduction to demonstrate how these skills will be used to solve a system of equations. You may wish to review finding the equation of a line and interpreting the slope and intercepts. The article used at the beginning of the activity will be referred to in Lesson 1. The data used in this activity can be found at the end of Lesson 1.

It is important to note that there are a number of prerequisite skills needed for this activity as well as the entire module. Students should be able to find the slope of a line, the x- and y-intercepts, and the equation of the line in both point-slope and slope-intercept form. Students are only expected to fit a line to the data using an "eyeball" fit. If students know how to draw a median-fit or least-squares regression line, they could use either one to draw the line. Students are expected to find the point of intersection of the two lines by estimating from the graph. The algebra will be developed in Lesson 1.

Technology

Graphing calculators are optional. Students should be encouraged to draw a line by hand, since this is a skill that is used throughout the module.

STUDENT PAGE 3

INTRODUCTORY ACTIVITY

Men's and Women's Olympic Times

Do you think that women's times will ever catch up to men's times in Olympic races?

"Women may outrun men, researchers suggest."

OBJECTIVE

Observe trends in data sets.

Analyzing trends in records for sporting events is of great interest to many people. Often these trends can be determinded by graphing. In this unit, you will learn how to find relationships between two sets of data by studying their graphs.

EXPLORE

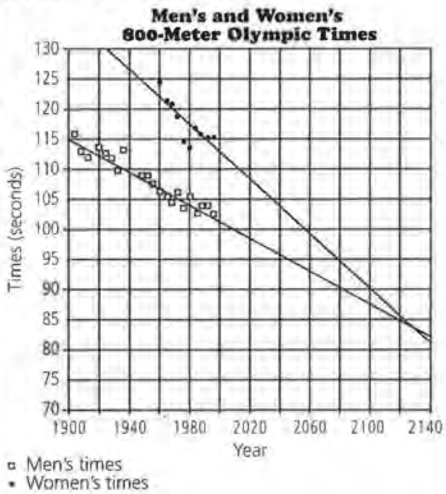
Within a few generations, female runners may start beating male runners in world-class competitions. An analysis of world records for a variety of distances found that women's rates have been improving about twice as quickly as men's rates.

Solution Key

1. Times for men and women in the 200-, 400-, 800-, and 1500-meter events

2. 0.16; twice as fast as men

3. and 4. Possible answers:



5. Based on Problems 3 and 4 graph, point of intersection: approximately (2125, 84)

6. The point represents the location where the two lines intersect. The ordered pair gives the year that men and women will run the 800-meter race in the same time. This assumes that trends will continue at the same rate of change.

STUDENT PAGE 4

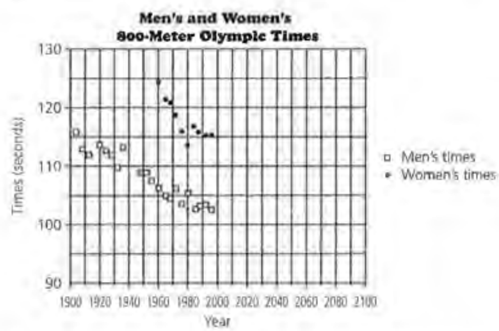
If this trend continues, the top female and male runners may start performing equally well between the years 2015 and 2055 in the 200-, 400-, 800-, and 1500-meter events. These findings were reported in a letter in the journal *Nature*.

> "None of the current women's world-record holders at these events could even meet the men's qualifying standard to compete in the 1992 Olympic games," researchers Brian Wipp and Susan Ward wrote. "However, it is the rates of improvement that are so strikingly different—the gap is progressively closing." But other researchers said they doubted the projections because they believed women's rate of improvement would decrease.
>
> Source: Staff and Wire Services, *Milwaukee Journal*, January, 1992

1. What data could you collect that would help you answer the question on page 3?

2. If you found that the men's times in an event were decreasing 0.08 second per year, by what amount must the women's times decrease to prove the researchers' claim?

The graph below shows men's and women's times for the 800-meter race.



3. On the graph on *Activity Sheet 1*, draw a line that you think represents the trend for the men's times.

4. On the same graph, draw a line that you think represents the trend for the women's times.

5. Extend the two lines. Estimate their point of intersection.

6. What does this point represent in terms of the data?

LESSON 1

Systems of Equations

Materials: rulers, *Activity Sheets 2* and *3*, *Lesson 1 Quiz*
Technology: graphing calculators with list capabilities
Pacing: 2–3 class periods

Overview

This lesson introduces students to three methods of solving a system of equations. First, the students are asked to estimate the point of intersection of two lines on a graph. Next, the students use a graphing calculator to make a table of values for each equation and look for a common ordered pair in the table. The remaining part of the lesson is devoted to solving the system algebraically. The equations are given in the form $y = mx + b$. The algebraic method asks the students to use substitution and set both equations equal to each other.

Teaching Notes

This lesson assumes that students know how to write the equation of a line and are able to interpret the slope of the line. For review of these topics, you may wish to see *Exploring Linear Relations* for examples and development of these concepts. You may wish to present the graphical approach and the table approach on one day and the algebraic approach on another day.

When drawing a line that best summarizes the data, students do not need to draw the median-fit or least-squares regression line. The intent is for the students to draw an "eyeball" fit line. This does mean that students' equations may be different but their slopes should be approximately the same.

Technology

It is suggested that students use a graphing calculator to make a table of values for Problem 4. They could also use a spreadsheet instead of a graphing calculator. The focus should be to have students start the table by counting by 1s and then change the increment value (**ΔTbl**) to smaller values until the y-values for both equations are equal for a given x-value.

Steps for the TI-83:

1. Enter the equation $y = 0.97x - 1893$ into [Y_1 =].

   Enter the equation $y = -1.1x + 2210$ into [Y_2].

2. [2nd] [TBLSET]
   Tbl Start = 1975
   ΔTbl = 1

3. [2nd] [TABLE]
   Use [▼]

4. [2nd] [TBLSET]
   Tbl Start = 1982
   ΔTbl = .1

Follow-Up

Students could investigate other Olympic events, such as swimming events, in which both men and women compete. Students can also do research on the history of women's competing in the Olympic Games.

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LESSON 1

Systems of Equations

Does your family subscribe to a daily paper?

If your family subscribes, who reads the paper?

What sections do people read the most often?

We often want to observe trends in related events. In this lesson, you will investigate the relationships between morning and evening newspaper circulation, as well as several other sets of data.

OBJECTIVES

Find the point of intersection of two lines on a graph.

Solve a system of equations.

INVESTIGATE

Trends in circulation of daily newspapers have been changing in recent years. Some cities in the United States have both an evening and a morning paper while others have one daily newspaper. A recent trend for evening newspapers has been to either stop publishing or to convert to a morning paper. The graph on page 6 shows the total circulation in millions of morning and evening newspapers across the United States.

STUDENT PAGE 6

Solution Key

Discussion and Practice

1. **a.** Morning newspaper circulation is increasing.

Evening newspaper circulation is decreasing.

b. Evening papers may go out of business and morning papers will continue to increase in circulation.

c. Owners of newspapers and advertisers; owners may want to know whether to keep the paper in production or not.

2. 1975: 61 million; 1980: 62 million; 1985: 63 million; 1990: 62 million; 1995: 58 million; yes

3. **a.** Slope of morning newspaper line is about 1; slope of evening newspaper line is about −1.

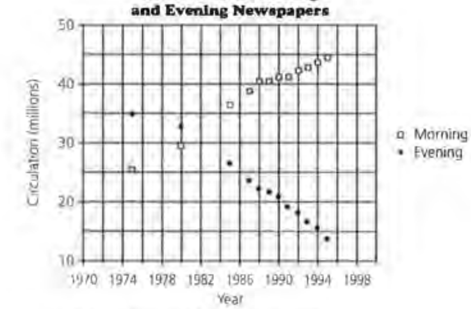
b. Possible answers: Morning newspaper, $y = 1x - 1950$; evening newspaper, $y = -1x + 2208$

c. Possible answers: Morning newspaper circulation: about 50 million; evening newspaper circulation: about 8 million; extend the lines and estimate from the graph or use the equations in part b.

d. Use the graph to see if the lines intersect near 32 million.

e. Year: 1982; circulation: 30 million; estimate the point where the two lines intersect on the graph.

Circulation of Morning and Evening Newspapers



Source: *Statistical Abstract of the United States, 1995*

Discussion and Practice

1. Refer to the graph above.

a. What trends do you observe in the plots?

b. Describe what you think will happen in the next few years if the trends continue. This is often called ***extrapolation***, or predicting beyond the given information.

c. Who might be interested in these trends? Why?

2. Use the graph to find the total combined circulation of morning and evening newspapers for the years 1975, 1980, 1985, 1990, and 1995. Has the total circulation remained about the same since 1975?

3. On the graph on *Activity Sheet* 2, draw a line that you think best represents each set of data.

a. Find the slope of each line. Describe the trends you observe in terms of the slopes.

b. Write an equation of each line.

c. Predict the circulation for evening and morning newspapers in 2000. How did you make your prediction?

d. Tanya claims that in 1982 both morning and evening newspapers had a circulation of about 32 million. How can you evaluate her claim?

e. When do you estimate that the circulation was the same for morning and evening newspapers? Approximately how large a circulation did each have when they were equal? Explain how you found your answer.

STUDENT PAGE 7

4. **a.** Answer depends on what equations are used.

b. Compare values under Y1 and Y2 and find the years in which the Y1 values become greater than the Y2 values. Answers may vary depending on equations used, but the two years will probably include 1982.

c. The year when evening and morning newspapers have the same circulation

A set of two or more equations in the same variables is a ***system of equations.*** Finding the point where two or more lines intersect, or cross, is called ***solving a system of equations.*** A solution of a system of equations in two variables is a set of values that makes all the equations in the system true.

To solve some systems of equations, you can graph each equation and find the point where the graphs intersect.

Sometimes you can find the point where the lines intersect by constructing a table of values. Tanya drew lines on her graph and found the following equations: $Y1 = 0.97X - 1893$ and $Y2 = -1.1X + 2210$. She used her calculator to make the following table of values.

X	Y1 = 0.97X − 1893	Y2 = −1.1X + 2210
1985	32.45	26.5
1986	33.42	25.4
1987	34.39	24.3
1988	35.36	23.2
1989	36.33	22.1
1990	37.3	21
1991	38.27	19.9
X = 1991		

4. Construct a table of values for the years from 1975 to 1985 for each of your equations.

a. Look at a table of values for each equation. Is any ordered pair the same for the two equations?

b. How can you determine from the table between what two years the ordered pairs will be the same? List the years.

c. Change the increment for your table to 0.1. Are any of the ordered pairs the same? If not, continue to change the increment until you find the value for X for which the values in Y1 and Y2 are the same. What does the ordered pair tell you?

5. It is the same.

STUDENT PAGE 8

You can also find the solution to the system algebraically. Let C represent the circulation and T the year. Rewrite the equations for this system as shown, letting C_m represent morning newspaper circulation and C_e represent evening newspaper circulation.

$$C_m = 0.97T - 1893 \qquad C_e = -1.1T + 2210$$

You would like to know when $C_m = C_e$.

Since C_m equals $0.97T - 1893$, you can substitute $0.97T - 1893$ for C_e in the second equation and solve for T.

$0.97T - 1893 = -1.1T + 2210$	The two equations are equal.
$+1.1T \qquad\qquad +1.1T$	Add $1.1T$ to both sides.
$2.07T - 1893 = 2210$	
$+1893 \qquad +1893$	Add 1893 to both sides.
$2.07T = 4103$	Divide both sides by 2.07.
$T = 1982.1$	

To find C_m, substitute 1982.1 for T in the equation.

$$C_m = 0.97(1982.1) - 1893$$
$$C_m = 29.64$$

The solution is (1982.1, 29.64).

The solution (1982.1, 29.64) means that at the beginning of 1982 the circulation of both morning and evening newspapers was about 29.6 million.

5. Use 1982.1 for T in the equation for the C_e and solve for C_e. How does your answer compare to the solution (1982.1, 29.64)?

Summary

Some problems have relationships that can be described by a set of two equations. To find values that make both equations true, you have to solve the two equations at the same time. You can do this

- by studying the graph to see where the lines intersect,
- by looking at a table of values for each equation to see where they match, or
- by using algebra and substituting from one equation into the other.

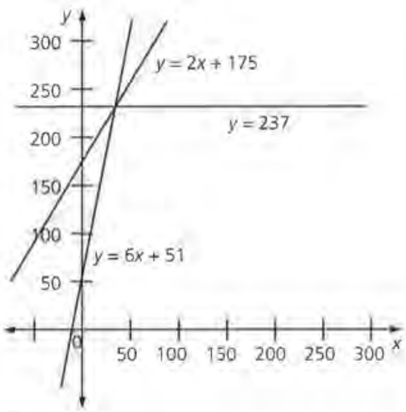
Practice and Applications

6. **a.** $x = 9$

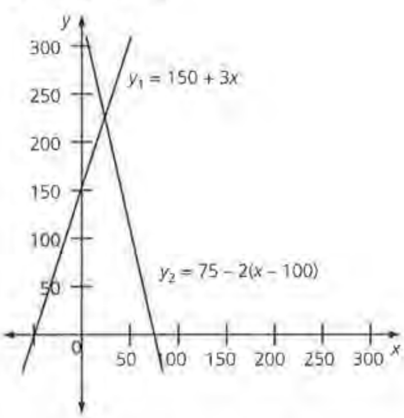
b. $4x + 51 = 175$

c. $x = 31$

d. All three lines intersect at the point (31, 237).



7. **a.** (25, 75)



b. Yes; $x = 25$, $y_1 = 225$, $y_2 = 225$

c. $x = 25$, $y_1 = y_2 = 225$

STUDENT PAGE 9

Practice and Applications

6. Consider an equation with a variable on both sides such as $6x + 51 = 2x + 175$.

a. You already know how to solve an equation with a variable listed only once. Solve $6x + 51 = 105$.

b. It seems reasonable to try to turn an equation with a variable on both sides into one you know how to solve. Eliminate the term $2x$ in $6x + 51 = 2x + 175$ by subtracting $2x$ from both sides of the equation. Write the new equation.

c. Solve your new equation.

d. Graph $y = 6x + 51$, $y = 2x + 175$, and $y = 237$ on the same set of axes. What observation can you make?

7. Consider the equations $y_1 = 150 + 3x$ and $y_2 = 75 - 2(x - 100)$.

a. Graph the equations on the same grid. Estimate the point of intersection of the two graphs.

b. Use your calculator to make a table of values (x, y) for each equation. Are any of the ordered pairs in the table the same? If not, continue to change the ΔTbl until you find an ordered pair that is the same for each equation.

c. Solve the system using algebra. For what x-value will $y_1 = y_2$? What are the values of y_1 and y_2 for this x-value?

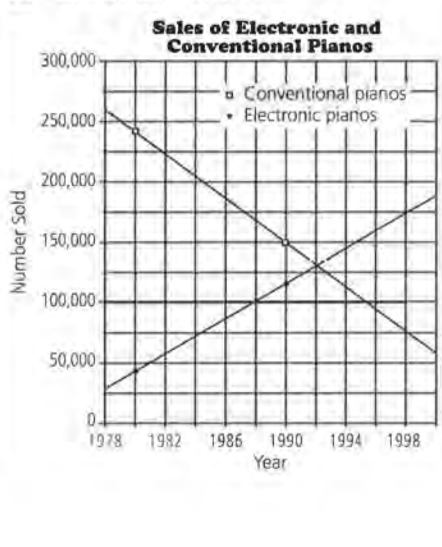
8. According to the American Music Conference, the sales of electronic pianos and keyboards have nearly overtaken sales of conventional pianos. In 1980, approximately 45,000 electronic pianos were sold; and in 1990, approximately 115,000 electronic pianos were sold. In 1980, approximately 240,000 conventional pianos were sold; and in 1990, approximately 150,000 conventional pianos were sold.

a. Assume the trends for both sales are linear. Make a plot of the data and estimate when the sales of electronic keyboards will or did overtake those of conventional pianos.

b. Find an equation for each line, and then solve the system algebraically. How does your solution compare to the estimate you made in part a?

c. Find the x-intercept, or the zero, for the equation for conventional pianos. What does this value indicate?

8. **a.** Possible answer:



a. An estimate of when electronic piano sales overtook conventional piano sales is 1992.

Answers to parts b and c are based on part a graph.

b. Electronic pianos: $y = 7000x - 13{,}815{,}000$; conventional pianos: $y = -9000x + 18{,}060{,}000$; point of intersection: (1992.1875, 130,312.5)

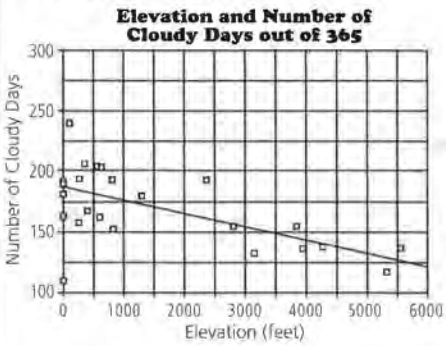
c. x-intercept = 2006.7; this is the first year when people will buy no conventional pianos.

STUDENT PAGE 10

9. a. City at sea level; Lander, WY

b. Anchorage, AK; Tampa, FL

c. Possible answer:



d. Based on part c graph, $y = -0.01x + 183.7$

9. Does elevation, or the height above sea level, have any effect on the weather? Are the numbers of clear days and cloudy days affected by the elevation of a city? The table below contains the elevation and the numbers of clear and cloudy days per year for selected cities in the United States.

City	Elevation (feet)	Number of Clear Days per Year	Number of Cloudy Days per Year
Albany, NY	275	67	190
Albuquerque, NM	5311	144	120
Anchorage, AK	114	64	238
Boise, ID	2838	111	153
Boston, MA	15	90	179
Burlington, VT	332	68	207
Helena, MT	3828	94	153
Lander, WY	5557	117	133
Milwaukee, WI	672	77	201
Minneapolis, MN	834	78	192
Mobile, AL	211	92	157
Nashville, TN	590	88	161
Newark, NJ	7	84	188
New Orleans, LA	4	83	162
Portland, OR	21	73	189
Raleigh, NC	434	93	167
Rapid City, SD	3162	123	130
Scotts Bluff, NB	3957	115	133
St. Louis, MO	535	69	202
Salt Lake City, UT	4221	114	134
San Antonio, TX	788	91	152
Springfield, MO	1268	103	178
Spokane, WA	2356	82	192
Tampa, FL	19	93	111

Source: *Statistical Abstract of the United States*, 1995

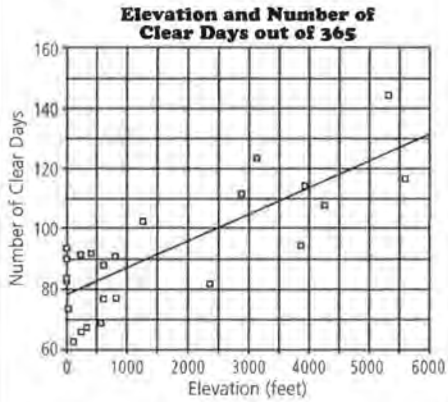
a. What would it mean if a city had an elevation of zero? Which city has the highest elevation?

b. Which city has the greatest number of cloudy days? The least?

c. On the first graph on *Activity Sheet 3*, draw a line that best fits the data for (elevation, cloudy days).

d. Write an equation for the line you drew in part c. This equation expresses the relationship between elevation and number of cloudy days per year.

e. Possible answer:



Answers to parts f and g are based on parts c and e graphs.

f. $y = 0.0089x + 78.4$

g. Elevation of approximately 5570 feet; sample explanation: I solved the equation $-0.01x + 183.7 = 0.0089x + 78.4$.

10. Check students' graphs.

a. (5, 16)

b. (–2.5, –2.5)

c. (8, 8)

d. (3, 12)

11. The values of the variables that make both equations true

12. a. The angular distance north or south of the equator, measured in degrees

STUDENT PAGE 11

e. On the second graph on *Activity Sheet 3*, draw a line that best fits the data for (elevation, clear days).

f. Write an equation for the line you drew in part e. This equation expresses the relationship between elevation and number of clear days per year.

g. For what elevation will the expected number of cloudy days and the number of clear days be the same? Explain how you got your answer.

10. Find the intersection point for each system of equations both by graphing and by finding the solution algebraically.

a. $y = 12 + 2(x - 3)$
$y = 16 + 4(x - 5)$

b. $y = x$
$y = -5x - 15$

c. $y = 2x - 8$
$y = 0.5x + 4$

d. $y = 12 + 2(x - 3)$
$y = 12 + 4(x - 3)$

11. When you solve a system of equations algebraically, what are you finding?

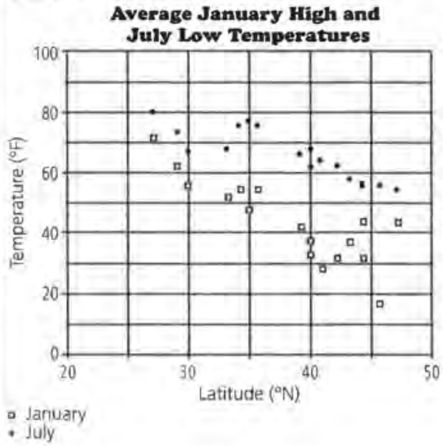
12. Below is a list of selected United States cities, their latitudes, and their average January high temperatures and average July low temperatures in degrees Fahrenheit (°F).

City	Latitude (°N)	January (°F)	July (°F)
Atlanta, GA	33	51	69
Baltimore, MD	39	41	67
Jackson, MI	30	57	68
Bismarck, ND	46	18	56
Boise, ID	43	37	59
Cleveland, OH	40	33	61
Chicago, IL	41	29	63
Detroit, MI	42	31	61
Houston, TX	29	62	73
Key West, FL	27	72	80
New York, NY	40	37	69
Oklahoma City, OK	35	47	77
Portland, ME	44	31	57
Portland, OR	44	44	56
Las Vegas, NV	36	55	76
Los Angeles, CA	34	55	76
Tacoma, WA	47	44	54

Source: *World Almanac and Book of Facts*, 1992

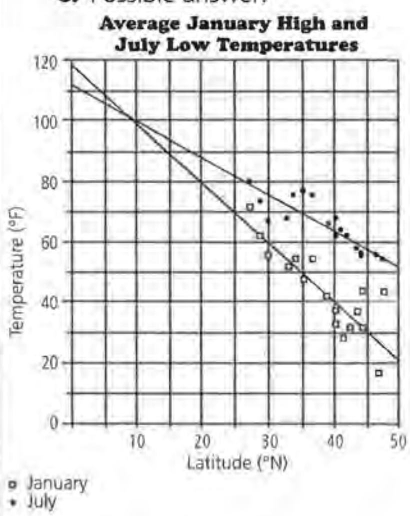
a. What is *latitude?* If you don't know, refer to a dictionary for the definition.

(12) b.



False; the greater the degree of latitude, the colder it is in January and July.

c. Possible answer:



Estimate for intersection point: (8, 102)

Answers to parts d, e, and f are based on part c graph.

d. January: $y = -1.92x + 117.02$; July: $y = -1.19x + 111.6$

e. Average July low temperature drops 11.9 degrees.

f. Point of intersection: (7.42, 102.76)

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b. Make a scatter plot of (latitude, average January high temperature). On the same graph, plot (latitude, average July low temperatures). Look at the plot, and then decide if the following statement is true: "The greater the degree of latitude, the warmer the city is in January and the colder in July."

c. Draw lines that you think best fit the data for (latitude, January temperature) and (latitude, July temperature). Extend each line and estimate their point of intersection from the graph.

d. Write an equation for each line.

e. For each 10° that the latitude increases, how does the average July low temperature change?

f. Use algebra to find the point of intersection of the two lines.

g. What does the point of intersection tell you about the relationship between temperature and latitude?

13. In 1975 there were 75 Division 1A field-goal kickers in football who kicked soccer style, while in 1990 there were 135. In 1975 there were 120 field-goal kickers who kicked straight on, and in 1990 there were only 2. When do you think the number of soccer-style kickers overtook the number of straight-on kickers? Explain how you made your decision.

14. The table below gives the estimated circulation figures for the magazines *Time*, *Newsweek*, and *U.S. News & World Report* in 1985 and 1991.

Magazine	1985	1991
Time	4.7 million	4.1 million
Newsweek	3.1	3.2
U.S. News & World Report	2.0	2.2

Source: *Magazine Publishers of America*

a. Assume that the rate of change for the three magazines is constant. Find the rate of change for each magazine.

b. Will the circulation for *Newsweek* overtake that of *Time*? If so, when? Create equations from the information to help you answer the question.

c. Answer the same question about *U.S. News & World Report* and *Newsweek*.

g. The point of intersection tells the latitude where the average high January temperature and the average low July temperature are equal.

13. Possible answer: 1978.7; I used the data to write the equations $y = 4x - 7{,}825$ and $y = -7.867x + 15{,}656.7$. Then I solved the equations.

14. a. *Time* rate of change = –0.1; *Newsweek* rate of change = 0.017; *U.S. News & World Report* rate of change = 0.033

b. Yes; equation for *Time*: $y = -0.1(x - 1985) + 4.7$; equation for *Newsweek*: $y = 0.017(x - 1985) + 3.1$; point of intersection: (1998.7, 3.33)

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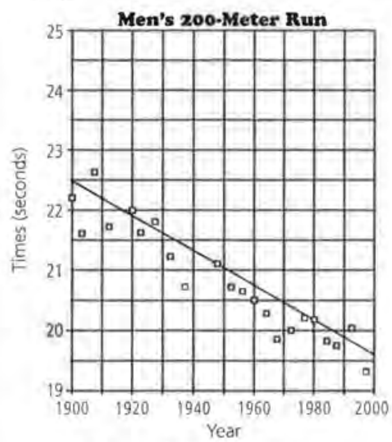
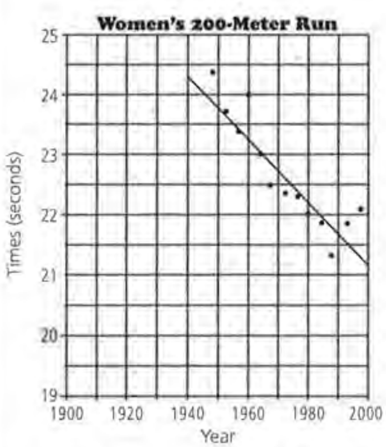
c. Equation for *U.S. News & World Report:* $y = 0.033(x - 1985) + 2.0$
Point of intersection (2053.7, 4.27)

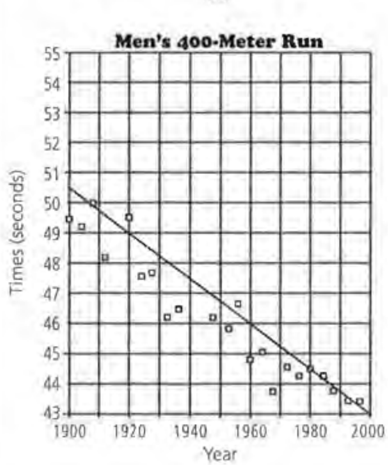
d. No; rates will probably not remain constant as people find other means to read about the news.

15. Possible lines are given and equations are based on those lines.

a.







d. Do you think it is safe to assume the rate of change is constant? What are some factors that might affect this assumption?

15. The following tables contain the times for the top female and male runners in the Olympic events referred to in the Introductory Activity of this unit. In groups, select an event and plot the male and female times over the given years.

a. Draw a line to represent the male data and a line to represent the female data.

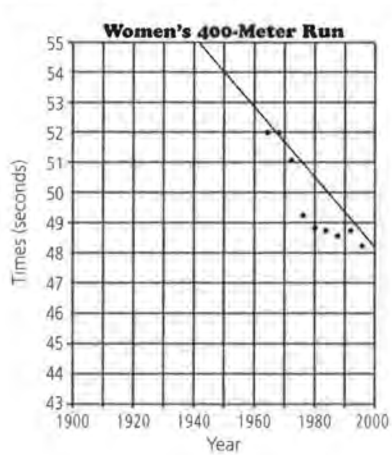
b. Write an equation for each line.

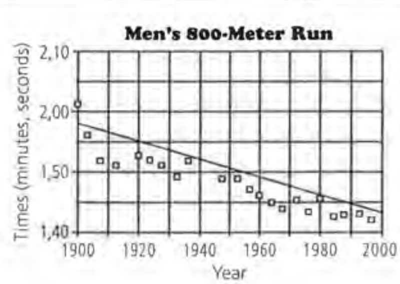
c. On page 3, the authors stated that women have been improving about twice as quickly as men. Do you agree with this statement? Explain your reasoning.

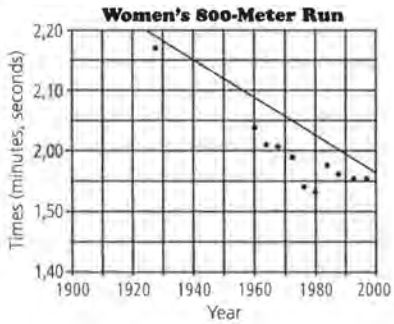
d. Will the top male and female runners be performing equally well in each event between the years 2015 and 2055? Show plots and use algebra to justify your claim.

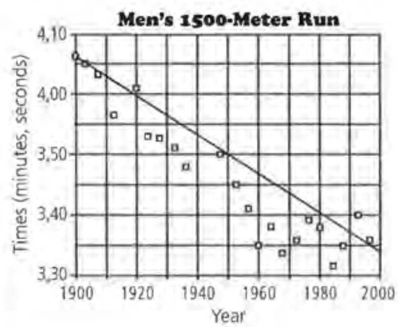
Times for the Olympic 200-Meter Run (seconds)

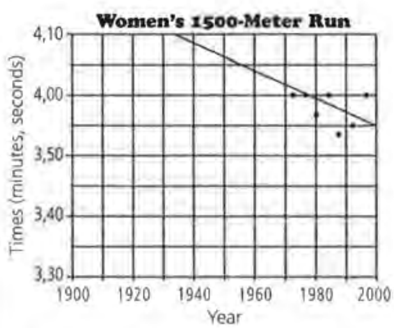
	Male		Female	
1900	Walter Tewksbury, US	22.2s		
1904	Archie Hahn, US	21.6s		
1908	Robert Kerr, Canada	22.6s		
1912	Ralph Craig, US	21.7s		
1920	Allan Woodring, US	22.0s		
1924	Jackson Scholz, US	21.6s		
1928	Percy Williams, Canada	21.8s		
1932	Eddie Tolan, US	21.2s		
1936	Jesse Owens, US	20.7s		
1948	Mel Patton, US	21.1s	Francina Blankers-Koen, Netherlands	24.4s
1952	Andrew Stanfield, US	20.7s	Marjorie Jackson, Australia	23.7s
1956	Bobby Morrow, US	20.6s	Betty Cuthbert, Australia	23.4s
1960	Livio Berruti, Italy	20.5s	Wilma Rudolph, US	24.0s
1964	Harry Car, US	20.3s	Edith McGuire, US	23.0s
1968	Tommie Smith, US	19.83s	Irena Szewinska, Poland	22.5s
1972	Valeri Borzov, USSR	20.00s	Renate Stecher, E. Germany	22.40s
1976	Donald Quarrie, Jamaica	20.23s	Barbel Eckert, E. Germany	22.37s
1980	Pietro Mennea, Italy	20.19s	Barbel Wockel, E. Germany	22.03s
1984	Carl Lewis, US	19.80s	Valerie Brisco-Hooks, US	21.81s
1988	Joe Deloach, US	19.75s	Florence Griffith-Joyner, US	21.34s
1992	Mike Marsh, US	20.01s	Gwinn Torrence, US	21.81s
1996	Michael Johnson, US	19.32s	Marie-Jose Perec, France	22.12s











b. Possible answers:

Equation of men's 200-meter run: $y = -0.028x + 75.7$

Equation of women's 200-meter run: $y = -0.055x + 131$

Equation of men's 400-meter run: $y = -0.075x + 193$

Equation of women's 400-meter run: $y = -0.117x + 282.2$

Equation of men's 800-meter run: $y = -0.142x + 387$

Equation of women's 800-meter run: $y = -0.321x + 757.7$

Equation of men's 1500-meter run: $y = -0.333x + 880$

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Times for the Olympic 400-Meter Run (seconds)

	Male		Female	
1896	Thomas Burke, US	54.2s		
1900	Maxey Long, US	49.4s		
1904	Harry Hillman, US	49.2s		
1908	Wyndham Halswelle, GB	50.0s		
1912	Charles Reidpath, US	48.2s		
1920	Bevil Rudd, S. Africa	49.6s		
1924	Eric Liddell, GB	47.6s		
1928	Ray Barbuti, US	47.8s		
1932	William Carr, US	46.2s		
1936	Archie Williams, US	46.5s		
1948	Arthur Wint, Jamaica, BWI	46.2s		
1952	George Rhoden, Jamaica, BWI	45.9s		
1956	Charles Jenkins, US	46.7s		
1960	Otis Davis, US	44.9s		
1964	Michael Larrabee, US	45.1s	Betty Cuthbert, Australia	52.0s
1968	Lee Evans, US	43.86s	Colette Besson, France	52.0s
1972	Vincent Matthews, US	44.66s	Monika Zehrt, E. Germany	51.08s
1976	Alberto Juantorena, Cuba	44.26s	Irena Szewinska, Poland	49.29s
1980	Viktor Markin, USSR	44.60s	Marita Koch, E. Germany	48.88s
1984	Alonzo Babers, US	44.27s	Valerie Brisco-Hooks, US	48.83s
1988	Steven Lewis, US	43.87s	Olga Bryzgina, USSR	48.65s
1992	Quincy Watts, US	43.50s	Marie-Jose Perec, France	48.83s
1996	Michael Johnson, US	43.49s	Marie-Jose Perec, France	48.25s

Times for the Olympic 800-Meter Run (seconds)

	Male		Female	
1896	Edwin Flack, Australia	2m11s		
1900	Alfred Tysoe, GB	2m1.2s		
1904	James Lightbody, US	1m56s		
1908	Mel Sheppard, US	1m52.8s		
1912	James Meredith, US	1m51.9s		
1920	Albert Hill, GB	1m53.4s		
1924	Douglas Lowe, GB	1m52.4s		
1928	Douglas Lowe, GB	1m51.8s	Lina Radke, Germany	2m16.8s
1932	Thomas Hampson, GB	1m49.8s		
1936	John Woodruff, US	1m52.9s		
1948	Mal Whitfield, US	1m49.2s		
1952	Mal Whitfield, US	1m49.2s		
1956	Thomas Courtney, US	1m47.7s		
1960	Peter Snell, New Zealand	1m46.3s	Lyudmila Shevtsova, USSR	2m4.3s
1964	Peter Snell, New Zealand	1m45.1s	Ann Packer, GB	2m1.1s
1968	Ralph Doubell, Australia	1m44.3s	Madeline Manning, US	2m0.9s
1972	Dave Wottle, US	1m45.9s	Hildegard Falck, W. Germany	1m58.55s
1976	Alberto Juantorena, Cuba	1m43.50s	Tatyana Kazankina, USSR	1m54.94s
1980	Steve Ovett, GB	1m45.40s	Nadezhda Olizayrenko, USSR	1m53.42s
1984	Joaquim Cruz, Brazil	1m43.00s	Doina Melinte, Romania	1m57.6s
1988	Paul Ereng, Kenya	1m43.45s	Sigrun Wodars, E. Germany	1m56.10s
1992	William Tanui, Kenya	1m43.66s	Ellen Van Langen, Netherlands	1m55.54s
1996	Vebjoern Rodal, Norway	1m42.58s	Svetlana Masterkova, Russia	1m55.54s

Equation of women's 1500-meter run: $y = -0.221x + 677$

c. In the 200-, 400-, and 800-meter runs, the women's rate of change was about twice that of men. In the 1500-meter run, the men's rate of change is about twice the women's rate of change.

d. Answers will vary.

16. a. Olympic times in particular events; specifically, the slopes of the best-fit lines for the data

b. In order to predict the future, we must assume that the rates of change will remain constant. Other researchers are probably saying that it is impossible to maintain these decreases in time.

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Times for the Olympic 1500-Meter Run (minutes, seconds)

	Male		Female	
1896	Edwin Flack, Australia	4m33.2s		
1900	Charles Bennet, GB	4m6.2s		
1904	James Lightbody, US	4m5.4s		
1908	Mel Sheppard, US	4m3.4s		
1912	Arnold Jackson, GB	3m56.8s		
1920	Albert Hill, GB	4m1.8s		
1924	Paavo Nurmi, Finland	3m53.6s		
1928	Harry Larva, Finland	3m53.2s		
1932	Luigi Beccali, Italy	3m51.2s		
1936	Jack Lovelock, New Zealand	3m47.8s		
1948	Henri Eriksson, Sweden	3m49.8s		
1952	Joseph Barthel, Luxembourg	3m45.2s		
1956	Ron Delany, Ireland	3m41.2s		
1960	Herb Elliott, Australia	3m35.6s		
1964	Peter Snell, New Zealand	3m38.1s		
1968	Kipchoge Keino, Kenya	3m34.9s		
1972	Pekka Vasala, Finland	3m36.3s	Lyudmila Bragina, USSR	4m01.4s
1976	John Walker, New Zealand	3m39.17s	Tatyana Kazankina, USSR	4m05.48s
1980	Sebastian Coe, GB	3m38.4s	Tatyana Kazankina, USSR	3m56.6s
1984	Sebastian Coe, GB	3m32.53s	Gabriella Dorio, Italy	4m03.25s
1988	Peter Rono, Kenya	3m35.96s	Paula Ivan, Romania	3m53.96s
1992	Fermin Casho Ruiz, Spain	3m40.12s	Hassiba Boulmerka, Algeria	3m55.30s
1996	Noureddine Morceli, Algeria	3m35.78s	Svetlana Masterkova, Russia	4m00.83s

16. The researchers Brian Whipp and Susan Ward wrote, "None of the current women's world-record holders at these events could even meet the men's qualifying standard to compete in the 1992 Olympic games. However, it is the rates of improvement that are so strikingly different—the gap is progressively closing."

a. What numerical information were Whipp and Ward using to make their claim?

b. Other researchers doubted the projections. List some of the reasons they might have given for their skepticism.

LESSON 2

Lines with the Same Slope

Materials: rulers, *Activity Sheet 4*
Technology: graphing calculators (optional)
Pacing: 1 class period

Overview

This lesson presents data sets that have approximately the same rate of change, or slope. The lesson opens with a data set comparing the percent of homes wired for cable to the percent of homes subscribing to cable. Students are asked to take values from the graph and place them in a table. Students then find the difference between the percent wired for cable and the percent that subscribe to cable. These percents will not be exactly equal but students should begin to make the connection that for parallel lines the difference is constant. The goal of this lesson is for students to conclude that lines with the same slope are parallel.

Teaching Notes

The focus of the lesson is on the slope of the lines. Having completed Lesson 1, students should be able to interpret the slope and conclude that lines with the same slope are parallel. In some of the examples, the lines that students draw will not have slopes that are exactly equal. Some students will want to say that the lines are not parallel. Suggest that students find the point of intersection and interpret this point.

Technology

The list feature of the graphing calculator can be used for Problem 1. Students can enter the years into L1 and the percent wired into L2 and percent subscribed into L3. L4 can be defined as the difference between L2 and L3. You might have students make a scatter plot of the ordered pairs (years, difference) or (L1, L4). The graph should be an approximately horizontal line showing, graphically, that the difference is approximately constant. Students can also use the graphing calculator to find the median-fit or least-squares regression line for each set of data and then compare the slopes of these lines.

Follow-Up

Have students use an almanac to update the cable data and determine if the rate of change for each line has remained about the same.

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LESSON 2

Lines with the Same Slope

Do two lines always have to intersect? How can you tell?

If two lines do not intersect, what characteristics will their graphs have? What is true about their equations?

OBJECTIVES

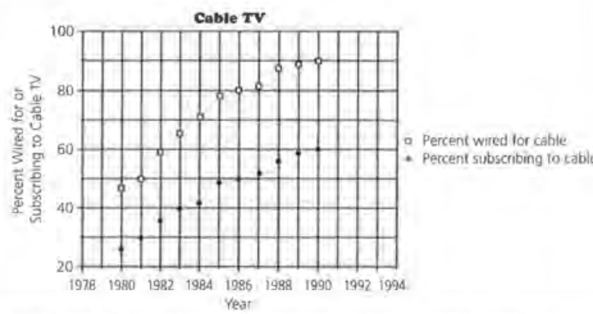
Recognize that parallel lines have the same slope or rate of change.

Recognize systems that do not have a solution.

In this lesson, you will learn about parallel lines and their equations. You will also see they can help you answer the questions above.

INVESTIGATE

The scatter plot below shows the percent of homes that are wired for cable and the percent of homes subscribing to a cable TV service for the years 1980 to 1990.



Source: "Cable TV." Copyright 1991 by Consumers Union of U.S., Inc., Yonkers, NY 10703-1057. Reprinted by permission from *Consumer Reports*, September, 1991

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Solution Key

Discussion and Practice

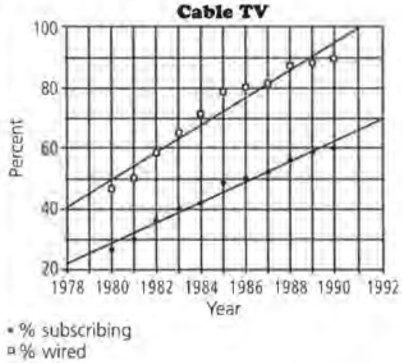
1. **a.** Percent wired for cable and percent subscribing to cable are increasing at about the same rate.

b. See table below.

c. From 1984, the difference remained about the same.

2. **a.** Possible answer:

Cable TV



b. It appears that they will never be the same.

c. They have approximately the same rate of change; or the lines are nearly parallel, diverging very slightly.

Discussion and Practice

1. Refer to the graph on page 16.

a. What trends do you observe in the graph?

b. Use the graph to complete a table like the following.

Year	Percent Wired	Percent Subscribing	Difference Between Percent Wired and Percent Subscribing
1980	____	____	____
1981	____	____	____
1982	____	____	____
1983	____	____	____
1984	____	____	____
1985	____	____	____
1986	____	____	____
1987	____	____	____
1988	____	____	____
1989	____	____	____
1990	____	____	____

c. Describe how the difference between the percent of homes wired for cable and the percent of homes subscribing to cable has changed over the years from 1980 to 1990.

2. Use the first graph on *Activity Sheet 4* for this problem.

a. Draw a line for the percent wired for cable as a function of the year and one for the percent subscribing to cable as a function of the year.

b. Estimate when the percent of homes wired for cable will be the same as the percent subscribing to cable.

c. It appears that the two lines have approximately the same slope. What does this tell you about the two lines?

d. For each of the lines you drew, find the rate of change, or *slope*. Are the lines parallel? Why or why not?

Year	Percent Wired	Percent Subscribing	Difference Between Percent Wired and Percent Subscribing
1980	47	26	21
1981	50	30	20
1982	59	35	24
1983	65	40	25
1984	71	42	29
1985	78	49	29
1986	80	50	30
1987	81	52	29
1988	88	56	32
1989	89	58	31
1990	90	60	30

d. Based on part a graph, slope for % wired line is ≈4.5; slope for % subscribing line is ≈3.5; the two lines are not parallel because the two slopes are not equal.

3. a. 10 pounds
   b. 5 pounds
   c. They are equal.

4. a. No; the algebra was correct.
   b. Both lines have the same slope. The lines are parallel and will never intersect, so they will not have a point of intersection.
   c. No; the lines do not intersect so there is no solution.

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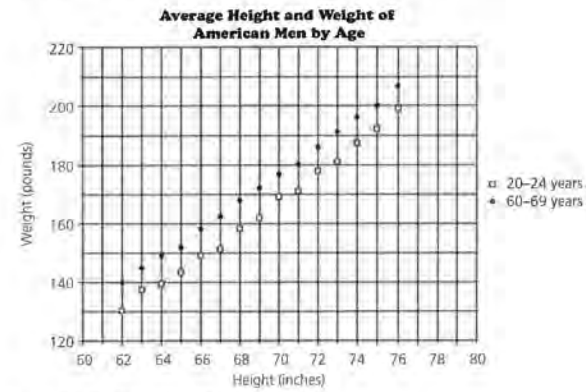
3. The scatter plot below shows the average weight of American men by height and age for men 20–24 years old and 60–69 years old.



Source: *The World Almanac and Book of Facts*, 1992

a. For each height, find the average difference in weight between a 20–24-year-old and a 60–69-year-old.

b. For each 1-inch increase in height, what is the increase in weight for a 20–24-year-old and a 60–69-year-old?

c. What do the values you found in part b tell you about the rate of change in weight for the two age groups?

4. Sam wanted to find where the two lines $y_1 = 2x - 10$ and $y_2 = 15 + 2(x - 4)$ intersect. His solution is shown here.

$$\begin{aligned} y_1 &= 2x - 10 \\ y_2 &= 15 + 2(x - 4) \\ 2x - 10 &= 15 + 2(x - 4) \\ 2x - 10 &= 15 + 2x - 8 \\ 2x - 10 &= 7 + 2x \\ -2x \quad & \quad -2x \\ \hline -10 &= 7 \end{aligned}$$

a. Did he make a mistake? If so, where?

b. Describe the graphs of the two lines. How does the graph help you understand the problem?

c. Can Sam conclude that $x = -10$ and $y = 7$? Explain.

STUDENT PAGE 19

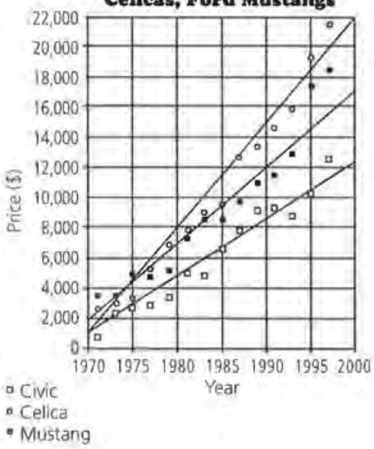
Practice and Applications

5. **a.** The lines are parallel, because they have the same slope, 5.

b. The lines will intersect, because they have different slopes, –2 and 2.

6. Possible answer:

Prices for Honda Civics, Toyota Celicas, Ford Mustangs



Summary

If two lines have the same slope, they are parallel and will not have a point of intersection. If two equations have the same slope, you have to determine whether they represent the same line or parallel lines. You can do this by investigating the points that work in the equations or the graphs of the lines they represent. Or you can find the solution algebraically.

- If two distinct lines have the same slope, or rate of change, then they are parallel and have no points in common.
- If two distinct lines intersect, their slopes are different, and they have one point in common.

Practice and Applications

5. Make a conjecture about the relation between the graphs of the lines represented by the pairs of equations in each problem. Verify your conjecture in some way.

a. $C = 250 + 5(x - 1980)$
$C = 120 + 5(x - 1965)$

b. $r = 18 + -2(x - 4)$
$r = 18 + 2(x - 4)$

6. Below are the list prices in dollars for new cars for the years 1971–1997. On the same set of axes, make a plot of years and prices for the Honda Civic, the Toyota Celica, and the Ford Mustang. Then fit a line to each set of data.

Year	Honda Civic	Chevrolet Camaro	Toyota Celica	Ford Mustang	Mercury Cougar	BMW	Corvette
1971	1395	3790	2847	3783	4069	5845	6327
1973	2150	3829	3159	3723	4045	8230	7007
1975	2799	4739	3694	4906	6121	10,605	9424
1977	2849	5423	5252	4814	6225	14,840	11,508
1979	3649	6021	6904	5339	6423	20,185	12,550
1981	5199	8142	7974	7581	8762	24,605	16,141
1983	4899	9862	8824	8466	10,725	24,760	19,368
1985	6479	10,273	9549	8441	11,825	20,970	26,901
1987	7968	11,674	12,608	9948	14,062	24,070	28,874
1989	9140	13,199	13,528	11,145	15,903	25,620	32,445
1991	9405	13,454	14,658	11,873	16,890	26,700	33,410
1993	8730	15,379	15,983	12,847	17,833	32,205	36,230
1995	10,130	17,536	19,410	17,550	18,960	34,220	37,955
1997	12,449	18,930	20,825	18,525	19,150	35,060	38,160

Source: *Kelley Blue Book*

a. Prices are increasing at different rates.

b. Based on the graph, slope of Civic line ≈390$/yr; slope of Celica line ≈700$/yr; slope of Mustang line ≈510$/yr

c. The Celica is the most expensive, then the Mustang, and finally the Civic.

7. a. Possible answer:

Attendance at Disney World and Universal Studios will never be the same, as the lines appear to be nearly parallel, diverging only slightly.

Answers to parts b and c are based on part a graph.

b. Slope of the Disney line ≈1.0; slope of the Universal line ≈0.67

c. Disney prediction: 36 million; Universal prediction: 10 million

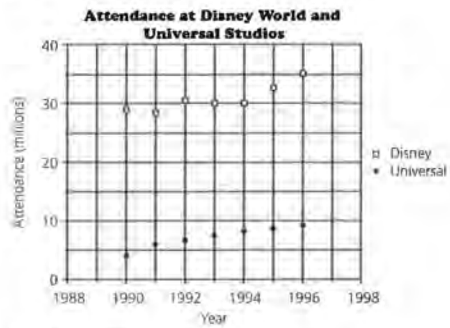
STUDENT PAGE 20

a. Are the prices for the Honda Civic, the Toyota Celica, and the Ford Mustang increasing at approximately the same rate?

b. How do the slopes of the lines compare?

c. If the rates of change stay approximately the same, predict the relation between the car prices in the future.

7. The scatter plot below shows the attendance (in millions of people) for Disney World and Universal Studios Florida.



Source: *Amusement Business Magazine*

a. On the second graph on *Activity Sheet 4*, fit a line to each set of data. Will the attendance at Disney World and Universal Studios ever be the same if the trend continues? How did you decide?

b. Describe the slope for each line in terms of the data.

c. If the rate of change for each line remains the same, predict the attendance for Disney World and Universal Studios Florida in the year 2000.

ASSESSMENT

Assessment for Unit I

Materials: rulers, graph paper, *Activity Sheet 5*
Technology: graphing calculators (optional)
Pacing: 1 class period or homework

Overview

Problems 1 and 2 assess students' understanding of Lesson 1. Problem 3 assesses the objectives of Lesson 2, and Problem 4 asks students to solve systems of equations algebraically.

Teaching Notes

This assessment can be used in a number of ways. It can be used strictly as an assessment that is completed by students in one class period. It could also be used as a take-home test or as additional practice on the objectives of Lessons 1 and 2. Students will be asked throughout the rest of the module to solve systems of equations, so it is important that students are able to find the intersection point of two equations.

Technology

Graphing calculators are optional. You may wish to allow students to use graphing calculators with Problem 3.

Follow-Up

Students can use ads.from the newspaper to collect data on the cost of lumber in their location. These prices could be used as a comparison of the prices presented in Problem 2. Students could use an almanac to update the data on life expectancies. They could also find the data on life expectancies for women.

STUDENT PAGE 21

Solution Key

1. **a.** Slope of incandescent line = 5.2; slope of fluorescent line = 1; for every additional 1000 hours of lighting, the slope expresses the additional amount of money it costs to burn this type of bulb.

b. Equation of incandescent line: $y = 5.2x + 1$; equation of fluorescent line: $y = x + 15$; the y-intercept is the initial cost of the bulb.

c. (3.3, 18.3); both bulbs cost the same amount if they are used for about 3 hours and 20 minutes.

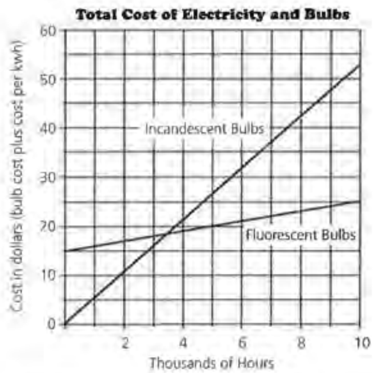
ASSESSMENT

Assessment for Unit I

OBJECTIVE

Apply knowledge of systems of equations.

1. Probably most of the light bulbs in your house are *incandescent* bulbs. These bulbs use electricity to superheat a filament. You may also have some *fluorescent* light bulbs. These bulbs use a small electric charge to ionize mercury vapor, which then gives off light. Below is a graph comparing the costs of incandescent light bulbs and fluorescent light bulbs. Incandescent bulbs cost about $1 each and fluorescent bulbs cost about $15 each. One thousand hours of wattage is denoted by "kwh."



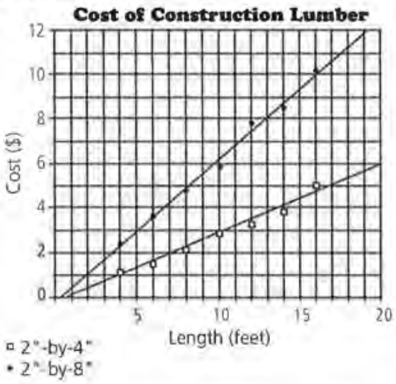
Source: *Milwaukee Journal Sentinel*, February 4, 1996

a. Find the slope of each line. Describe the slope in terms of the data.

b. Write an equation for each line. Describe the y-intercept of each line in terms of the data.

c. Solve the system of equations. Then interpret your answer.

2. a. Possible answer:



Equation of 2-by-4 line: $y = 0.31x - 0.25$; equation of 2-by-8 line: $y = 0.64x - 0.28$

Answers to parts b, c, and d are based on part a answer.

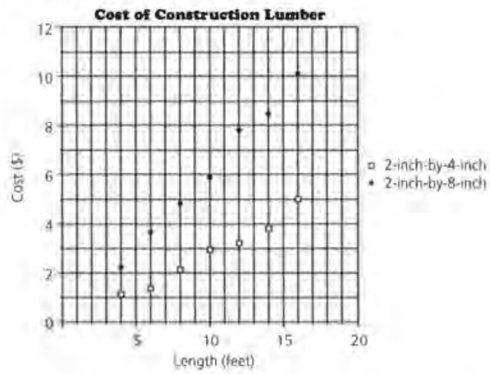
b. 2-by-4 line: For each additional foot in length, the cost of the lumber increases by 31 cents; 2-by-8 line: For each additional foot in length, the cost of the lumber increases by 64 cents.

c. \$1.92

d. (0.9, −0.22); this point is very close to (0, 0), which means that for either size, to buy nothing costs nothing.

STUDENT PAGE 22

2. The graph below shows the costs of lumber at a local lumber store. The prices are for 2-inch-by-4-inch and 2-inch-by-8-inch boards of various lengths.



a. On the graph on *Activity Sheet 5*, draw a line for each set of data and find an equation for each line.

b. Describe the slope of each line in terms of the data.

c. Use your equation to predict the cost of a 2-inch-by-4-inch board that is 7 feet long.

d. Solve the system of equations. Interpret your answer.

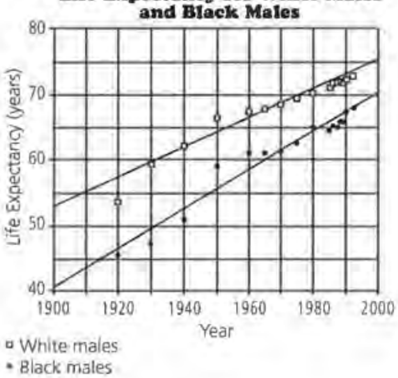
3. The life expectancy of males born in certain years is given in the table below.

Year	Life Expectancy (years) White Male	Life Expectancy (years) Black Male	Year	Life Expectancy (years) White Male	Life Expectancy (years) Black Male
1920	54.4	45.5	1980	70.7	65.3
1930	59.7	47.3	1985	71.9	64.8
1940	62.1	51.5	1986	72.0	66.8
1950	66.5	59.1	1987	72.2	65.0
1960	67.4	61.1	1988	72.3	66.7
1965	67.6	61.1	1989	72.5	66.7
1970	68.0	61.3	1990	72.9	67.0
1975	69.5	63.7	1992	73.2	67.8

Source: *World Almanac and Book of Facts*, 1994

3. a. Possible answer:

Life Expectancy for White Males and Black Males



Equation for white-male line: $y = 0.23x - 384$; equation for black-male line: $y = 0.295x - 520$

b. Based on part a answer, slope of white-male line = 0.23; slope of black-male line = 0.295

Possible comparison paragraph: Life expectancy of white males is higher than that of black males at this time. The rate of change for life expectancy of black males is 0.295, which is greater than the 0.23 for white males. This means that if the trend continues, in about the year 2114 black males and white males will have the same life expectancy of 102 years. This seems unlikely.

4. a. $(1.8\overline{3}, 1.\overline{33})$

b. (9.6, 4.8)

c. $(1.\overline{841269}, 6.\overline{64})$

STUDENT PAGE 23

a. Plot the year and the life expectancy for white males and the year and life expectancy for black males and find equations for the lines that best fit the data.

b. Find the slope of each line and write a paragraph comparing the life expectancies of white and black males. Indicate what might be predicted for future life expectancies for each group.

4. Solve each system of equations.

a. $y = -2x + 5$
$y = 4x - 6$

b. $y = \frac{1}{2}x$
$y = -\frac{3}{4}x + 12$

c. $y = 6 + 0.35x$
$y = 0.2 + 3.5x$

UNIT II

Graphing Inequalities

INTRODUCTORY ACTIVITY

Estimating the Number of Raisins

Materials: $\frac{1}{2}$-ounce boxes of raisins (one for each student), graph paper, rulers
Technology: graphing calculators (optional)
Pacing: $\frac{1}{2}$ class period

Overview

The purpose of this activity is to have students begin to think about what it means for a point to be above or below a line. Students are asked to estimate the number of raisins in a $\frac{1}{2}$-ounce box of raisins and to compare their estimates to the actual counts.

Teaching Notes

When students are working through this activity, stress that a point below the line $y = x$ represents a point where the x-coordinate is greater than the y-coordinate. If students struggle with this concept, have them list ordered pairs and compare the x- and y-coordinates. The first problem in Lesson 3 presents another activity that is very similar to this one. You do not have to introduce the $<$ and $>$ symbols, since they are first used in Lesson 3.

The module *Exploring Linear Relations* in the *Data-Driven Mathematics* series contains an activity in which students estimate ages of famous people. This activity could be used in place of the activity here or as another example.

Technology

Students can use graphing calculators to make the scatter plot. The estimates can be entered into L1 and the actual counts into L2. The line Y = X can then be graphed, and students can use their graph for the problems in this activity.

Follow-Up

You may wish to have students construct a line graph of the actual number of raisins in each box. They could find the average number and write a description of the graph and how their number compared to the class average.

STUDENT PAGE 27

INTRODUCTORY ACTIVITY

Estimating the Number of Raisins

How many raisins do you suppose there are in a small box?

How did you make your estimate?

Do you think your estimate is greater than or less than the actual number of raisins in the box?

In this unit, you will investigate data that are less than or greater than a given standard, such as the weight or number of raisins in a box of a given size.

OBJECTIVE

Compare an estimate to an actual count.

EXPLORE

Each of you should have your own $\frac{1}{2}$-ounce box of raisins. Look carefully at your box of raisins without opening it. Then write your estimate of the number of raisins contained in the box. Now open your box and count the raisins. How close were you to the actual number? Was your estimate high or low?

Solution Key

1. Answers will vary.
2. Answers will vary.
3. On the line $y = x$
4. The estimate was less than the actual count.
5. The estimate was greater than the actual count.

STUDENT PAGE 28

1. Collect from each student in class his or her estimate for the number of raisins and the actual number. Record the data in a table similar to the one below.

Student	Estimate	Actual Number
1	______	______
2	______	______
3	______	______
•	______	______
•	______	______
•	______	______

2. Make a scatter plot of the ordered pairs (estimate, actual number) for the class data.
3. Where will the point lie if your estimate is correct? Draw the line that represents 100% accurate estimates. Write an equation for the line.
4. What does it mean if a point is above this line?
5. Describe, in terms of estimating raisins, a point found below the line.

LESSON 3

Shading a Region

Materials: graph paper, rulers, *Activity Sheet 6*
Technology: calculators (optional)
Pacing: 1 class period

Overview

This lesson begins with an activity similar to the estimating-raisins introductory activity. It is used to introduce the use of the symbols $<$ and $>$ to describe a region that is below or above the line $y = x$.

Teaching Notes

The main focus of the lesson is to have students understand how to describe a region on a graph with the symbols $<$, $\leq$, $>$, and $\geq$. Be sure students know the difference between the symbols $<$ and $\leq$ and the symbols $>$ and $\geq$, and that they can graphically represent inequalities involving these symbols. It is not appropriate to have students draw a median-fit or least-squares regression line. Initially, Problem 8 may be difficult for some students. You may wish to review graphing the lines of the type $x = 5$ and $y = 3$ before students work through Problem 8.

Technology

Graphing calculators are not needed. Students should draw the line $y = x$ and shade the region by hand.

Follow-Up

Students could generate a list of situations that involve the comparison of two variables in which it would be appropriate to draw the line $y = x$.

STUDENT PAGE 29

LESSON 3

Shading a Region

How good were you at estimating the number of raisins in a $\frac{1}{2}$-ounce box?

How good do you think you are at estimating the number of calories in certain fast-food items?

When comparing prices between two stores, how can you tell whether one store is generally higher priced or lower priced than the other?

In this lesson, you will use inequalities to investigate these questions.

OBJECTIVE

Find the solutions to an inequality using the line $y = x$.

INVESTIGATE

Many fast-food chains base their advertising on the quality and prices of their food or on claims that their food is nutritious and tasty.

Discussion and Practice

A single hamburger at McDonald's has 270 calories. On page 30 is a list of some other items from McDonald's.

Solution Key

Discussion and Practice

1. Estimates will vary.

Item	Estimated Calories	Actual Calories
Small Bag of French Fries		210
Filet o' Fish		360
6 Chicken McNuggets		290
Chicken Sandwich		510
Big Mac		530
Small Chocolate Shake		340
Small Diet Coke		0
Apple Pie		260
Cheeseburger		320
Quarter Pounder		430

2. **a.** Answers will vary.

b. On the line $y = x$

c. The estimate was less than the actual value or the actual value was greater than the estimate. The actual value is greater than the estimate.

d. Answers will vary.

e. The estimate is greater than the actual value or the actual value is less than estimate; $y < x$

STUDENT PAGE 30

Item	Estimated Calories	Actual Calories
Small Bag of French Fries	______	______
Filet o' Fish	______	______
6 Chicken McNuggets	______	______
Chicken Sandwich	______	______
Big Mac	______	______
Small Chocolate Shake	______	______
Small Diet Coke	______	______
Apple Pie	______	______
Cheeseburger	______	______
Quarter Pounder	______	______

1. Copy the table above and estimate the number of calories to the nearest 10 calories in each item. Then your teacher will give you the correct number of calories to the nearest 10 calories.

2. To help you determine how well you estimated the number of calories, complete the following.

a. Make a scatter plot of the ordered pairs (estimated calories, actual calories).

b. Where would the point lie if your estimate were correct? Draw the line that represents 100% accurate estimates. Write an equation for that line.

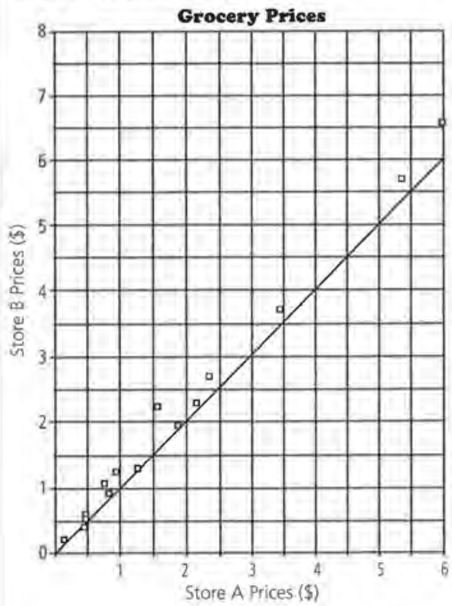
c. What does it mean if a point is above this line? Shade the region above the line. Make a list of five ordered pairs in the shaded region. What is true about the relationship between the actual calories (y-value) and the estimated calories (x-value) in each ordered pair you wrote?

d. A point in the region shaded above the line can be described with the inequality $y > x$. Verify that the ordered pairs you listed above satisfy this inequality.

e. Describe a point found in the region below the line in words. Write a symbolic description with an inequality.

STUDENT PAGE 31

3. a. Possible answer:



b. On the line $y = x$

c. Below the line $y = x$; $y > x$

d. Store A has cheaper prices because many of the points are above the line $y = x$, which means that the x-coordinate (Store A) is less than the y-coordinate (Store B).

4. **a.** $y = x$

b. $y < x$

3. Listed below are randomly selected items and their prices from two grocery stores.

Item	Store A	Store B
Soda (12-pack)	$3.48	$3.68
Head of lettuce	$0.98	$1.29
Salad dressing (24 oz)	$2.38	$2.69
Punch (liter)	$0.88	$0.98
Catsup (28 oz)	$1.58	$2.25
Peanut butter (28 oz)	$5.38	$5.68
1 package of Kool Aid	$0.19	$0.22
1 box of Jell-O	$0.47	$0.48
Can of corn (11oz)	$0.49	$0.59
Box of rice	$1.28	$1.35
Pizza sauce (32 oz)	$1.88	$1.98
Baked beans (21 oz)	$0.78	$1.09
Mushroom soup (10.5 oz)	$0.49	$0.55
Breakfast cereal (21.3 oz)	$2.18	$2.39
Coffee (39 oz)	$5.98	$6.55

a. Make a scatter plot of the data with the prices from Store A on the x-axis and prices from Store B on the y-axis.

b. Where will the point for items lie if the prices at the two stores are the same? Draw a line representing these prices.

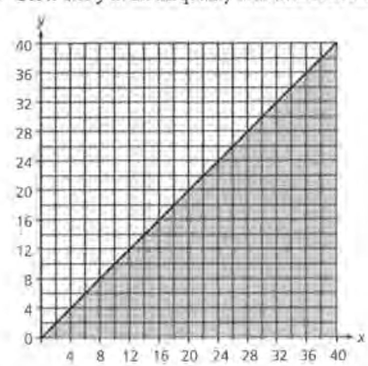
c. Describe the region containing most of the ordered pairs.

d. Which store do you think has the cheaper prices? Why?

4. Refer to the graph below.

a. Write an equation for the line drawn on the graph.

b. Use x and y in an inequality that describes the shaded region.

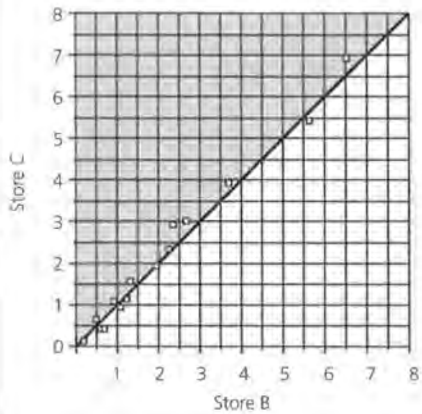


Practice and Applications

5. **a.** Any point above the line $y = x$

b. Any point below the line $y = x$

c.



Store B is slightly cheaper than Store C, because there are more points above the line $y = x$, which means that prices in store B are less than prices in store C.

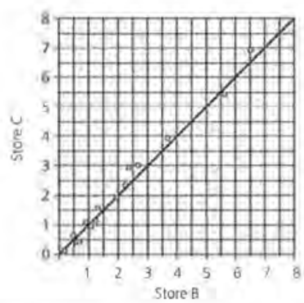
STUDENT PAGE 32

Summary

To compare two quantities with the same units, you can draw the line $y = x$. This line divides the plane into two regions. One region contains points where y is greater than x, and the other region contains points where y is less than x. When graphing the inequality $y > x$ or the inequality $y < x$, the line $y = x$ is usually shown as a dashed line to indicate that the points on the line do not satisfy the inequality.

Practice and Applications

5. In the graph that follows, the prices from Store C are plotted against the prices from store B.



a. Identify a point where Store B has lower prices than Store C does. Describe the costs for that item.

b. Identify a point where Store B is more expensive than Store C. Describe the costs for that item.

c. Make a sketch of the plot and shade in the area where Store B has lower prices than Store C. In general, which store has lower prices? How can you tell?

STUDENT PAGE 33

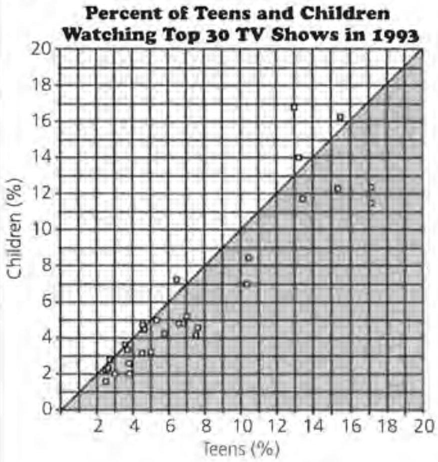
6. The following data are the percents of teens and children who watched America's favorite prime time network television programs in 1992–93, according to the Nielsen Media Research.

Rank	Program	%Teens	%Children
1	*60 Minutes*	2.8	2.9
2	*Roseanne*	15.3	12.2
3	*Home Improvement*	13.4	11.8
4	*Murphy Brown*	6.6	4.9
5	*Murder, She Wrote*	2.6	2.4
6	*Coach*	10.4	8.4
7	*NFL Monday Night Football*	7.5	4.1
8	*CBS Sunday Night Movie*	3.9	2.6
9	*Cheers*	7.0	5.1
10	*Full House*	13.0	16.9
11	*Northern Exposure*	3.9	2.0
12	*Rescue 911*	6.4	7.2
12	*20/20*	4.6	4.8
14	*CBS Tuesday Night Movie*	5.9	4.2
15	*Love and War*	4.5	3.1
16	*Fresh Prince of Bel Air*	17.1	12.4
16	*Hangin' With Mr. Cooper*	13.2	14.0
16	*Jackie Thomas Show*	10.3	7.0
19	*Evening Shade*	3.6	3.7
20	*Hearts Afire*	3.8	3.6
20	*Unsolved Mysteries*	4.7	4.5
22	*PrimeTIME LIVE*	2.5	1.6
23	*NBC Monday Night Movie*	7.6	4.6
24	*Dr. Quinn, Medicine Woman*	5.3	5.0
25	*Seinfeld*	5.0	3.2
26	*Blossom*	17.1	11.5
26	*48 Hours*	3.0	2.0
28	*ABC Sunday Night Movie*	6.9	4.9
29	*Matlock*	2.5	2.2
30	*The Simpsons*	15.5	16.2

Source: *The World Almanac and Book of Facts*, 1994

STUDENT PAGE 34

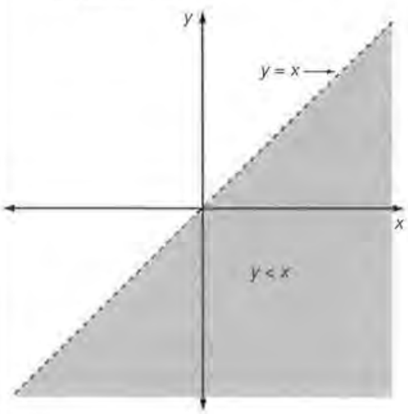
6. a. Possible answer:



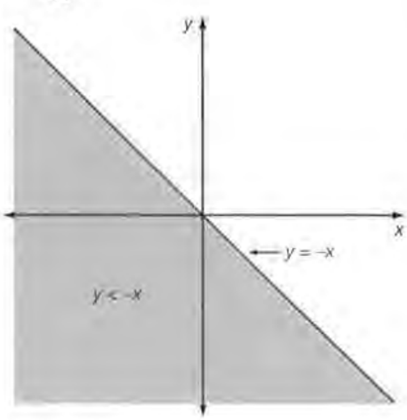
b. $t > c$, where t = % of teens and c = % of children watching

c. *Blossom,* the greatest distance from the line $y = x$

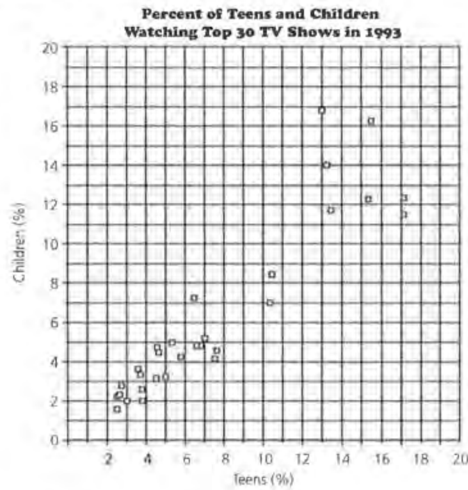
7. a. The y-coordinate is less than the x-coordinate.



b.



A scatter plot of the percent of children and the percent of teens who watched the top thirty programs is given below and on *Activity Sheet 6.*



a. On the graph on *Activity Sheet 6,* draw a line to help determine where the percent of teens watching is greater than the percent of children watching. Indicate the region by shading on the plot.

b. Write an inequality to describe this region.

c. For which program is the difference between the percent of children and the percent of teens who watch the greatest? How can you tell from the graph?

7. Consider the equations $y = x$ and $y = -x$.

a. Describe the graph of $y < x$ in words and with a picture.

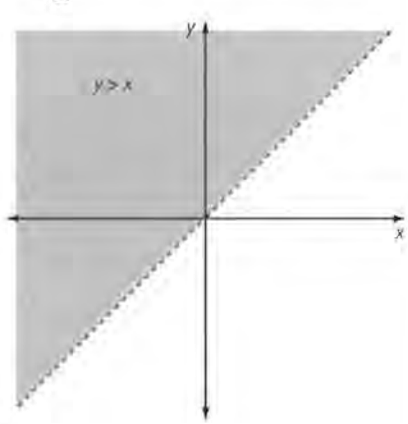
b. Graph the line $y = -x$. Shade the region that describes the set of all points that satisfy the inequality $y < -x$.

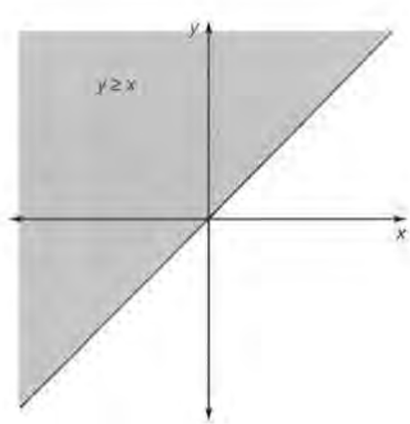
c. Graph the inequalities $y > x$ and $y \geq x$. How can you show that the two inequalities are different?

8. Graph each inequality.

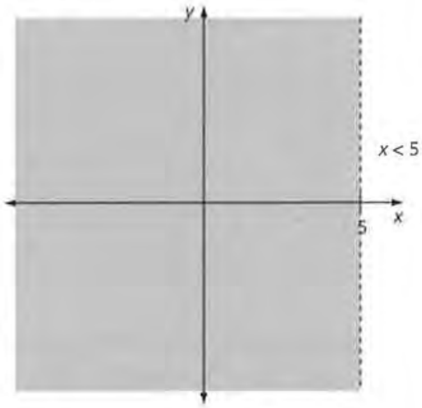
a. $x < 5$ b. $y \geq -3$ c. $x \geq 2$ d. $y < 0$

c.

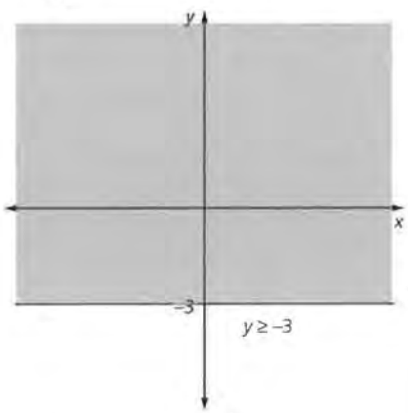




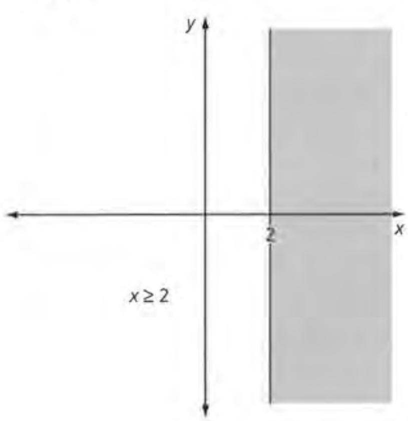
8. a.



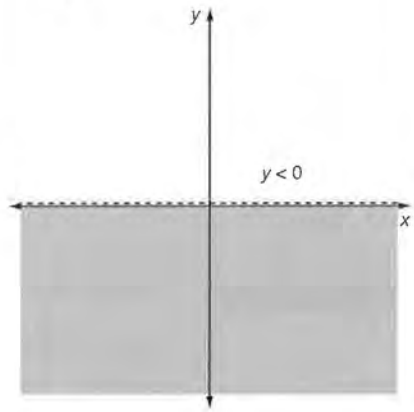
b.



c.



d.



LESSON 4

Graphing Inequalities

Materials: graph paper, rulers, *Activity Sheet 7, Lesson 4 Quiz*
Technology: graphing calculators (optional)
Pacing: 1 class period

Overview

This lesson extends the students' understanding of the use of the symbols < and > to describe a region. The lesson begins by presenting a graph and a line of best fit. Students are asked to graph the line; and as the lesson progresses, they are asked to shade the region that is below the line and describe this region with an inequality. At the end of the lesson, students should be able to graph a given inequality and to describe a shaded region with an inequality.

Teaching Notes

All of the inequalities given in this lesson are in the form $y < mx + b$ or $y > mx + b$. The main emphasis is placed on developing an understanding of the relationship between the inequality and the shaded region.

In presenting this lesson, you may wish to use the data on curl-ups, since they are not used in the lesson. Students should draw a line that fits the data and then discuss the meaning of the points that are in the region below the line. This region can then be described with an inequality in the form $y < mx + b$.

Technology

It is important that students graph and shade the proper region by hand. If students are using graphing calculators, they should be encouraged to make a sketch of their graph or print the results.

Follow-Up

Ask students to think of situations where "benchmarks" or levels are set to achieve an award that changes for different ages.

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LESSON 4

Graphing Inequalities

How old do you have to be to drive a car?

How many points do you have to earn in track to qualify for a letter?

OBJECTIVE

Graph and interpret a linear inequality.

Many situations involve inequalities. You will get a grade of B if you have more than a certain number of points; renting a car can be more economical if you drive more than a certain number of miles; selecting the best telephone company and package depends on the range of calls you make per month. In this lesson, you will learn to find a region that contains a set of points that satisfy a statement of inequality.

INVESTIGATE

Each year, students are encouraged to participate in the National Physical-Fitness Tests to demonstrate how physically fit they are and to qualify for a national award. The qualifying standards for boys for the Presidential Award are given below.

Physical-Fitness Qualifying Standards for Boys, The Presidential Award

Ages	Curl-Ups (1 minute)	Shuttle Run (seconds)	Sit and Reach (inches)	1-Mile Run (min:sec)	Pull-Ups
6	33	12.1	3.5	10:15	2
7	36	11.5	3.5	9:22	4
8	40	11.1	3.0	8:48	5
9	41	10.9	3.0	8:31	5
10	45	10.3	4.0	7:57	6
11	47	10.0	4.0	7:32	6
12	50	9.8	4.0	7:11	7
13	53	9.5	3.5	6:50	7
14	56	9.1	4.5	6:26	10
15	57	9.0	5.0	6:20	11
16	56	8.7	6.0	6:08	11
17	55	8.7	7.0	6:06	13

Source: *The Orlando Sentinel*, November 17, 1991

Solution Key

Discussion and Practice

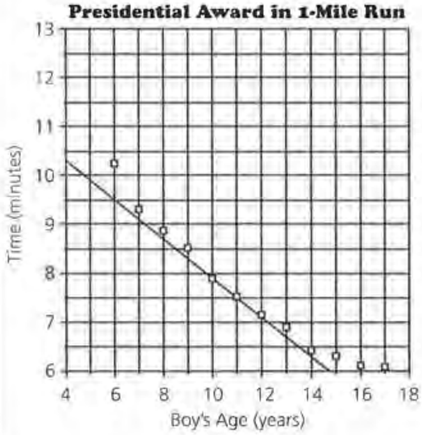
1. Be sure students realize that the seconds in the data have been changed to fractions of minutes.

a. As the boy's age increases, the time needed to earn a Presidential Award in the 1-mile run decreases.

b. No; the units on the *x*- and *y*-axes are different, and the graph has a negative slope.

c.

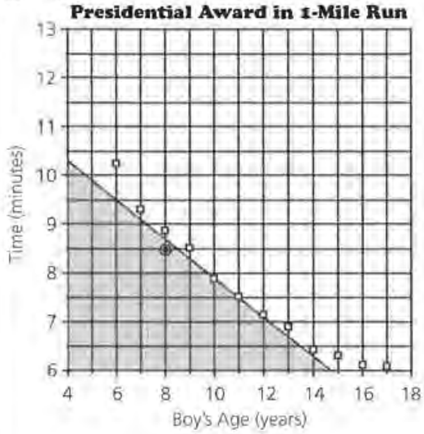
Presidential Award in 1-Mile Run



d. Substitute the value of 8 for *x*. The equation gives a value of 8.7 or 8 minutes and 42 seconds. This is faster than the 8 minutes 48 seconds from the table.

2. a.

Presidential Award in 1-Mile Run

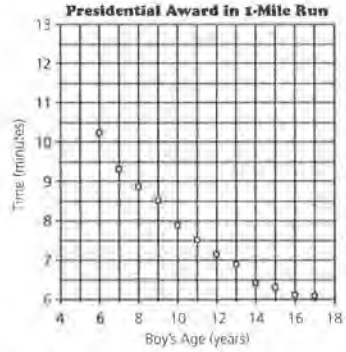


STUDENT PAGE 36

Discussion and Practice

1. Below is a scatter plot of the physical-fitness qualifying standards for boys aged 6 to 17 to earn a Presidential Award in the 1-mile run.

Presidential Award in 1-Mile Run



a. What trend do you see in the plot?

b. Would the line $y = x$ make sense in this plot? Explain.

c. Suppose that the equation of a line that seems to fit the data is $y = -0.4x + 11.9$. Graph this line on the graph on *Activity Sheet 7*. Describe how you graphed the line.

d. How can you determine the qualifying standard for an 8-year-old boy using the equation? How does this value compare with the time listed in the table?

2. If an 8-year-old boy ran the mile in 8 minutes and 30 seconds (8.5 min), he would have exceeded the Presidential standards. That is, he would have run the mile in a time less than or equal to the established standard.

a. On the graph for Problem 1c, plot the point representing the boy's time. Shade the region of the graph that contains this point. What does this region represent?

b. List three ordered pairs that lie in the shaded region.

c. Write an inequality for the shaded region. Show how each ordered pair you listed in part b satisfies the inequality.

d. If a 15-year-old boy ran the mile in 7 minutes and 10 seconds, would he qualify for a Presidential Award? How can you use your graph to determine your answer?

All the points that are below the Presidential Award marks.

b. Answers will vary.

c. $y < -0.4x + 11.9$; demonstrations of how each ordered pair satisfies the inequality will vary.

d. No; the point (15, 7 minutes 10 seconds) is above the line.

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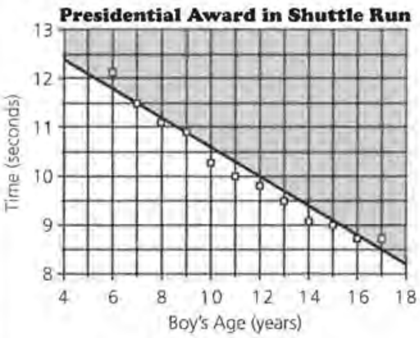
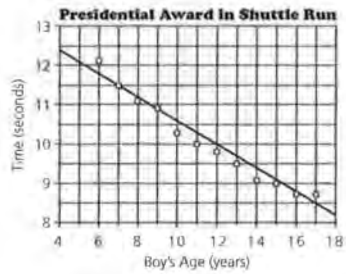
3. **a.** Possible answer: $y = -0.03x + 13.6$

b.



c. Based on part a equation, $y > -0.3x + 13.6$

3. Shown below is the graph of boy's age in years and time in seconds necessary to earn a Presidential Award in the shuttle run.



a. Write an equation for the line drawn on the plot.

b. Sketch the plot and the line, and then shade the region that represents the times for boys who would not earn an award for the shuttle run.

c. Write an inequality that represents the shaded region drawn in part b.

Summary

- If two variables have the same unit, you can plot them and use the graph of $y = x$ to determine whether the x-variable is greater than or less than the y-variable. The region above the graph for $y = x$ can be represented by the inequality $y > x$. The regions below the graph for $y = x$ can be represented by the inequality $y < x$.
- If two variables do not have the same unit but seem to have a linear relationship, you can draw a line summarizing the relationship. This line can be used to determine an inequality for the relationship. The area on one side of the line is greater than the relationship; the area on the other side is less. To find which side is greater than the relationship, you must investigate what the ordered pairs on each side represent and whether or not they satisfy the inequality. The inequality $y > ax + b$ represents the region above the graph of $y = ax + b$, and the inequality $y < ax + b$ represents the region below the graph of $y = ax + b$.

Practice and Applications

4. **a.** Shaded above a dashed line
b. Shaded below a dashed line
c. Shaded below a solid line

5. **a.** As the girl's age increases, the time needed to qualify for a Presidential Award in the shuttle run decreases. Times start to level off around the age of 15.

b.

Presidential Award in Shuttle Run

c. Possible answer:

Presidential Award in Shuttle Run

$y = -0.2x + 13.4$

d. Based on part c graph,
$y < -0.2x + 13.4$

6. Times needed to qualify for a Presidential Award in the shuttle run for boys are less than for girls. At the early ages, there is not much difference; but as ages increase, the gap between the times increases.

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Practice and Applications

4. If the equation of a line is $y = sx + d$, describe each region.

a. $y > sx + d$

b. $y < sx + d$

c. $y \leq sx + d$

5. The data for the physical fitness qualifying standards for the Presidential Award for girls are in the table below.

Physical-Fitness Qualifying Standards for Girls, The Presidential Award

Ages	Curl-Ups (1 minute)	Shuttle Run (seconds)	Sit and Reach (inches)	1-Mile Run (min:sec)	Pull-Ups
6	32	12.4	5.5	11:20	2
7	34	12.1	5.0	10:36	2
8	38	11.8	4.5	10:02	2
9	39	11.1	5.5	9:30	2
10	40	10.8	6.0	9:19	3
11	42	10.5	6.5	9:02	3
12	45	10.4	7.0	8:23	2
13	46	10.2	7.0	8:13	2
14	47	10.1	8.0	7:59	2
15	48	10.0	8.0	9:08	2
16	45	10.1	9.0	8:23	1
17	44	10.0	8.0	8:15	1

Source: *The Orlando Sentinel*, November 17, 1991

a. Look carefully at the data for age and time for the shuttle run. Describe the relationship between age and time.

b. Make a scatter plot of the girls' Presidential Award qualifying times for the shuttle run.

c. Draw a line to fit the data. Find an equation for the line.

d. Write an inequality that represents the region that describes those girls who would qualify for an award.

6. Compare the times for boys and the times for girls to receive a Presidential Award for the shuttle run. What are your conclusions?

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7. a. Possible answer:

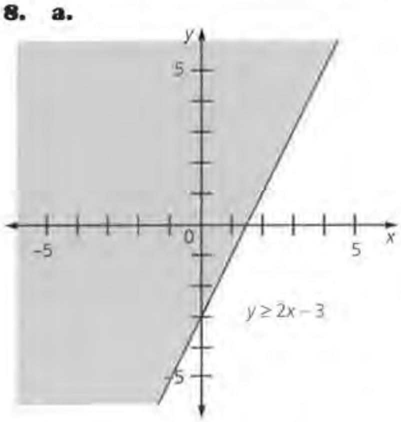
As the week number increases, the time walking increases.

Answers to parts b and c are based on part a graph.

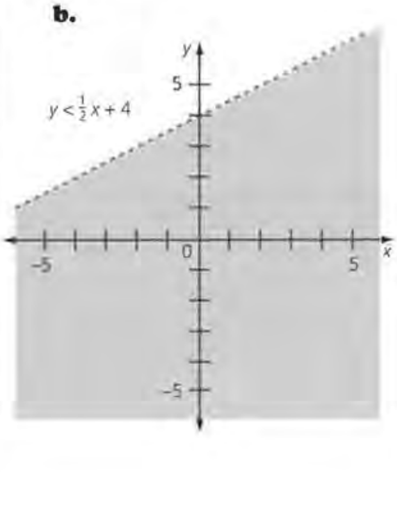
b. $y = 7.2x$; this descibes the relationship between the week number and the number of minutes a person should walk.

c. The region below the line represents the values where a person walks less than the amount suggested in the chart; $y < 7.2x$

8. a.



b.



7. Walking is one of the most effective ways to exercise. Here is a basic 12-week plan to get started on a walking program. Walk three days a week for the amount of time shown on the chart. At the end of 12 weeks, maintain the time shown for the twelfth week.

Minutes per Day Walking

Week	Day 1	Day 2	Day 3
1	10	10	10
2	12	12	16
3	15	15	20
4	15	20	25
5	20	25	35
6	30	25	45
7	35	35	50
8	40	40	60
9	45	45	60
10	45	45	70
11	45	45	80
12	45	45	90

Source: *Men's Health Magazine*

a. Make a scatter plot of (week number, minutes walking) for day 3. Draw a line to summarize the relationship. Describe any trends that you see in the graph.

b. Find an equation for the line. What relationship does the line describe?

c. What does the region below the line represent? Write an inequality that describes this region.

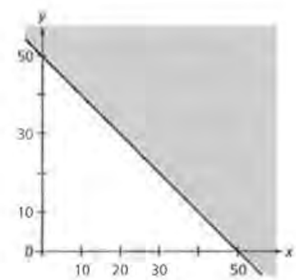
8. Graph each inequality.

a. $y \geq 2x - 3$ **b.** $y < \frac{1}{2}x + 4$

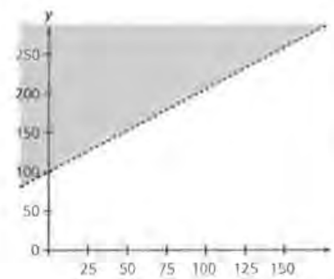
c. $y \leq 5 - 0.5(x - 2)$ **d.** $y > 30 + 4(x - 2)$

9. Write an inequality to describe each graph.

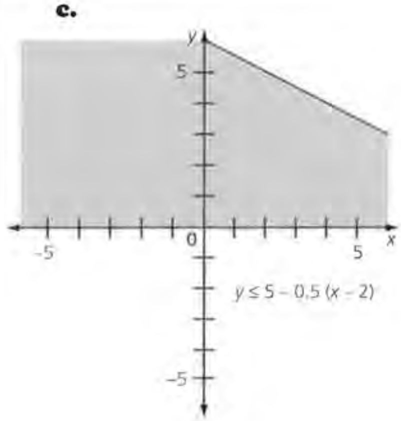
a.



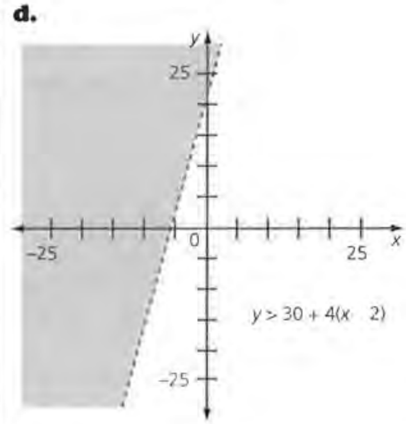
b.



c.



d.



9. a. $y \geq -x + 50$ **b.** $y > x + 100$

Assessment for Unit II

Materials: graph paper, rulers
Technology: graphing calculators (optional)
Pacing: 1 class period or homework

Overview

Problem 1 assesses students' understanding of the inequality $y \leq x$. Problem 2 asks students to draw a line that best fits the data, write its equation, and then change the equation to an inequality that represents a shaded region.

Teaching Notes

This assessment can be used in a number of ways. It can be used strictly as an assessment that is completed by students in one class period. It could also be used as a take-home test or as additional practice on the objectives of Lessons 3 and 4. The rest of the module uses inequalities extensively; as a result, students will need a good understanding of graphing a given inequality or describing a shaded region with an inequality.

Technology

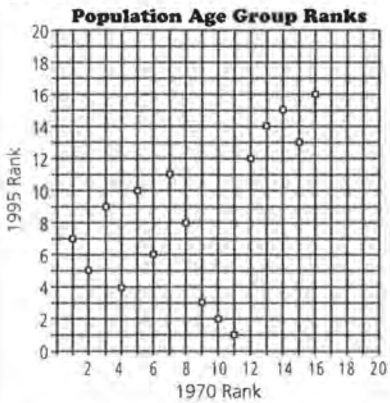
The use of graphing calculators is optional. You may wish to allow students to use a calculator to make the scatter plot asked for in Problems 1 and 2.

Follow-Up

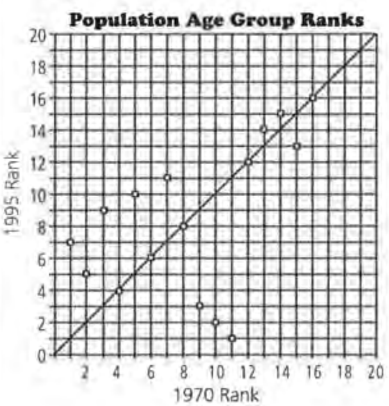
Students could use a current issue of *U.S. News & World Report* to update the data presented and then draw and find the equation of a line that relates the number of faculty to the enrollment. Students could compare this equation to the equation they found in Problem 2c.

Solution Key

1. a.



b.



$y = x$

c. Below the line $y = x$, 4 in this region

d. Above the line $y = x$, 7 in this region

e. Age group 35–39; changed from a rank of 11 to 1

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ASSESSMENT

Assessment for Unit II

OBJECTIVE

Solve problems by graphing inequalities.

1. Age groups in the United States can be ranked according to their percent of the total population. The 1970 and 1995 ranks for each age group used in the United States Census are given in this table.

Age Group	1970 Rank	1995 Rank
5 and younger	4	4
5–9	2	5
10–14	1	7
15–19	3	9
20–24	5	10
25–29	6	6
30–34	10	2
35–39	11	1
40–44	9	3
45–49	7	11
50–54	12	12
55–59	13	14
60–64	14	15
65–74	8	8
75–84	15	13
85 and older	16	16

Source: *Statistical Abstract of the United States*, 1996

a. Make a scatter plot of (1970 rank, 1995 rank).

b. Draw the line showing all the points where the rank in 1970 is the same as in 1995. Write an equation for the line.

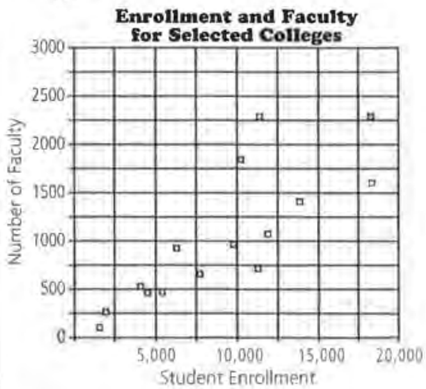
c. Where on the scatter plot are the age groups whose rank in 1995 is higher than the rank in 1970? "Higher" means that the rank moves closer to 1. How many of the age groups are there in this region?

d. Where on the scatter plot are the age groups whose rank in 1995 is lower than the rank in 1970? "Lower" means that the rank moves toward closer to 16. How many of the age groups are there in this region?

e. Which age group had the greatest change in rank? Describe this change.

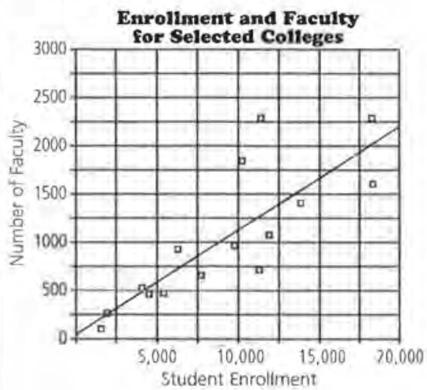
2. a. Yes; higher enrollment would mean more faculty are needed to teach.

b.



As student enrollment increases, the number of faculty increases.

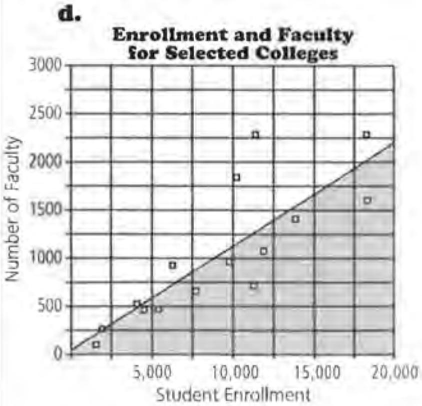
c. Possible answer:



$y = 0.11x + 51$

Answers to parts d and e are based on part c graph.

d.



$y < 0.11x + 51$

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2. Listed below are the 15 highest-ranked national universities among 204 schools that are research-oriented. The ranks were determined by the *U.S. News and World Report* magazine. The chart gives the enrollment and the number of faculty for each school for the 1992–93 school year.

Rank	College	Enrollment	Faculty
1	Harvard University	18,273	2,278
2	Princeton University	6,438	935
3	Yale University	11,129	712
4	Massachusetts Institute of Technology	9,798	975
5	California Institute of Technology	2,009	270
6	Stanford University	13,893	1,408
7	Duke University	11,426	2,304
8	Dartmouth College	5,475	468
9	University of Chicago	10,231	1,843
10	Cornell University	18,450	1,593
11	Columbia University School of Engineering and Applied Sciences	1,832	101
12	Brown University	7,593	663
13	Northwestern University	12,032	1,079
14	Rice University	4,033	542
15	Johns Hopkins University	4,613	473

Source: *US News and World Report, 1994 College Guide*

a. Would you expect to see some relation between the number of faculty members and the enrollment? Why or why not?

b. Make a scatter plot of (enrollment, number of faculty). Describe any pattern you see.

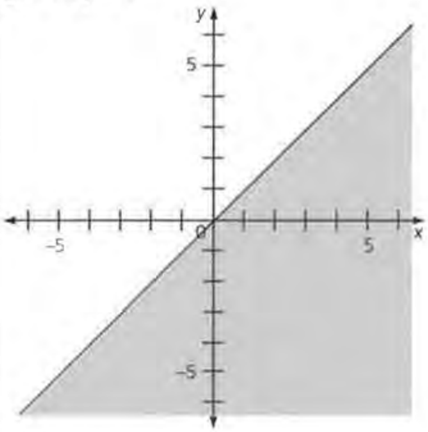
c. Draw a line that seems to fit the data. Write an equation of this line that relates the number of faculty members to the enrollment.

d. Shade the region where the number of faculty members is less than expected. Write an inequality to represent this region.

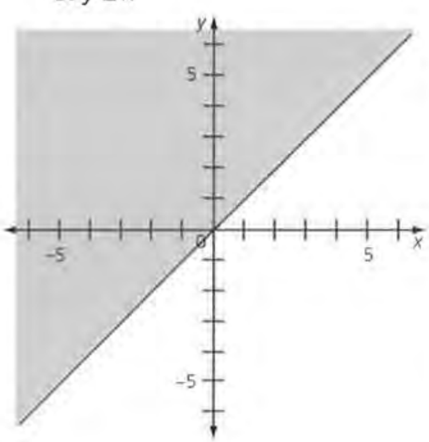
e. The University of Texas at Austin has an enrollment of 49,253 and 2,358 faculty members. Is the relationship better or worse than you expected? Tell how you made your decision.

e. Worse than expected. Substitute 49,253 into the equation. The predicted number of faculty is about 5470.

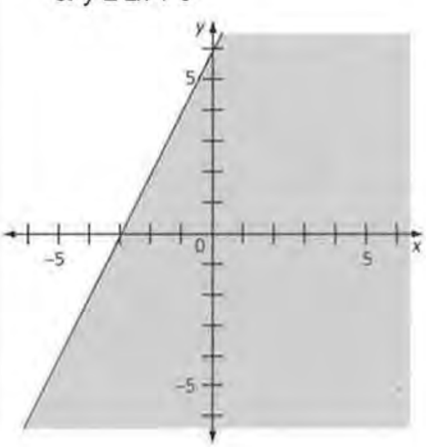
3. **a.** $y \le x$



b. $y \ge x$



c. $y \le 2x + 6$



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3. Graph each inequality

a. $y \le x$

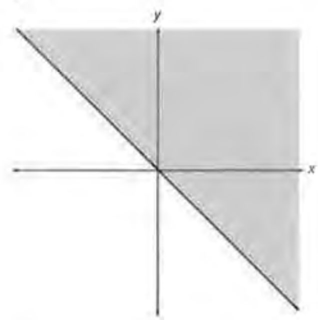
b. $y \ge x$

c. $y \le 2x + 6$

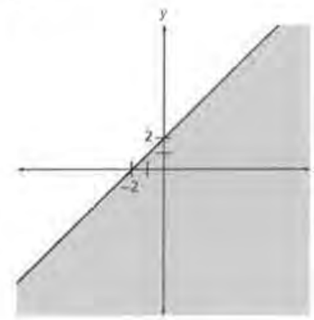
d. $y \ge -2x - 7$

4. Write an inequality to describe each graph.

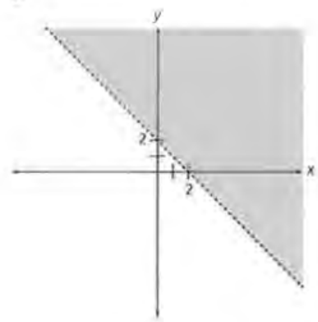
a.



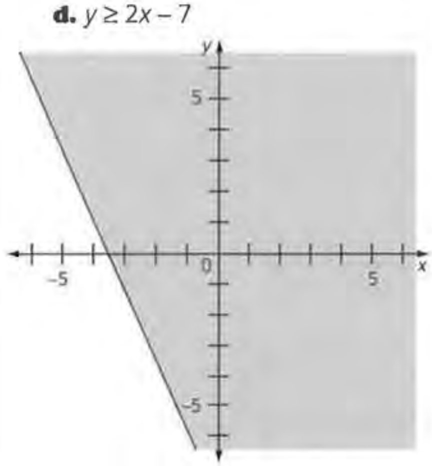
b.



c.



d. $y \ge 2x - 7$



4. **a.** $y \ge -x$

b. $y \le x + 2$

c. $y > -x + 2$

UNIT III

Solving Systems of Inequalities

INTRODUCTORY ACTIVITY

Fast Foods

Materials: none needed
Technology: calculators
Pacing: $\frac{1}{2}$ class period

Overview

This is a very brief activity to give students an opportunity to preview the next few lessons. Students are given the amount of cholesterol and sodium in a chicken sandwich and in 1 cup of 1% milk. They are asked to determine how many chicken sandwiches and how much milk they could consume and still remain under the recommended dietary limits.

Teaching Notes

This activity could be assigned as homework after students have finished the assessment for Unit II. Lesson 5 will build on the ideas presented in this activity, so there is no need to proceed too deeply with your students.

Technology

Calculators will help students with the necessary computation.

Solution Key

1. **a.** Yes; 2(50) + 2(10) = 120, which is less than 300.

b. Yes; 2(820) + 2(115) = 1870, which is less than 2400.

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INTRODUCTORY ACTIVITY

Fast Foods

Do you ever think about how many calories are in the food you eat?

Are you concerned about the amount of fat and salt in your diet?

OBJECTIVE

Investigate ordered pairs that satisfy a constraint.

Many people need to be aware of the amount of cholesterol and sodium in the food that they eat, because high cholesterol levels can lead to heart attacks and high sodium levels may be a cause of high blood pressure. If your daily dietary intake is about 2,500 calories a day, then your daily diet should include less than 300 mg of cholesterol and less than 2,400 mg of sodium.

EXPLORE

The amounts of cholesterol and sodium in a chicken sandwich and in 1 cup (8 fluid ounces) of 1% lowfat milk at a fast-food restaurant are listed below.

	Chicken Sandwich	1 Cup of 1% Milk
Cholesterol (mg)	50	10
Sodium (mg)	820	115

1. Suppose you ate 2 chicken sandwiches and drank 2 cups of 1% milk.

a. Would you stay under the dietary levels of 300 mg of cholesterol? Show how you determined your answer.

b. Would you stay under the dietary levels of 2,400 mg of sodium? Show how you determined your answer.

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2. **a.** Less than 6
   b. 2.9, just less than 3
3. **a.** Less than 30
   b. 20.9, just less than 21
4. (0, 20), representing no chicken sandwiches and 20 cups of milk; if at least one chicken sandwich must be eaten, then the ordered pair is (1, 13), representing 1 chicken sandwich and 13 cups of milk.

2. If you ordered just chicken sandwiches and no milk,
   a. how many sandwiches could you eat and stay under the dietary levels of 300 mg of cholesterol?
   b. how many sandwiches could you eat and stay under the dietary levels of 2,400 mg of sodium?
3. If you just ordered just 1% milk and no chicken sandwiches,
   a. how many cups of milk could you drink and stay under the dietary levels of 300 mg of cholesterol?
   b. how many cups of milk could you drink and stay under the dietary levels of 2,400 mg of sodium?
4. Find an ordered pair that represents the *greatest* number of chicken sandwiches and cups of 1% milk that you could consume and stay under dietary levels for cholesterol and sodium. Do you think one person actually might eat the combination that you found?

LESSON 5

Graphing Conditions

Materials: graph paper, rulers, *Activity Sheets 8* and *9*
Technology: graphing calculators (optional)
Pacing: 1 class period

Overview

The main objective of this lesson is to have students graph two inequalities on the same coordinate system. All of the inequalities in this lesson are in the form $x \leq a$ or $y \leq b$. The lesson begins with data from the 1996 NBA Championship Series. Students are asked to graph the inequalities *total points* ≥ 80 and *rebounds* ≥ 40. These inequalities form four regions on the graph, and students are asked to interpret each region and write an inequality that represents each region. The remainder of the lesson uses different data sets to emphasize the meaning of the region where the two given inequalities overlap.

Teaching Notes

As you present this lesson, you will want to remind students of the difference and similarities between the inequalities $x \leq 5$ and $x < 5$. In addition, the emphasis should be placed on the interpretation of the shaded region where the two inequalities overlap. Most students will not have too much difficulty with this lesson, but it is important that they be able to graph the given inequalities and understand that the solution set is the shaded region where the shading overlaps.

Technology

Graphing calculators are optional. Students can make their scatter plots on a calculator and use the **DRAW** function to draw the horizontal and vertical lines.

Follow-Up

Have students find the data for the most recent NBA Championship Series. The data can usually be found at the NBA Web site. Have students use the same criteria, *total points* ≥ 80 and *total rebounds* ≥ 40, to determine the MVP.

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LESSON 5

Graphing Conditions

Do you watch professional basketball or other sports on television?

Who is your favorite player?

Led by Michael Jordan, the Chicago Bulls won the 1996 NBA Championship. They defeated the Seattle Supersonics, led by Shawn Kemp, in six games. Jordan was voted the most valuable player by the media covering the game. If you had been allowed to vote for MVP, would you have voted for Jordan?

OBJECTIVE

Graph and interpret systems of inequalities in the form $x < a$ and $y < b$.

INVESTIGATE

The following tables contain the statistics for the 1996 Championship series. Only those players who played a total of at least 40 minutes in the series are listed.

1996 NBA Championship Series, Seattle

Player	Min	FG-A	FT-A	RB	AST	PF	PT
Kemp	242	49-89	42-49	86	13	28	140
Schrempf	238	35-80	21-24	39	16	18	98
Payton	274	40-90	19-26	44	44	22	108
Hawkins	230	25-55	24-26	28	6	21	80
Perkins	190	23-61	17-21	37	12	13	67
Wingate	43	5-10	4-4	3	0	13	15
McMillan	51	3-7	2-3	11	6	6	11
Brickowski	68	2-9	0-0	14	3	16	5

Min (minutes played), FG-A (field goals made-field goals attempted), FT-A (Free throws made-free throws attempted), RB (rebounds), AST (assists), PF (personal fouls), PT (total points)

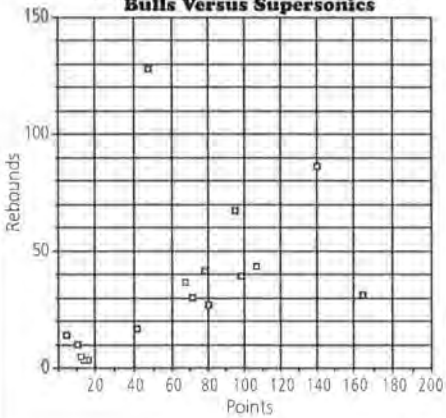
Source: *Chicago Tribune*

Solution Key

Discussion and Practice

1.

1996 NBA Championship Series
Bulls Versus Supersonics

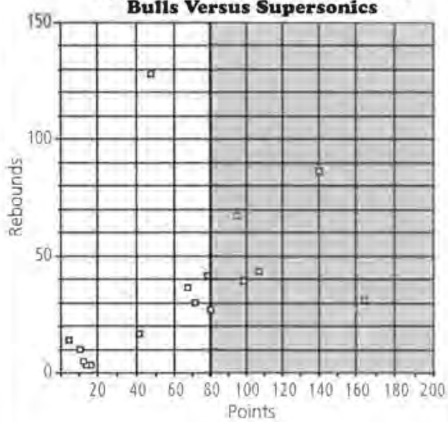


2. Most of the data points lie roughly along a line.

3. Yes, generally, as the number of points increase the number of rebounds increases. Exceptions are Michael Jordan and Dennis Rodman.

4.

1996 NBA Championship Series
Bulls Versus Supersonics



5. See the graph in Problem 4; Kemp, Schrempf, Payton, Hawkins, Pippen, Jordan

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1996 NBA Championship Series, Chicago

Player	Min	FG-A	FT-A	RB	AST	PF	PT
Pippen	248	34-99	17-24	69	32	22	94
Rodman	225	17-35	11-19	129	15	27	45
Longley	170	27-47	16-22	31	13	23	70
Harper	115	12-32	11-12	17	10	8	39
Jordan	252	51-123	56-67	32	20	17	164
Kukoc	176	30-71	8-10	41	21	14	78
Brown	49	6-12	2-4	3	5	2	17
Wedington	42	7-10	1-2	5	1	7	14

Min (minutes played), FG-A (field goals made-field goals attempted), FT-A (Free throws made-free throws attempted), RB (rebounds), AST (assists), PF (personal fouls), PT (total points)

Source: *Chicago Tribune*

Discussion and Practice

Most Valuable Player

Two categories that are important considerations in determining the MVP are total points and rebounds.

Use the first grid on *Activity Sheet 8* for Problems 1–7.

1. Make a scatter plot of the ordered pairs (total points, rebounds) of all the players on both teams.
2. What observations can you make from the scatter plot?
3. Do you think there is an association between total points and rebounds? Explain your answer.
4. Suppose you felt that the most valuable player should have scored at least 80 points in the six games. Draw a line on your scatter plot showing all of the ordered pairs whose x-coordinate, or total points, is 80.
5. The line that you have drawn separates the scatter plot into two regions. Shade the region that shows the location of all the ordered pairs for total points greater than or equal to 80. Identify the players that are in this region.
6. Suppose you also felt that the most valuable player is one that had 40 or more rebounds. Draw a line on your scatter plot showing all the ordered pairs whose y-coordinate, or total rebounds, is 40.
7. Shade the region that shows the location of all the ordered pairs with at least 40 rebounds. List all the players who had more than 40 rebounds.

6.

1996 NBA Championship Series
Bulls Versus Supersonics

7. See the graph in Problem 6; Kemp, Payton, Pippen, Rodman, Kukoc

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8. a. *x*-coordinate less than 80 and *y*-coordinate less than 40

b. *x*-coordinate greater than 80 and *y*-coordinate less than 40

c. *x*-coordinate greater than 80 and *y*-coordinate greater than 40

9. a. $t < 80$ and $r > 40$

b. $t > 80$ and $r < 40$

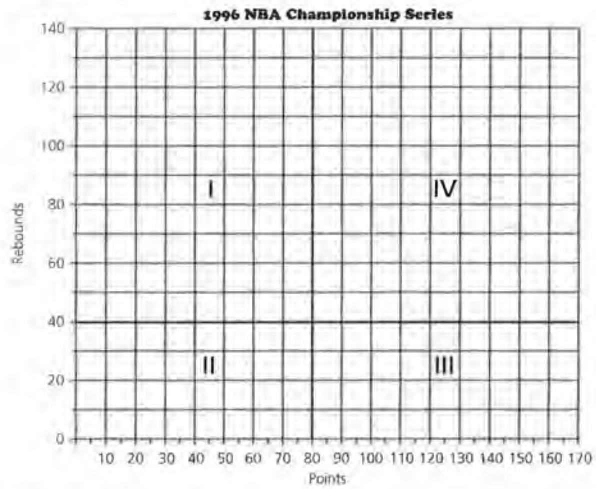
c. $t > 80$ and $r > 40$

10. a. Region IV

b. Kemp, Payton, Pippen

c. No; Jordan did not satisfy both conditions.

Your scatter plot should be separated into four nonoverlapping regions.



The points in region I are those whose *x*-coordinate is less than 80 and whose *y*-coordinate is greater than 40.

8. Describe the ordered pairs that are in each other region.

a. Region II **b.** Region III **c.** Region IV

If you let *t* represent the total points and let *r* represent the total rebounds, then region II could be represented by the inequalities $t < 80$ and $r < 40$.

9. Write a pair of inequalities to represent each other region.

a. Region I **b.** Region III **c.** Region IV

10. Consider the region that satisfies the conditions that the most valuable player should have at least 80 points and at least 40 rebounds.

a. Which region satisfies these conditions?

b. Identify the players in the region.

c. Based on these two conditions, do you agree that Michael Jordan should have been voted the MVP of the Championship series? Why or why not?

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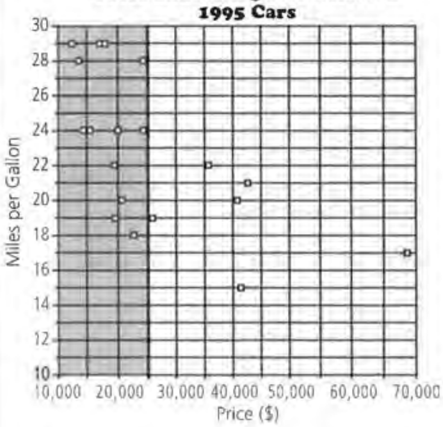
11. a.

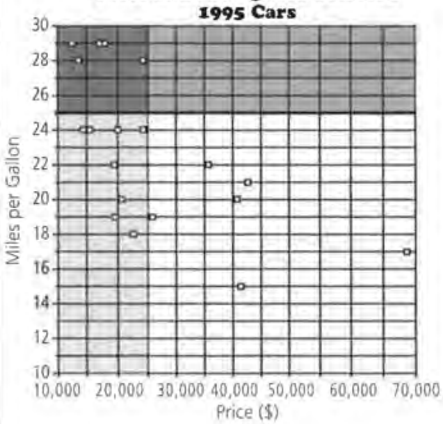
Price and Mileage for Selected 1995 Cars



b. Let c = price of a car; $c \le 25{,}000$

c.

Price and Mileage for Selected 1995 Cars



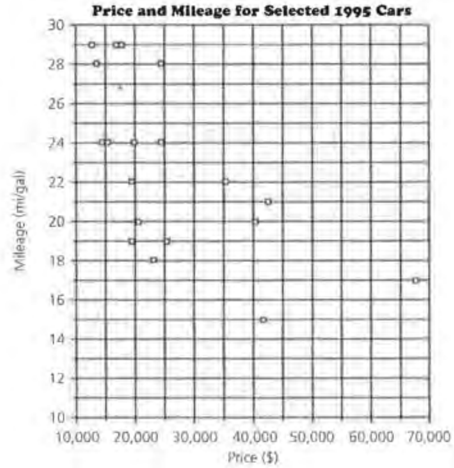
d. Let m = miles per gallon; $m \ge 25$

e. 5 cars

Buying a Car

The scatter plot below shows the prices and the miles per gallon for a sample of 1995 cars. A car buyer decides that the car she will purchase must cost no more than $25,000 and get at least 25 miles per gallon.

Price and Mileage for Selected 1995 Cars



11. Use the second grid on *Activity Sheet 8* for this problem and the next.

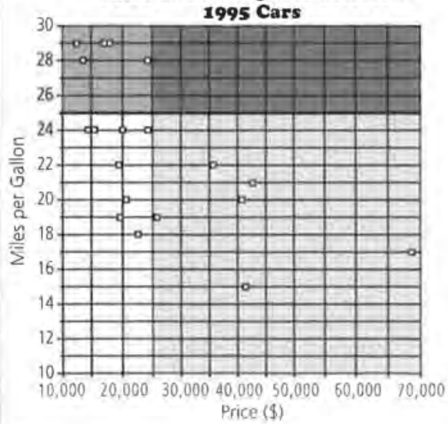
- **a.** Shade the region of the graph that represents all the cars that cost no more than $25,000.
- **b.** Write an inequality to represent this region.
- **c.** Shade the region of the graph that represents all the cars that get at least 25 miles per gallon.
- **d.** Write an inequality to represent this region.
- **e.** How many cars satisfy both conditions?

12. Write a pair of inequalities and sketch a region that represents each set of conditions.

- **a.** Costs at least $25,000 and gets at least 25 miles per gallon
- **b.** Costs no more than $25,000 and gets no more than 25 miles per gallon

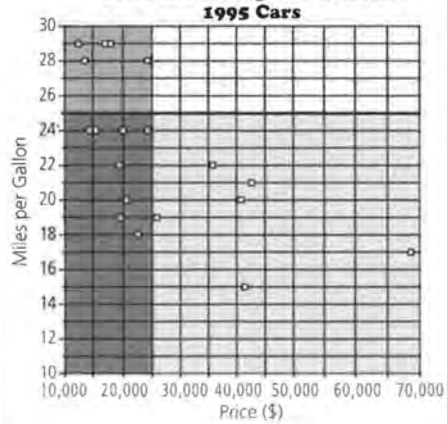
12. a. $c \ge 25{,}000$ and $m \ge 25$

Price and Mileage for Selected 1995 Cars



b. $c \le 25{,}000$ and $m \le 25$

Price and Mileage for Selected 1995 Cars



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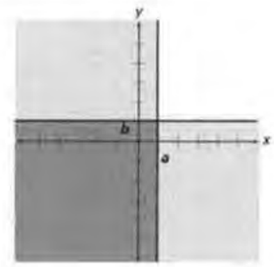
Summary

The graph of the the conditions $x \le a$ and $y \le b$ is the doubly-shaded region shown at the right.



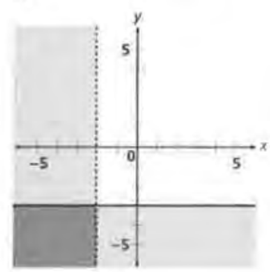
Practice and Applications

13. Graph each pair of inequalities.

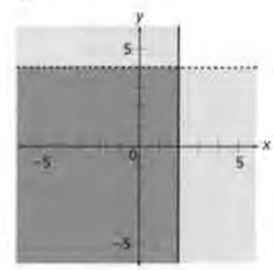
a. $x < 5$, $y > 2$ **b.** $x > 1$, $y < 3$ **c.** $x \ge 0$, $y \ge 0$

14. Write a pair of inequalities to represent the doubly-shaded region of each graph.

a.



b.

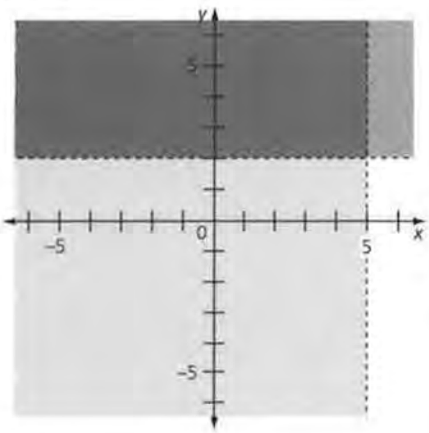


c.

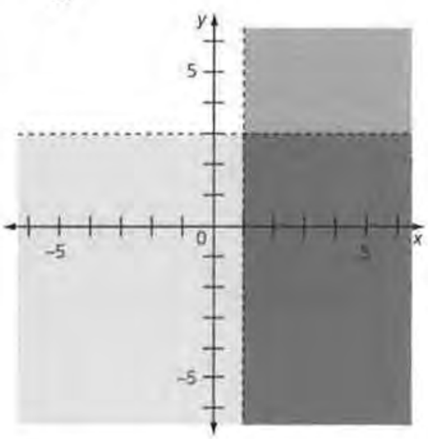
d.

Practice and Applications

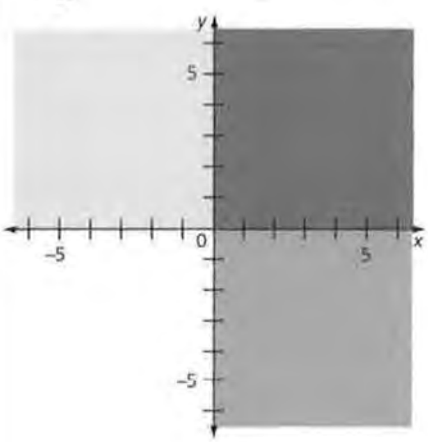
13. a.



b.



c.



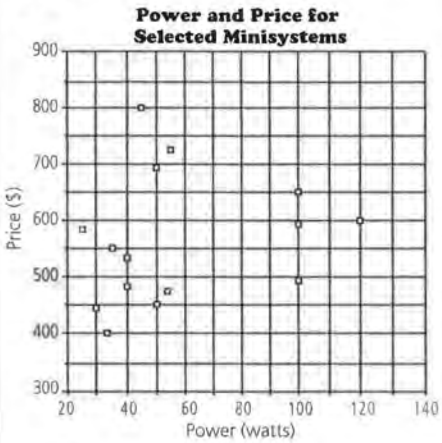
14. a. $x < -2$ and $y \le -3$

b. $x \le 2$ and $y < 4$

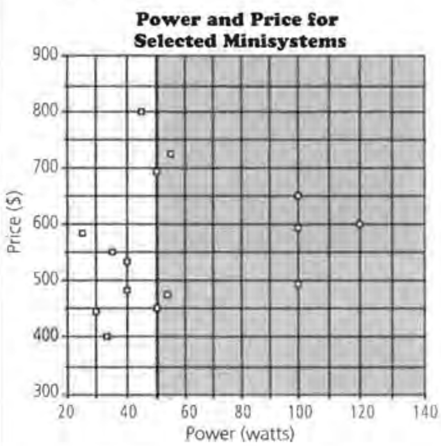
c. $x \le 4$ and $y \le 0$

d. $x \le 0$ and $y \le 0$

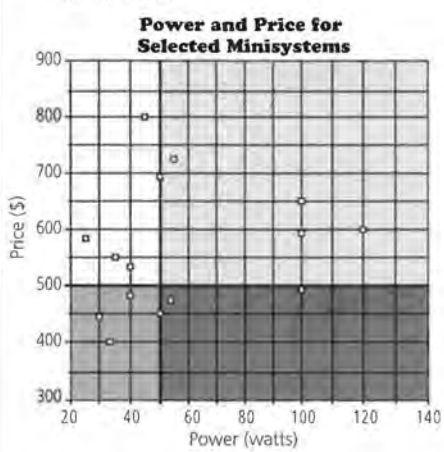
15. a.



b. $W \geq 50$



c. $C \leq 500$



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15. The table below contains the watts per channel and suggested retail price in 1996 for selected stereo minisystems. A consumer wants a stereo minisystem with at least 50 watts per channel and wishes to pay no more than $500.

Model	Power (watts)	Price ($)
Yamaha GX-5	35	550
JVC MX-C550	40	530
AIWA NSC-V51G	40	480
AIWA NSX-999	120	600
Onkyo PCS-707	25	580
JVC MX-C330	30	440
Denon D-500	45	800
Sony MHC-C605SS	100	590
AIWA NSX-V90	50	690
Pioneer CCS-404	100	650
Sony MHC-405SS	100	490
Fisher DCS-M37	50	450
Pioneer CCS-204	33	400
Kenwood UD-303	53	470
Kenwood UD-403	55	730

Source: *Consumer Reports*, Febuary 1996

a. On the grid on *Activity Sheet 9*, make a scatter plot of the ordered pairs (power, price).

b. Shade the region of the graph that represents all minisystems that have 50 or more watts per channel. Write an inequality to represent this region.

c. On the scatter plot, shade the region of the graph that represents all mininsystems that cost $500 or less. Write an inequality to represent this region.

d. Which minisystems satisfy both conditions?

e. Write a pair of inequalities to represent the region where cost is more than $500 and watts per channel is greater than 50.

d. Sony MHC-405SS, Fisher DCS-M37, Kenwood UD-303

e. Let W = watts and C = cost; $C \geq 500$ and $W \geq 50$

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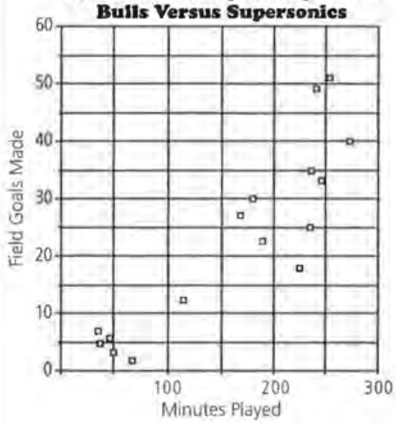
16. a. New Zealand, Denmark, Sweden, Finland, Norway, Switzerland, Netherlands

b. J = judicial system rating and A = anticorruption rating; $J \geq 8$ and $A \geq 8$

c. $J \leq 4$ and $A \geq 6$; Singapore

17.

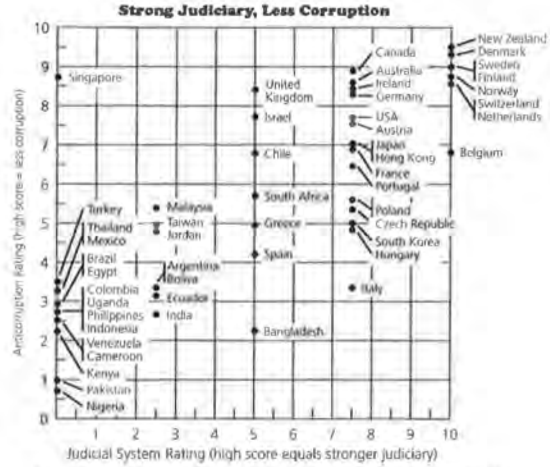
1996 NBA Championship Series Bulls Versus Supersonics



Conditions will vary, but minutes should be at least 150 minutes and field goals made should be at least 25.

16. The scatter plot below shows the relationship between a country's judicial-system rating and anticorruption rating.

Strong Judiciary, Less Corruption



Source: *Economic Freedom of the World 1975–95; Transparency International 1996*

a. Identify the countries that have a rating of 8 or higher on both scales.

b. Write a pair of inequalities to represent the region of the graph where countries have a rating of 8 or higher on each scale.

c. Write a pair of inequalities to represent the region of the graph where countries have a judicial-system rating of 4 or lower and an anticorruption rating of 6 or higher. Identify the countries in this region.

Extension

17. Use the categories of total minutes played and field goals made from the 1996 NBA Championship series data presented in this section to decide which players are the most valuable. Your work should include a scatter plot and the conditions, written as inequalities, used to make your decisions.

LESSON 6

Systems of Inequalities

Materials: graph paper, rulers, *Activity Sheets 10* and *11*, *Lesson 6 Quiz*
Technology: graphing calculators (optional)
Pacing: 2 class periods

Overview

This lesson begins by presenting data on the number of calories and calories from fat for selected items from McDonald's. Students are asked to graph a line through the origin that best represents the data. The students are then asked to compare actual data values to values on the line. From this comparison, the inequalities in the form $y \leq mx$ are developed.

Teaching Notes

The beginning part of the lesson is very similar to Lesson 4. Students draw a line and then compare ordered pairs above and below the line. This lesson extends Lesson 4 by having students investigate the relationship between two inequalities both in the form $y \leq mx$. The main focus is on the understanding of the shaded regions and the solution set to two given inequalities. This is why all the given equations pass through the origin.

Technology

If your students are using graphing calculators, they should not use the **MED-MED** line or **LINReg** options on the calculator. When using these options, the calculator will find a line that has a y-intercept other than zero. Except for Problem 18, the intent of the lesson is to use lines whose y-intercepts are zero.

Follow-Up

Have students list all the items that they eat in one day. Have them use a nutrition guide to determine the calories and calories from fat for all the items. Have them make a scatter plot of their data and draw a line through the origin that fits their data. Have them interpret the slope of their line. They can then add the line $y = 0.30x$ and compare their line to this line.

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LESSON 6

Systems of Inequalities

How often do you eat breakfast, lunch, or dinner at a fast-food restaurant?

How healthy do you think the food is at these restaurants?

OBJECTIVE

Graph and interpret systems of inequalities in the form $y < mx$.

Many people think that the food at fast-food restaurants is high in calories and saturated fats. In this lesson, you will investigate how to use inequalities to evaluate how the number of calories from fast food compares to suggested dietary recommendations.

INVESTIGATE

Calories from Fat

With the increase in spending at fast-food restaurants comes a greater awareness of dietary concerns. In 1989, the National Research Council published Recommended Daily Allowances stating that no more than 30% of a person's daily caloric intake should come from fats. The table on page 55 shows the calories and calories from fat for selected items from McDonald's in a recent year.

Solution Key

Discussion and Practice

1. As calories increase, calories from fat also increase.

2. **a.** 0 calories and 0 calories from fat

b. Possible answer:

Calories and Calories from Fat in Fast Foods

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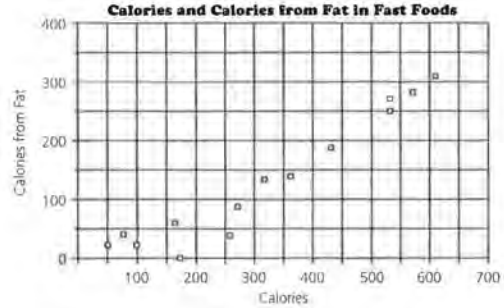
Item	Calories	Calories from Fat
Hamburger	270	90
Cheeseburger	320	130
Quarter Pounder	430	190
Quarter Pounder with Cheese	530	270
Arch Deluxe	570	280
Arch Deluxe with Bacon	610	310
Big Mac	530	250
McGrilled Chicken Classic	260	35
Garden Salad	80	35
Fajita Chicken Salad	160	60
Lite Vinaigrette Dressing	50	20
Apple Bran Muffin	170	0
Apple Danish	360	140
1% Lowfat Milk	100	20

Source: *McDonald's Nutrition Facts*

Below is a scatter plot of the ordered pairs (calories, calories from fat).

Calories and Calories from Fat in Fast Foods



Discussion and Practice

1. What patterns do you observe in the scatter plot above?

2. Refer to the scatter plot above.

a. What does the point (0, 0) mean in terms of the data?

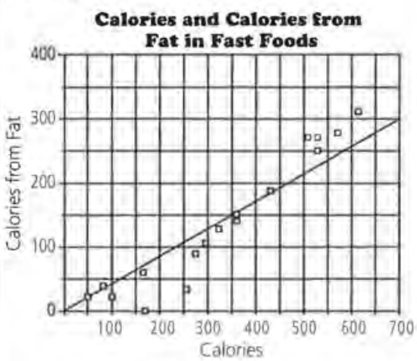
b. On the first graph on *Activity Sheet 10,* draw a line that passes through the point (0, 0) that you feel summarizes the data.

3. **a.** Possible answer: $y = 0.43x$
b. Based on part a, for every 100 calories, 43 calories come from fat.

4. **a.** Based on Problem 2 graph, 95 calories from fat
b. Prediction is less than actual amount.

5. Based on Problem 3, predicted calories from fat: 155, 220, 125

6. **a.**



b. Filet o' Fish below the line, McChicken Sandwich above the line, and Egg McMuffin below the line

c. The actual number of calories from fat is greater than the predicted number of calories from fat.

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3. Refer to Problem 2b.
   a. Write an equation for the line you have drawn.
   b. Explain the meaning of the slope of this line in terms of the data.

4. The number of calories in a small bag of French fries is 220.
   a. Use your line and predict the number of calories from fat for a small bag of French fries.
   b. If the actual number of calories from fat is 110, how accurate was your prediction?

5. The following items were not shown on the scatter plot above. Use the equation for the line you drew and predict the number of calories from fat for each of the given number of calories.

Item	Calories	Predicted Calories from Fat
Filet o' Fish	360	______
McChicken Sandwich	510	______
Egg McMuffin	290	______

6. The actual numbers of calories and calories from fat are listed in the table below.

Item	Calories	Actual Calories from Fat
Filet o' Fish	360	150
McChicken Sandwich	510	270
Egg McMuffin	290	110

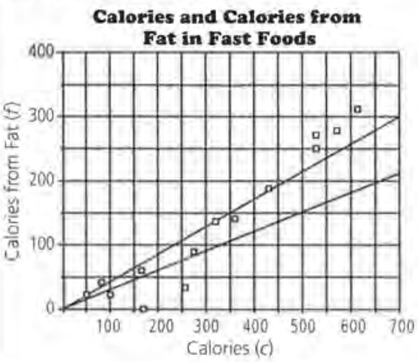
   a. On the graph for Problem 2, add the ordered pairs (calories, calories from fat) for the three items listed in the table above.
   b. Are the ordered pairs above, below, or on the line that you have drawn?
   c. Explain what it means if a point is above the line.

7. **a.** Mc Grilled Chicken Classic, Apple Bran Muffin, 1% Lowfat Milk

b. Calories from fat are less than 30% of calories.

c. Yes

8. Possible answer:



9. **a.** Hamburger, Cheeseburger, Fajita Chicken Salad, Apple Danish

b. They are above the recommended daily amount but below the general McDonald's average.

c. Let c = calories from fat and x = calories; $c < 0.43x$ and $c > 0.3x$

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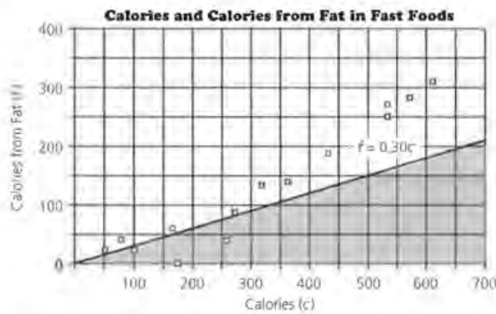
The National Research Council suggests that we should get no more than 30% of our calories from fat. To express this recommendation as an inequality,

let c = number of calories and
let f = number of calories from fat.
Then, $f \le 0.30c$.

The graph of the inequality is shown below.



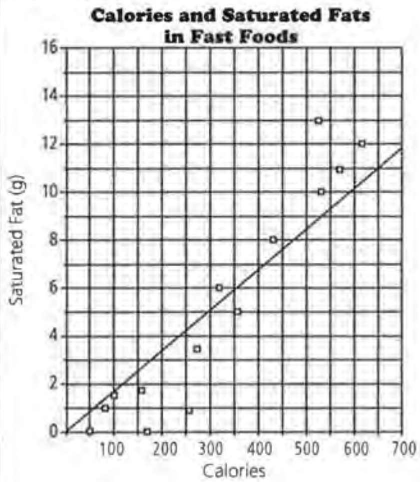
7. Refer to the graph above.
   a. Identify the items that are in the shaded region.
   b. What do the items in this region represent?
   c. Could you make a meal out of these items?

8. Draw the line you drew in Problem 2 on the plot of the inequality given on the second graph on *Activity Sheet 10*.

9. Refer to the line in Problem 8.
   a. Identify the items that lie in the unshaded region but below the original line that you drew.
   b. What do the items in this region represent?
   c. Write a pair of inequalities to describe this region.

10. As the number of calories increases, the number of grams of saturated fat increases.

11. Possible answer:



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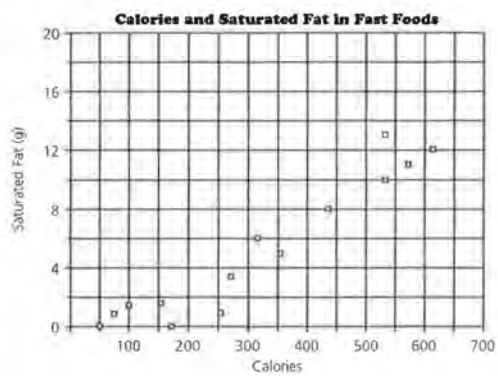
Saturated Fats

The following table contains the number of calories and the amount of saturated fat in selected items from McDonald's.

Item	Calories	Saturated Fat (g)
Hamburger	270	3.5
Cheeseburger	320	6
Quarter Pounder	430	8
Quarter Pounder with Cheese	530	13
Arch Deluxe	570	11
Arch Deluxe with Bacon	610	12
Big Mac	530	10
McGrilled Chicken Classic	260	1
Garden Salad	80	1
Fajita Chicken Salad	160	1.5
Lite Vinaigrette Dressing	50	0
Apple Bran Muffin	170	0
Apple Danish	360	5
1% Lowfat Milk	100	1.5

Source: *McDonald's Nutrition Facts*

Below is a scatter plot of the ordered pairs (calories, saturated fat) of the items in this table.



10. Describe any trends that you observe.

11. On the first graph on *Activity Sheet 11*, draw a line that passes through the origin and summarizes the data.

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12. Consider the line you drew in Problem 11.

a. Write an equation for the line.

b. Describe the region below the line.

c. Write an inequality to represent this region.

13. The Research Council suggests that no more than 10% of our calories should come from saturated fats.

a. Write an inequality to describe this relationship.

b. On the graph for Problem 11, graph the inequality from part a.

c. Identify the items that are in the shaded region.

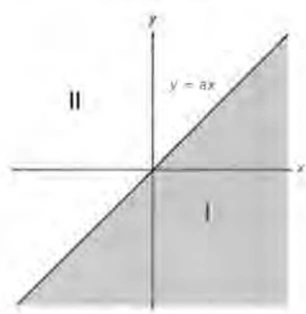
14. The plot from Problem 13b is divided into three regions.

a. Describe each region.

b. Write a set of inequalities for each region.

Summary

An inequality in the form $y \le ax$ separates the coordinate system into two regions. In the graph below, region I represents the ordered pairs that satisfy the inequality and region II represents the ordered pairs that do not satisfy the inequality.



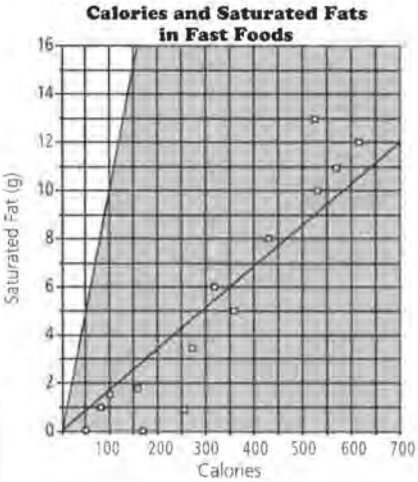
12. a. Possible answer: $y = 0.017x$

b. Based on part a, grams of saturated fat are less than 1.7% of calories.

c. Let c = calories and s = grams of saturated fat; $s < 0.017c$

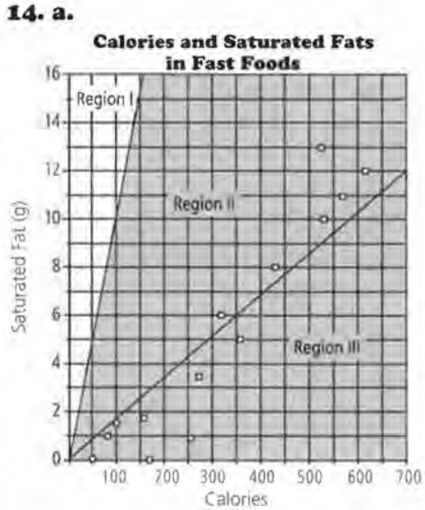
13. a. $s \le 0.10c$

b.



c. All the items

14. a.



Region I contains ordered pairs such that the grams of saturated fat comprise more than 10% of calories. Region II contains ordered pairs such that the grams of saturated fat comprise less than 10% of calories and more than 1.7% of calories. Region III contains ordered pairs such that the grams of saturated fat comprise less than 1.7% of calories.

b. Region I: $s > 0.1c$; region II: $s < 0.1c$ and $s > 0.017c$; region III: $s < 0.017c$

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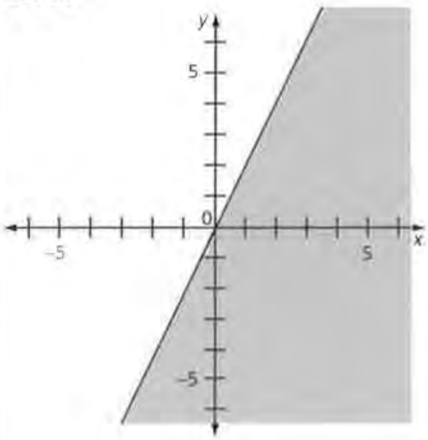
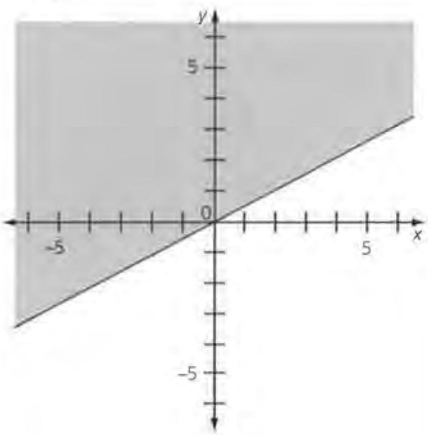
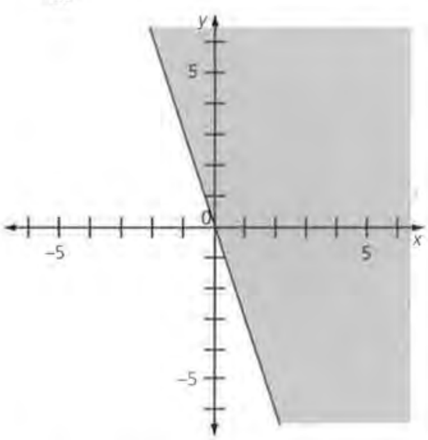
15. a.



b.



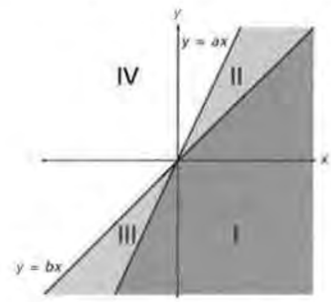
c.



16. a. $y \le -\frac{1}{3}x$

b. $y \ge 4x$

Given a system of two inequalities in the form $y \le ax$ and $y \le bx$, the coordinate system can be divided into four regions. In the graph below, region I represents the ordered pairs that satisfy both inequalities. Region II satisfies the inequality $y \le ax$ but not the inequality $y \le bx$. Region III satisfies the inequality $y \le bx$ but not the inequality $y \le ax$. Region IV represents the ordered pairs that satisfy neither inequality.



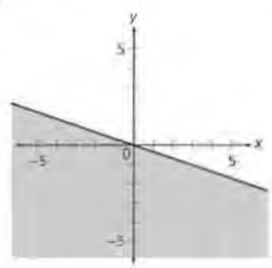
Practice and Applications

15. Graph each inequality.

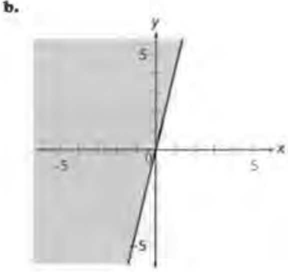
a. $y \le 2x$ **b.** $y \ge \frac{1}{2}x$ **c.** $y \ge -3x$

16. Write an inequality to represent the shaded region in each graph.

a.



b.

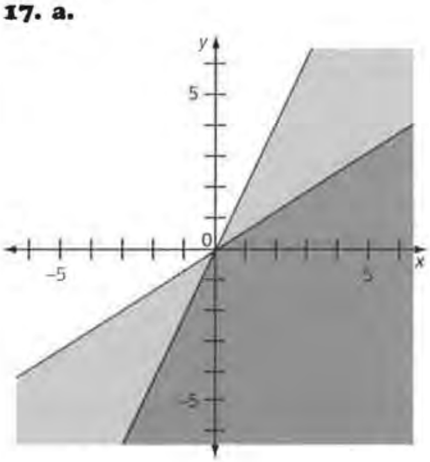


17. Graph each system of inequalities.

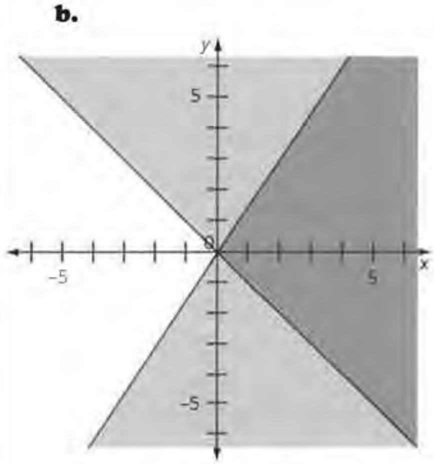
a. $y \le \frac{2}{3}x$
$y \le 2x$

b. $y \ge 1x$
$y \le 1.5x$

17. a.



b.



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18. a. Possible answer:

b. Based on part a graph, $y = 0.29x - 84$; for every increase of $10 in monthly gross income, monthly housing costs will increase by $2.90.

c. The person is paying more of his or her income for housing than is the average person in the sample.

d. $y \leq 0.20x$

e.

f. Ordered pairs where the monthly housing costs are 20% or less of monthly gross income

g. Based on part b, $y < 0.29x - 84$ and $y > 0.20x$

18. Many financial advisors suggest that a single person should spend no more than 20% of his or her gross monthly income on housing costs. These costs include rent or house payment and utility payments. The scatter plot below shows the relationship between gross monthly income and housing costs for a random sample of 15 single people.

a. On the second graph on *Activity Sheet 11*, draw a line that summarizes the data.

b. Find an equation for the line. Describe the line in terms of the data.

c. What does it mean if an ordered pair is above the line that you have drawn?

d. Write an inequality to represent the recommendation that a single person's housing cost should be no more than 20% of his or her gross monthly income.

e. On *Activity Sheet 11*, graph the inequality on the scatter plot.

f. What do the ordered pairs that lie in the shaded region represent?

g. Write a pair of inequalities to describe the ordered pairs not in the shaded region but below the original line that you drew.

LESSON 7

Applying Systems of Inequalities

Materials: graph paper, rulers, *Activity Sheets 12–14, Lesson 7 Quiz*
Technology: graphing calculators (optional)
Pacing: 2 class periods

Overview

In this lesson, students develop inequalities in the form $ax + by \geq c$ and investigate the idea of linear programming. The first investigation involves a student who wishes to burn calories by jogging and weight lifting. Students are given the number of calories burned for each activity and are then asked to determine the total number of calories burned for a given number of minutes for each activity. Students then generalize these relationships and form two inequalities. The two inequalities are graphed on the same coordinate system. The region formed by the overlap of the two inequalities is defined as the ***feasible region***. Students are then asked to find the ***corner points*** and use these points to find the total number of calories burned.

The next investigation involves how much money a conservation club can make from a cookie sale. The constraints given include the number of cookies to be made and the amount of money the club can spend. Inequalities developed are in the form $ax + by \leq c$. The main focus is on the meaning of the feasible region and finding the maximum profit.

The last investigation involves how many hamburgers and apple pies a person can eat and still remain under the daily dietary constraints of at most 2400 mg of sodium and at most 65 grams of fat. The students are again asked to write inequalities, graph the inequalities, describe the feasible region, and find and interpret the corner points.

Teaching Notes

This lesson can be very difficult for students. Students need to be able to graph an inequality in the form $ax + by \geq c$. You may wish to have students find the x- and y-intercepts of the inequality and use the intercepts to graph the inequality. These intercepts become two corner points of the feasible region. Students also need to be able to solve a system of inequalities to find the remaining corner point. Even though corner points are introduced in this lesson, the idea that one of the corner points will be the answer to the question or which point will maximize or minimize the constraint is not formally covered. The intent of the lesson is to give students a beginning understanding of feasible regions and corner points.

Technology

Students can use graphing calculators to help them graph the inequalities and find the corner points. All of the inequalities are in the form $ax + by \geq c$, so students will need to solve for y before entering them into the calculator. Once equations are entered, students can graph the equations and shade the correct regions. Graphing calculators can then be used to find the intersection of the two equations.

Follow-Up

You may wish to have students choose two menu items other than hamburger and apple pie and repeat Problems 22–28 using their choices.

Solution Key

Discussion and Practice

1. $j + w \geq 180$

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LESSON 7

Applying Systems of Inequalities

Do you ever have to change your plans because you don't have enough money or enough time?

OBJECTIVE

Graph and interpret systems of inequalities in the form $ax + by < c$

Suppose you want to buy some concert tickets but you have a limit on how much money you can spend. Or suppose you want to start an exercise program but have a limited amount of time you can exercise. When you have to make a decision, you are often faced with limits or constraints. In this lesson, you will investigate the use of inequalities to find the best solution to a problem when several conditions or constraints must be met. This process is known as *linear programming.*

INVESTIGATE

Linear programming was developed after World War II by mathematicians and economists to help find the best solutions to problems in industry, such as the question of which combination of products will produce the maximum profit.

Discussion and Practice

Burning Calories

Josh, a high-school sophomore, wants to begin an exercise program that combines jogging and weight lifting. Suppose Josh's coach wants him to exercise at least 180 minutes a week.

1. Let j = number of minutes jogging and w = number of minutes weight lifting. Write an inequality that shows the relationship between minutes jogging and minutes weight lifting and the condition of at least 180 minutes of exercise a week.

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2. On *Activity Sheet 12*, graph your inequality from Problem 1 on the first grid.

3. Find an ordered pair in the shaded region. Describe what the ordered pair represents.

Josh knows that for his weight and size jogging burns about 9 calories per minute and weight lifting burns approximately 4.5 calories per minute.

4. How many calories will Josh burn if he jogs
   a. for 90 minutes and lifts weights for 90 minutes each week?
   b. for 240 minutes and lifts weights for 30 minutes each week?
   c. for 60 minutes and lifts weights for 250 minutes each week?

5. If Josh wants to burn at least 1200 calories per week, could he reach this goal by jogging
   a. for 150 minutes and lifting weights for 100 minutes?
   b. for 100 minutes and lifting weights for 90 minutes?
   c. for 50 minutes and lifting weights for 150 minutes?

6. What do you think is the minimum number of minutes that Josh can jog and lift weights and still reach his goal of burning 1200 calories in a week? Remember he must exercise a total of at least 180 minutes a week.

You can use the following equation to find the total number of calories burned:

$9j + 4.5w = c$, where j = number of minutes jogging, w = number of minutes weight lifting, and c = number of calories burned

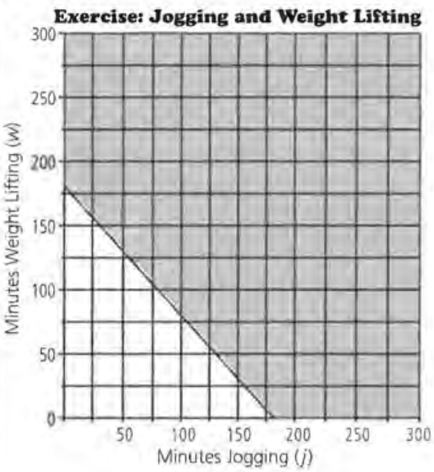
Since Josh wants to burn at least 1200 calories per week, you can use the following inequality:

$9j + 4.5w \geq 1200$

7. On *Activity Sheet 12*, graph this inequality on the second grid.

8. Describe all the ordered pairs in the shaded region.

2.



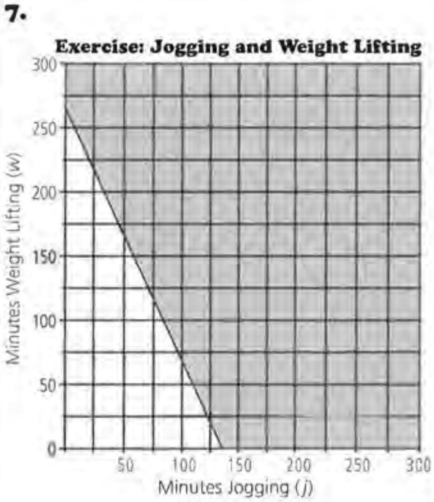
3. Possible answer: (100, 100); the *x*-coordinate is the number of minutes jogging and the *y*-coordinate is the number of minutes weight lifting.

4. **a.** 1215 calories
b. 2295 calories
c. 1665 calories

5. **a.** Yes; 1395 calories
b. Yes; 1305 calories
c. No; 1125 calories

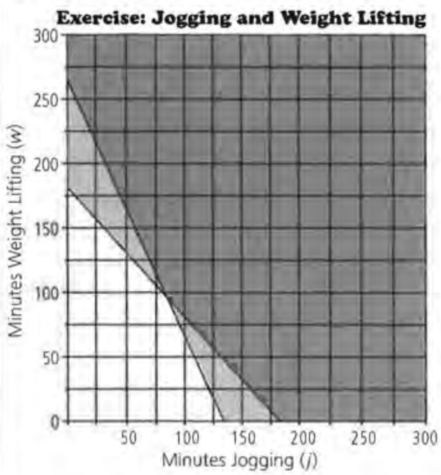
6. Possible answer: (100, 80)

7.



8. The *x*-coordinate is the number of minutes jogging and the *y*-coordinate is the number of minutes weight lifting; these two values satisfy the inequality $9j + 4.5w \geq 1200$.

9.



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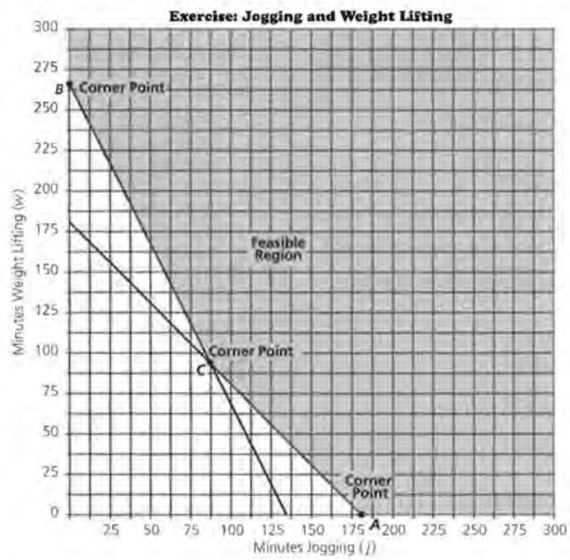
Josh wants to exercise at least 180 minutes and burn at least 1200 calories in a week. In order for him to determine the minimum number of minutes that he must jog and lift weights to reach this goal, he needs to consider the two inequalities together.

9. On *Activity Sheet 13*, graph the two inequalities $j + w \geq 180$ and $9j + 4.5w \geq 1200$ on the first grid.

The region that is represented by the overlap of the two inequalities is called the ***feasible region***. This region represents all the possible combinations of minutes jogging and weight lifting that meet the constraints of at least 180 minutes exercising and at least 1200 calories burned. Consider this graph.



Points A, B, and C are known as ***corner points*** of the feasible region. Point A is the x-intercept of the equation $j + w = 180$. Point B is the y-intercept of the equation $9j + 4.5w = 1200$. Point C is the point whose ordered pair satisfies both equations.

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10. a. The points in the feasible region satisfy both inequalities.

b. *A* (180, 0); *B* (0, 266.7); *C* (86.7, 93.3)

c. See table below.

d. Point *A*

11. Answers will vary.

12. Point *C*

13. $m + c \leq 144$

10. Refer to the graph on page 64.

a. The shaded region is the feasible region. Explain why this is the feasible region.

b. Find the ordered pairs for points *A*, *B*, and *C*. List them in a table like the one below. Show your work.

Point	Ordered Pair	Total Minutes Exercising	Total Calories Burned
A	______	______	______
B	______	______	______
C	______	______	______

c. Find the total number of minutes exercising and the total calories burned for each corner point. List the results in your table.

d. Of the three corner points, which one gives the greatest calorie burn for the fewest minutes exercising?

11. In your table, list three more ordered pairs that are in the feasible region. Find the total number of minutes exercised and the total calories burned for each ordered pair.

12. Suppose Josh wants to exercise for 180 minutes to stay in shape but does not want to lose weight. Of all the points listed in the table, which point gives the least number of calories burned for 180 minutes exercised? Compare answers within your group.

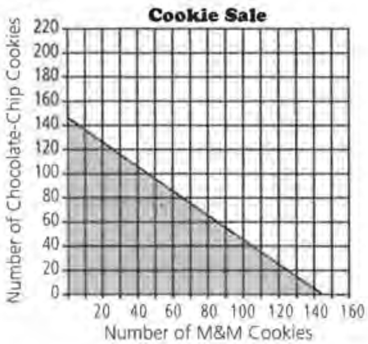
Cookie Sale

The Conservation Club at a high school decides to sell cookies to earn some money to buy trees to plant around school. They decide to make no more than 12 dozen, or 144, cookies. They also decide to make only two types of cookies, one with M&Ms® and the other with chocolate chips.

13. Let m = number of M&M cookies and let c = number of chocolate-chip cookies. Write an inequality that shows the relationship between the number of each type of cookie under the condition that club members are planning to make no more than 144 cookies.

Point	Ordered Pair	Total Minutes Exercising	Total Calories Burned
A	(180, 0)	180	1620
B	(0, 266.7)	266.7	1200
C	(86.6, 93.3)	180	1200

14.



15. Possible answer: (80, 40); 80 is the number of M&M cookies and 40 is the number of chocolate-chip cookies.

16. **a.** \$18

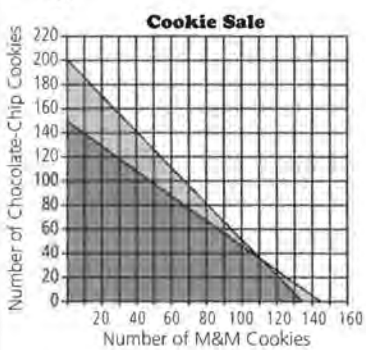
b. \$6.60

c. \$19.40

17. $0.15m + 0.10c = t$

18. **a.** $0.15m + 0.10c \leq 20$

b.



c. Answers will vary. Each ordered pair satisfies the two inequalities: $m + c \leq 144$ and $0.15m + 0.1c \leq 20$.

d. Answers will vary.

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14. On *Activity Sheet 13*, graph your inequality from Problem 13 on the second grid.

15. List an ordered pair in the shaded region of your graph in Problem 14. Describe what this ordered pair represents.

The Conservation Club members know that each M&M cookie will cost about \$0.15 to make and each chocolate-chip cookie will cost about \$0.10 to make.

16. About how much will it cost to make

a. 72 cookies of each kind?

b. 44 M&M cookies and 100 chocolate-chip cookies?

c. 100 M&M cookies and 44 chocolate-chip cookies?

17. Use m to represent the number of M&M cookies made, c the number of chocolate-chip cookies made, and t the total cost to write an equation that can be used to find the total cost of making all the M&M and chocolate-chip cookies.

18. The advisor of the club said that the club members can spend no more than \$20.00 on supplies for the cookies.

a. Express the total cost as an inequality that shows the condition that the total cost cannot exceed \$20.00.

b. Graph the inequality on the same grid you used for Problem 14.

c. In a table like the one below, list three ordered pairs in the feasible region. Describe what each point represents.

Ordered Pair	Number of M&M Cookies	Number of Chocolate-Chip Cookies	Total Cost
___	___	___	___
___	___	___	___
___	___	___	___
___	___	___	___
___	___	___	___
___	___	___	___

d. Find the total cost for each ordered pair and list it in your table.

e. Find the corner points of the feasible region. Add these points to your table and find the total cost.

e.

Ordered Pair	Number of M&M Cookies	Number of Chocolate-Chip Cookies	Total Cost
(0, 144)	0	144	\$14.40
($133.\overline{3}$, 0)	133	0	\$20.00
(112, 32)	112	32	\$20.00

19. a. $p = 0.45m + 0.40c$

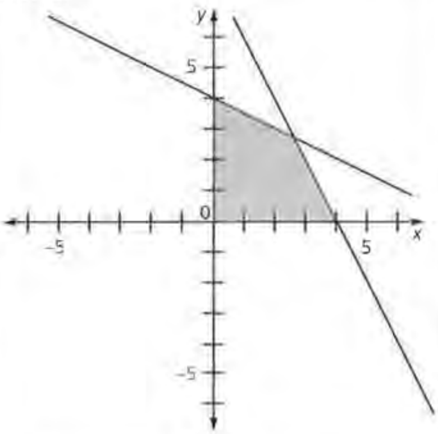
b. Answers for first three points will vary. Profit for (0, 144) = \$57.60; profit for ($133.\overline{3}$, 0) = \$60.00; profit for (112, 32) = \$63.20

c. (112, 32)

d. (112, 32)

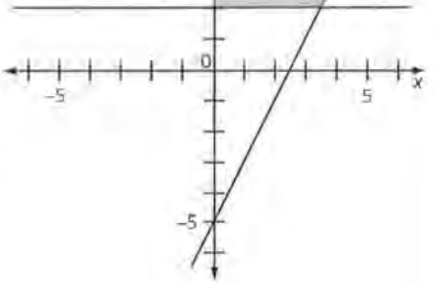
Practice and Applications

20. a.



b. See second column.

c.



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19. The Club decides to sell M&M cookies for \$0.60 each and chocolate-chip cookies for \$0.50 each. This means a profit of \$0.45 for each M&M cookie and \$0.40 for each chocolate-chip cookie.

a. Use m to represent the number of M&M cookies made, c the number of chocolate-chip cookies made, and p the total profit to write an equation that could be used to find the total profit.

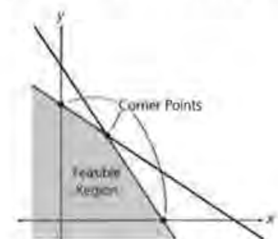
b. Find the profit for each ordered pair listed in your table in Problem 18.

c. Of the points listed in the table, which point gives the maximum profit?

d. Compare your answer with those of other members in your group. Which point in the feasible region gave the maximum profit?

Summary

Linear programming involves writing a system of inequalities based on various constraints. The graph of the system of inequalities determines the *feasible region.* This region represents all of the ordered pairs that satisfy the system of inequalities. The *corner points* are those points at which various quantities are maximized or minimized.

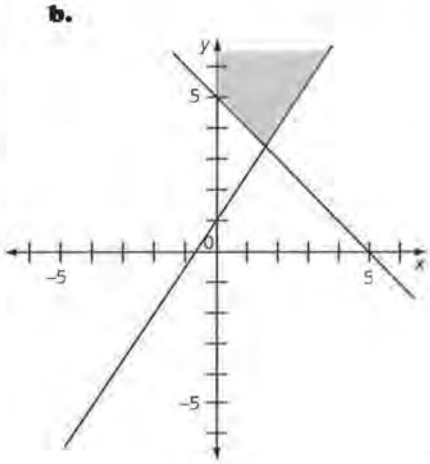


Practice and Applications

20. Graph the feasible region for each system of inequalities.

a.	**b.**	**c.**
$y \le -2x + 8$	$y \ge 5 - x$	$y \ge 2$
$y \le -0.5x + 4$	$y \ge 1.5x + 1$	$y \le 8$
$y \ge 0$	$y \ge 0$	$y \ge 2x - 5$
$x \ge 0$	$x \ge 0$	$x \ge 0$

b.



21. **a.** $p = 0.45m + 0.50c$

b. (0, 200), $100; (133.33, 0), $60; (112, 32), $66.40

c. (0, 200)

22. **a.** 1260 mg

b. 33 g

c. 1990 mg

d. 56 g

23. $s = 530h + 200a$

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21. Suppose the Conservation Club decides to sell each cookie for $0.60.

a. Write an equation for the total profit that the club could make from the sale of M&M and chocolate-chip cookies.

b. Find the profit for each ordered pair that you listed in Problem 18.

c. Of the points listed, which one gives the maximum profit?

The National Research Council suggests that we have at most 2400 mg of sodium and at most 65 g of fat each day. If you were going to eat at a fast-food restaurant, how many hamburgers and apple pies could you eat and stay within these dietary conditions?

22. A hamburger from McDonald's contains 530 mg of sodium and 10 g of fat. Apple pie contains 200 mg of sodium and 13 g of fat.

a. How many milligrams of sodium are contained in 2 hamburgers and 1 apple pie?

b. How many grams of fat are contained in 2 hamburgers and 1 apple pie?

c. How many milligrams of sodium are contained in 3 hamburgers and 2 apple pies?

d. How many grams of fat are contained in 3 hamburgers and 2 apple pies?

Could you eat 6 hamburgers and no apple pies and stay within the dietary conditions concerning sodium and total fat? What do you think is the maximum number of hamburgers and apple pies you could order and eat and still stay under the dietary conditions?

Let h = the number of hamburgers eaten,
let a = the number of apple pies eaten,
let s = amount of sodium in mg, and
let f = amount of fat in g.

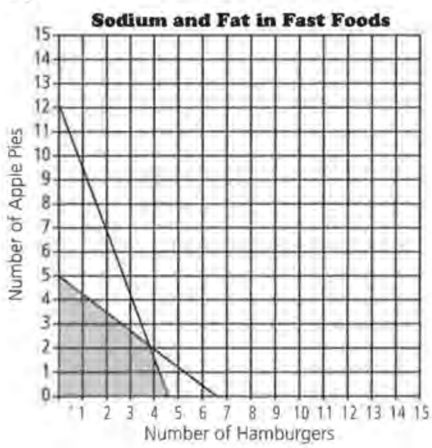
23. Write an equation that can be used to find the total amount of sodium for a given number of hamburgers and apple pies.

24. $f = 10h + 13a$

25. $530h + 200a \leq 2400$

26. $10h + 13a \leq 65$

27.



28. **a.** All the ordered pairs that satisfy the inequalities $530h + 200a \leq 2400$ and $10h + 13a \leq 65$

b. (0, 5), (3.7, 2.1), (4.5, 0)

c.

Corner Point	Sodium	Fat
(0, 5)	1000	65
(3.7, 2.1)	2400	65
(4.5, 0)	2400	45

d. Between 3 and 4 hamburgers and about 2 apple pies

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24. Write an equation that can be used to find the total amount of fat for a given number of hamburgers and apple pies.

25. The dietary conditions state that the amount of sodium should be less than 2400 mg per day. Write an inequality that shows the relationship between the amount of sodium from a given number of hamburgers and apple pies and the limit of 2400 mg of sodium per day.

26. The dietary conditions state that the amount of fat should be less than 65 grams each day. Write an inequality that shows the amount of fat less than or equal to 65 grams.

27. On the grid on *Activity Sheet 14*, graph your inequalities from Problems 25 and 26. Clearly show the feasible region.

28. Refer to your graph from Problem 27.

a. Describe the ordered pairs in the feasible region.

b. Find the corner points of the feasible region.

c. For each corner point, find the milligrams of sodium and the grams of fat.

d. What is the maximum number of hamburgers and apple pies that you could eat and stay under the dietary constraints of sodium and fat?

Assessment for Unit III

Materials: rulers, *Activity Sheet 15, End-of-Module Test*
Technology: graphing calculators (optional)
Pacing: 1 class period or homework

Overview

Problem 1 assesses students' understanding of the inequalities in the form $x \leq a$ and $y \leq b$ as presented in Lesson 5. Problem 2 assesses the objectives of Lessons 6 and 7. Students are asked to graph a pair of inequalities and find the corner points and feasible region.

Teaching Notes

This assessment can be used in a number of ways. It can be used strictly as an assessment that is completed by students in one class period. It could also be used as a take-home test or as additional practice on the objectives of Lessons 5–7.

Technology

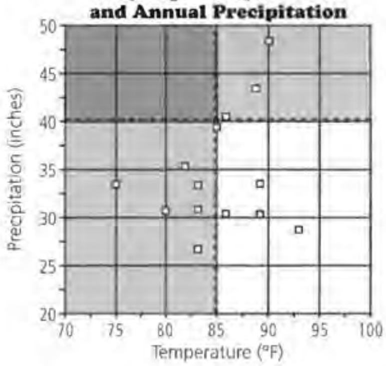
The use of graphing calculators is optional. Students may need a calculator to help them in their calculations of milligrams of riboflavin and vitamin B6.

STUDENT PAGE 70

Solution Key

1. a.

July High Temperatures and Annual Precipitation



b. See graph in Problem 1a; $t < 85$

c. See graph in Problem 1a; $p > 40$

d. No city

e. $t > 85$ and $p > 40$

ASSESSMENT

Assessment for Unit III

OBJECTIVE

Apply knowledge of systems of inequalities.

1. The following table contains the normal July high temperature (°F) and the normal annual precipitation (inches) for selected cities in the central United States.

City	High Temperature (°F)	Precipitation (inches)
Milwaukee, WI	80	30.9
Nashville, TN	90	48.5
Cleveland, OH	82	35.4
Cincinnati, OH	86	40.1
Omaha, NE	89	30.3
St. Louis, MO	89	33.9
Minneapolis, MN	83	26.4
Sault Ste. Marie, MI	75	33.5
Detroit, MI	83	31.0
Louisville, KY	88	43.6
Wichita, KS	93	28.6
Des Moines, IA	86	30.8
Indianapolis, IN	85	39.1
Chicago, IL	83	33.3

Source: *The World Almanac and Book of Facts*, 1994

a. On *Activity Sheet 15*, use the first grid to make a scatter plot of the ordered pairs (temperature, precipitation).

b. Shade the region of the graph that represents the cities that have a normal high temperature in July of less than 85°F. Write an inequality to represent this region.

c. Shade the region of the graph that represents the cities that have an annual precipitation of more than 40 inches. Write an inequality to represent this region.

d. Identify the cities that satisfy both conditions.

e. Write a pair of inequalities that represent the region where July temperature is higher than 85°F and annual precipitation is more than 40 inches.

STUDENT PAGE 71

2. **a.** 1.385

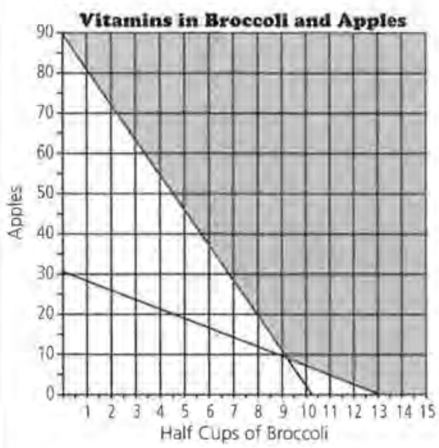
b. 1.43

c. $0.154B + 0.066A$ = number of mg of B6

d. $0.166B + 0.019A$ = number of mg of riboflavin

e. $0.154B + 0.066A \geq 2$; $0.166B + 0.019A \geq 1.7$

f.



g. The ordered pairs in the feasible region satisfy both inequalities.

h. (0, 89.5), (9.24, 8.74), (13, 0)

i.

Corner Point	B6 (mg)	Riboflavin (mg)
(0, 89.5)	5.907	1.7
(9.24, 8.74)	2	1.7
(13.0, 0)	2	2.158

j. 13 half cups of broccoli and no apples

2. Vitamins are an important part of everyone's diet. Vitamins B6 and riboflavin work together with the other B vitamins to help cells absorb and burn energy. A diet deficient in B vitamins often results in muscle weakness and in psychiatric problems. The U.S. Recommended Daily Allowance of vitamin B6 is 2 mg and of riboflavin is 1.7 mg. Listed below are the amounts of B6 and riboflavin in a half cup of broccoli and in 1 apple.

Food	B6 (mg)	Riboflavin (mg)
Broccoli (half cup)	0.154	0.166
Apple (1)	0.066	0.019

a. How many milligrams of riboflavin are contained in 4 cups of broccoli and 3 apples?

b. How many milligrams of vitamin B6 are contained in 4 cups of broccoli and 3 apples?

c. Write an expression that can be used to find the total number of milligrams of B6 for a given number of apples and number of half cups of broccoli.

d. Write an expression that can be used to find the total number of milligrams of riboflavin for a given number of apples and number of half cups of broccoli.

e. Use your expressions from parts c and d to write two inequalities that show that the recommended daily allowance of B6 is at least 2 mg and the recommended daily allowance of riboflavin is at least 1.7 mg.

f. On the second grid on *Activity Sheet 15*, graph your inequalities from part e and clearly show the feasible region.

g. Describe the ordered pairs in the feasible region.

h. Find the corner points of the feasible region.

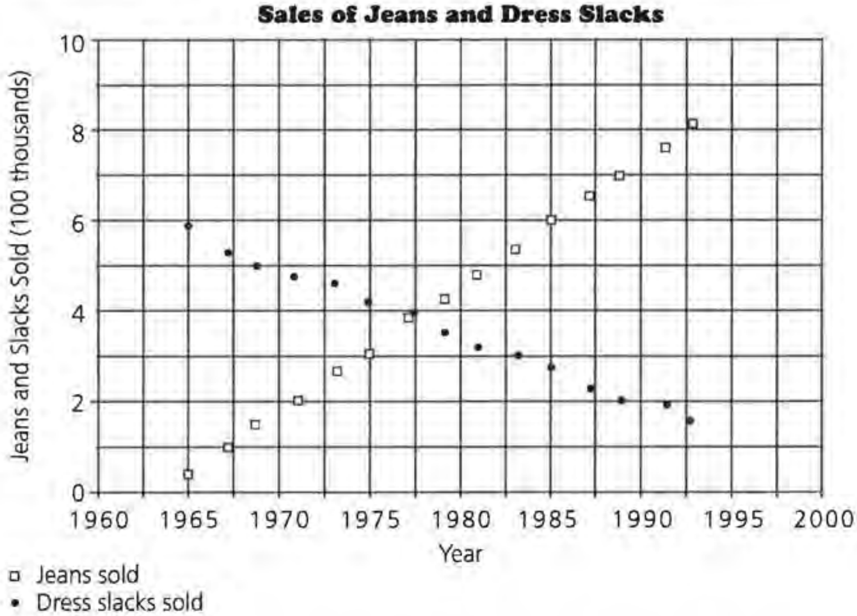
i. For each corner point, find the number of milligrams of vitamin B6 and riboflavin.

j. What is the least number of apples and half cups of broccoli a person could eat to reach the recommended daily allowance for B6 and riboflavin?

Teacher Resources

NAME ______________________________

1. The scatter plot below shows the trends in the number of pairs of jeans and dress slacks sold from 1965 to 1992.



a. Draw a line that bests fits the data for jeans sales and another line that best fits the data for dress-slacks sales.

b. Write an equation for each line.

c. Describe the trends in the lines in terms of the slope.

d. Use algebra to find the point of intersection of the two lines.

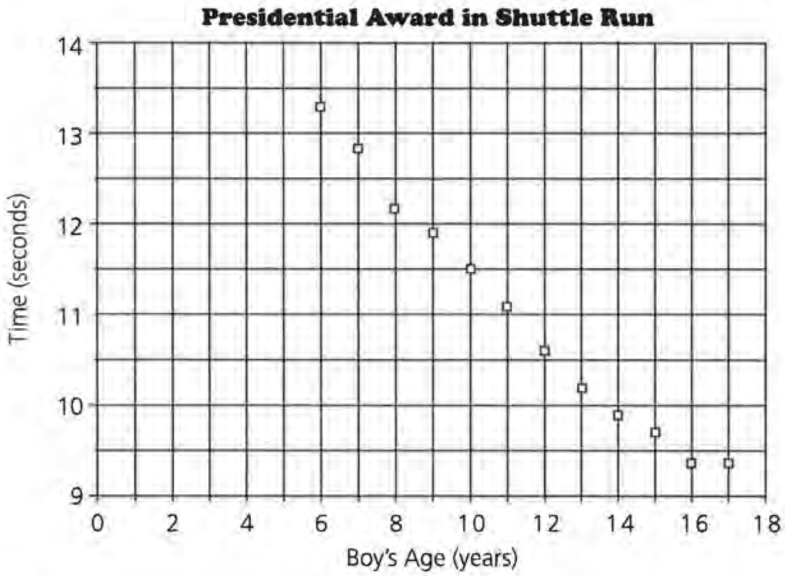
e. What does this point represent?

2. Find the point of intersection for the following system of equations by making a graph and by finding the solution algebraically.

$y = -2x + 6$ and $y = \frac{1}{2}x + 1$

NAME ______________________________

1. Shown below is a graph of boy's age and time in seconds to earn a Presidential Physical-Fitness Award in the shuttle run.



a. Draw a line that best represents the data.

b. Write an equation of the line.

c. Shade the region that represents the times for boys who would not earn an award.

d. Write an inequality that represents the shaded region.

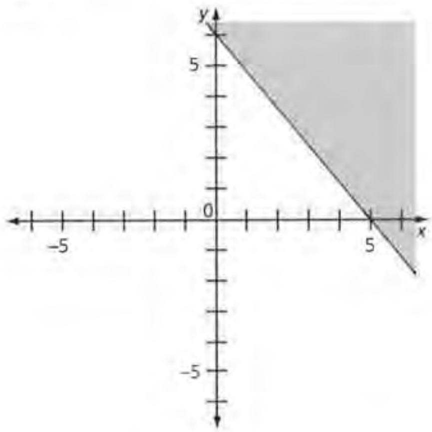
2. Graph each of the following inequalities.

a. $y \leq -2x - 3$

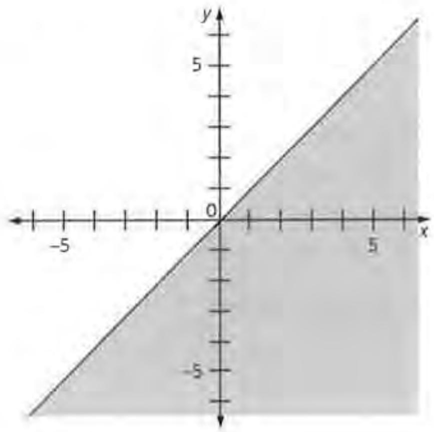
b. $y > 10 + 3(x - 2)$

3. Write an inequality for each graph.

a.

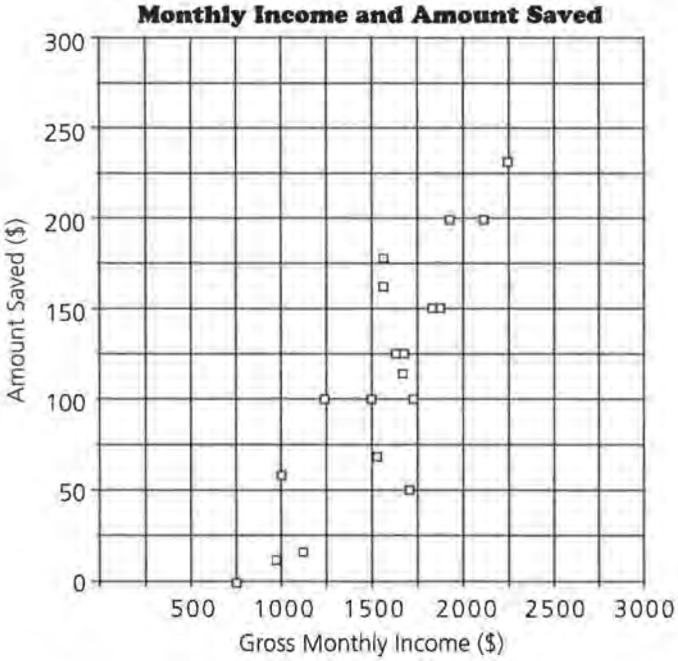


b.



NAME ______________________________

1. Some financial advisors suggest a person should save at least 10% of his or her gross income. The scatter plot below shows the relationship between gross monthly income and the amount saved by a sample of 15 people.



a. Through the origin, draw a line that summarizes the data.

b. Write an equation for the line.

c. Describe the slope in terms of the data.

d. Write an inequality indicating that a person should save at least 10% of his or her gross monthly income.

e. On the graph above, graph the inequality you wrote in part d.

f. What do the ordered pairs in the shaded region represent?

g. Write a pair of inequalities that describe the ordered pairs not in the shaded region but above the original line that you drew in part a.

2. Graph the following systems of inequalities.

a. $y \geq \frac{1}{2}x$ and $y \leq 2x$ **b.** $y \geq -2x$ and $y \leq \frac{2}{3}x$

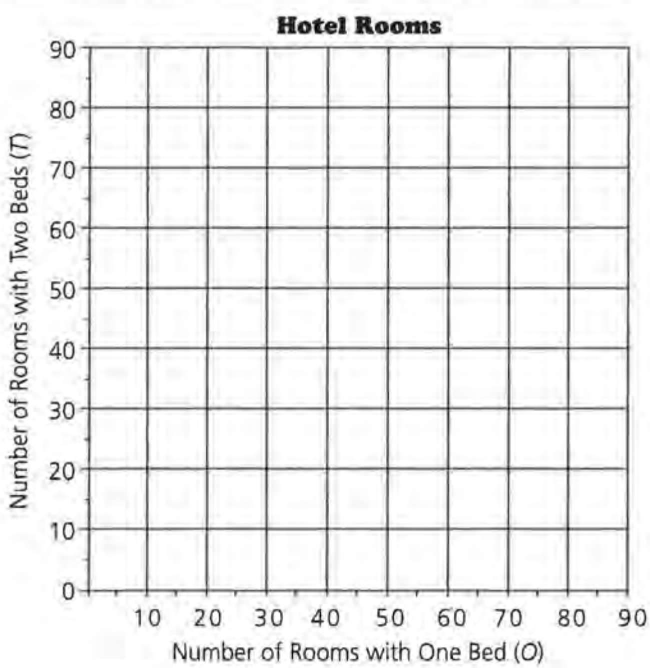
NAME ______________________________

While planning their wedding, Laura and Jim contacted a local hotel to inquire about reserving rooms for their guests from out of town. The hotel has two types of rooms. One type of room has one queen-size bed and the other type has two double beds. They decided that they would need at most 75 rooms for their guests and at least twice as many rooms with two beds as one bed. In order for Laura and Jim to receive a reduced rate for the reception to be held at the hotel, they must guarantee that at least 12 of each type of room will be rented.

1. Let O = number of rooms with one bed, and let T = number of rooms with two beds.

- **a.** Write an inequality that represents the constraint that Laura and Jim will need at most 75 rooms.
- **b.** Write an inequality that represents the constraint that Laura and Jim will need at least twice as many rooms with two beds as one bed.
- **c.** Write two inequalities that represent the constraint that Laura and Jim will need at least 12 of each type of room.

2. One the grid below, determine the feasible region by graphing the four inequalities from Problems 1a, b, and c.



3. Describe the points in the feasible region.

4. Find the corner points of the feasible region. Show how you found the points.

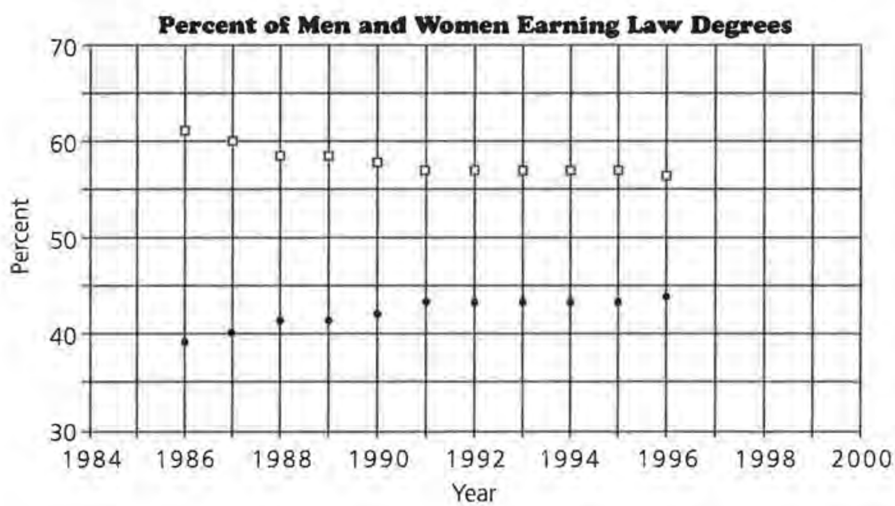
5. If a room with one bed costs $55 per night and a room with two beds costs $65 per night, find the cost at each corner point.

NAME ______________________________

1. The table at the right compares the percent of men and women earning law degrees in the United States from 1986 to 1996.

Year	Males	Females
1986	61%	39%
1987	60%	40%
1988	59%	41%
1989	59%	41%
1990	58%	42%
1991	57%	43%
1992	57%	43%
1993	57%	43%
1994	57%	43%
1995	57%	43%
1996	56%	44%

a. On the graph below, draw a line that you think best represents each set of data.



b. Find the slope of each line. Describe the trend you observe in terms of the slopes.

c. Write an equation for each line.

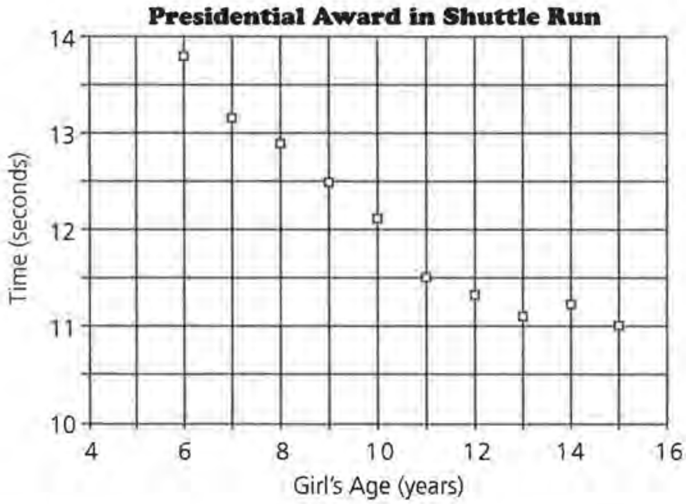
d. Predict the percent of women earning law degrees in the year 2000. Explain how you made your prediction.

e. For what year will the percent of men and the percent of women earning law degrees be the same? Show how you got your answer.

2. Find the intersection point for the following system of equations by making a graph and by finding the solution algebraically.

$y = -x + 8$ and $y = \frac{1}{2}x - 1$

3. Shown below is a graph of girl's age and time in seconds to earn a Presidential Physical-Fitness Award in the shuttle run.



a. Draw a line that best represents the data.

b. Write an equation for the line.

c. Write an inequality that represents the girls' times that are better than the standard.

4. Graph the inequality $y \leq -3x - 2$.

5. Graph the following systems of inequalities.

a. $y \geq -3$ and $x \leq 4$

b. $y \leq 3x$ and $y \geq 1.5x$

c. $x + y \leq 10$, $y \geq x$, and $y \geq 0$

6. Cindy wants to combine swimming and walking as part of an exercise routine. She wants to exercise a total of 4 hours per week (240 minutes). She wants to spend at least 60 minutes on each activity and would like to burn at least 1000 calories a week with this exercise routine. Swimming burns 10 calories per minute and walking burns 5 calories per minute.

Let S = number of minutes swimming and W = number of minutes walking

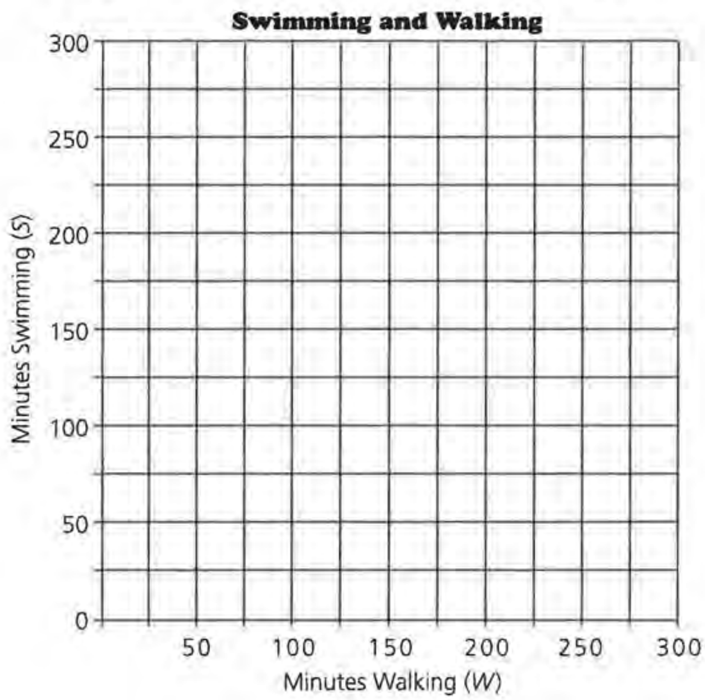
a. Write an inequality for each of the following constraints.

i. Cindy wants to exercise at most 240 minutes.

ii. Cindy wants to spend at least 60 minutes on each activity.

iii. Cindy wants to burn at least 1000 calories per week.

b. On the graph below, determine the feasible region by graphing the inequalities you wrote in part a.



c. Find the corner points. Show how you found the points.

1. a. Possible answer:

Answers to parts b, c, and d are based on part a graph.

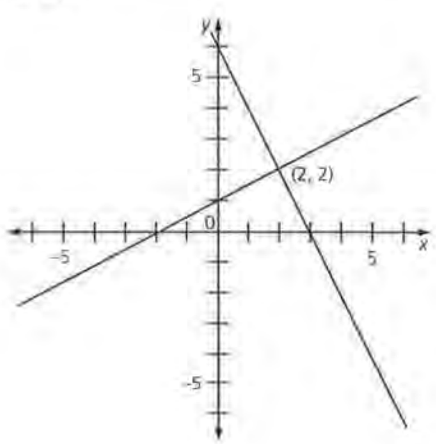
b. Jean-sales equation: $y = 0.28x - 549.8$
Dress-slacks sales equation: $y = -0.15\ x + 300$

c. Interpretation of jeans-sales slope: The number of jeans sold increases $0.28 \times 100{,}000 = 28{,}000$ each year.
Interpretation of dress-slacks slope: The number of dress slacks sold decreases $0.15 \times 100{,}000 = 15{,}000$ each year

d. The point of intersection is approximately (1976, 3.6)

e. This point represents the year when jeans sales and dress-slacks sales were equal.

2. (2, 2)

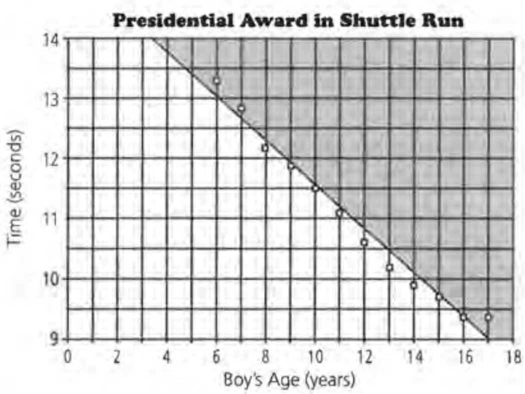


1. **a.** Possible answer:

Answers to parts b, c, and d are based on part a graph.

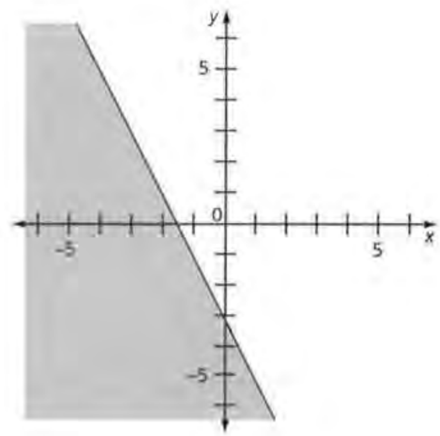
b. $y = -0.37x + 15.2$

c.

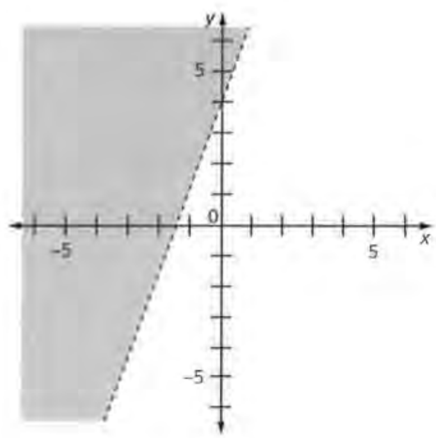


d. $y > -0.37x + 15.2$

2. **a.**



b.



3. **a.** $y \geq -\frac{6}{5}x + 6$

b. $y \leq x$

1. a. Possible answer:

b. There will be many different answers to this question. By having the students draw their line through the origin, the slopes could vary. You should expect equations whose slope is about 0.07, that is, $y = 0.07x$.

c. For each additional \$1, the amont saved is 7¢.

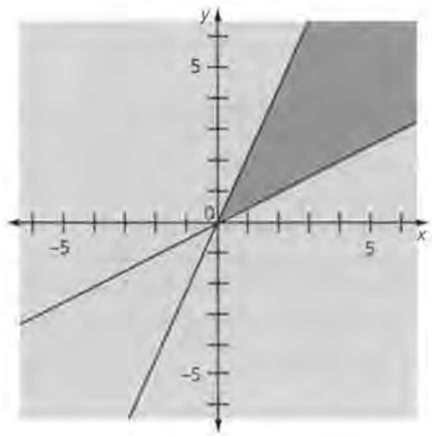
d. Amount saved $\geq 0.10 \times$ gross monthly income

e.

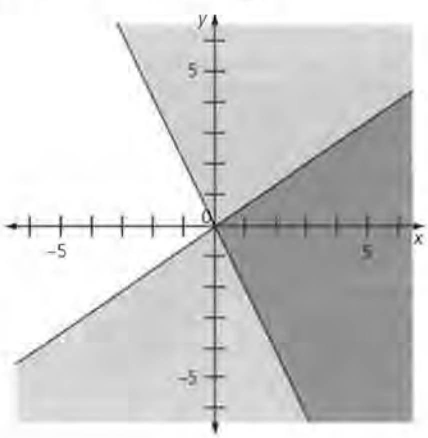
f. The points where the amount saved is greater than 10% of the gross monthly income.

g. $y \geq 0.07x$ and $y \leq 0.10x$

2. a. $y \geq \frac{1}{2}x$ and $y \leq 2x$



b. $y \geq -2x$ and $y \leq \frac{2}{3}x$

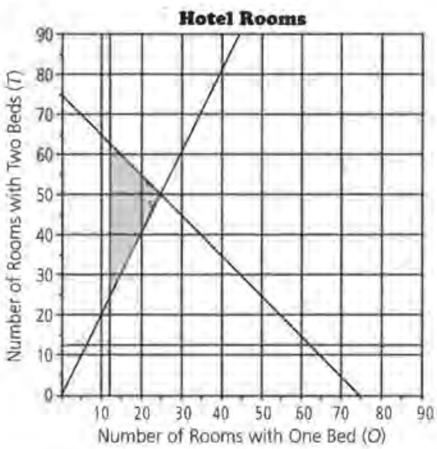


1. **a.** $O + T \leq 75$

b. $T \geq 2O$

c. $T \geq 12$ and $O \geq 12$

2.

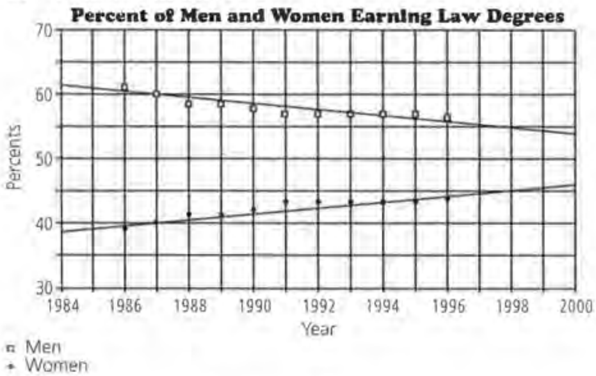


3. The points in the feasible region represent all the ordered pairs that satisfy the four inequalities.

4. (12, 24), (12, 63), and (25, 50); students' explanations will vary.

5. Point (12, 24): cost = 12(55) + 24(65) = $2220
Point (12, 63): cost = 12(55) + 63(65) = $4755
Point (25, 50): cost = 25(55) + 50(65) = $4625

1. **a.** Possible answer:



Answers to parts b, c, and d are based on part a graph.

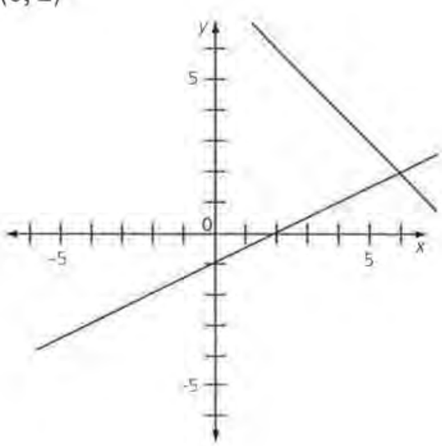
b. Slope of men's line: –0.436; slope of women's line: 0.436
Each year the percent of men earning a law degree drops about 0.44 percent; each year the percent of women earning a law degree increase about 0.44 percent.

c. Men's equation: $y = -0.436x + 926$; women's equation: $y = 0.436x - 826$

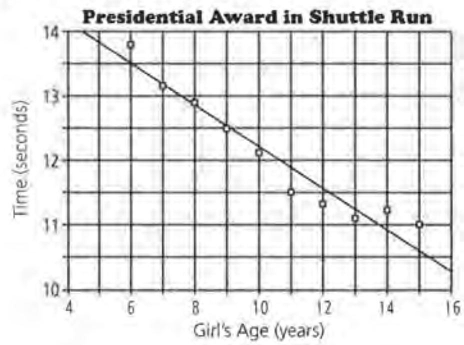
d. Approximately 46%

e. About the year 2010. Answers may vary somewhat, depending on what slope the students use. You may wish to suggest that they use at least 3 decimal places.

2. (6, 2)



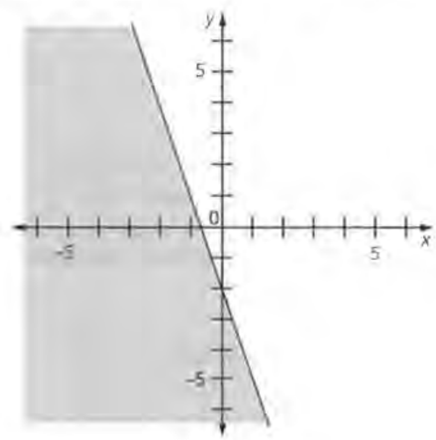
3. **a.** Possible answer:



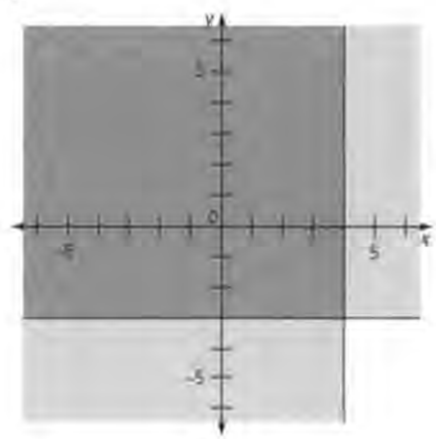
b. Based on part a graph, $y = -0.318x + 15.4$

c. Based on part a graph, $y \leq -0.318x + 15.4$

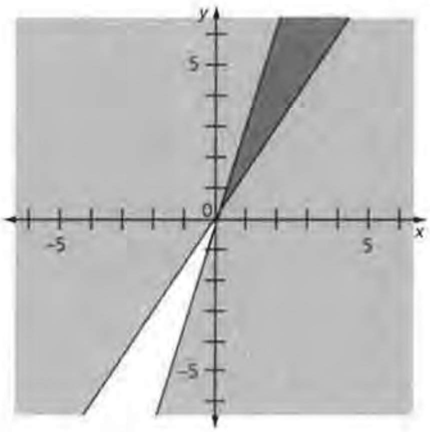
4.



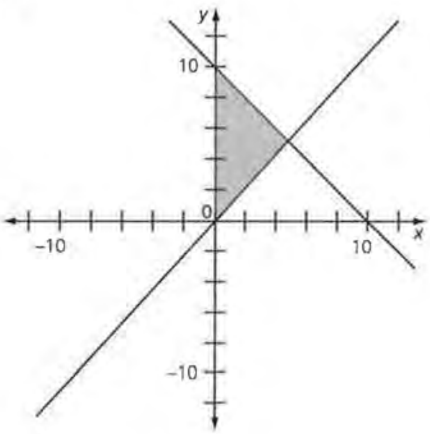
5. **a.**



b.



c.

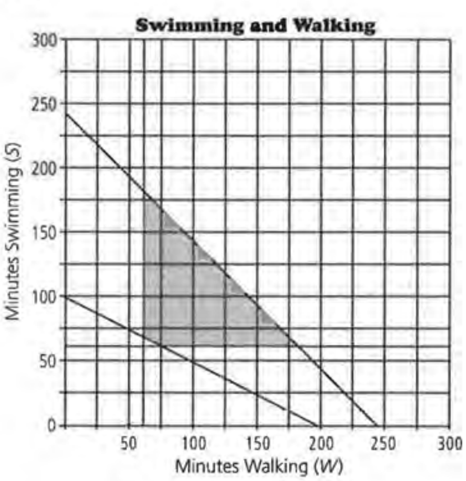


6. **a.** **i.** $S + W \leq 240$

ii. $S \geq 60$ and $W \geq 60$

iii. $10S + 5W \geq 1000$

b.

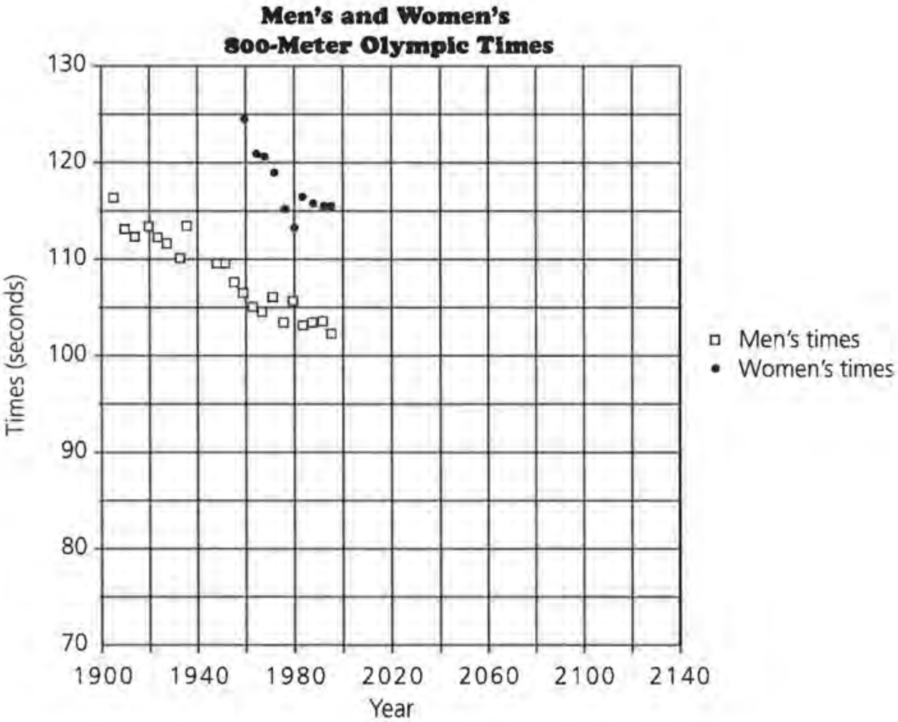


c. (60, 70), (60, 180), (180, 60), (80, 60)

ACTIVITY SHEET 1

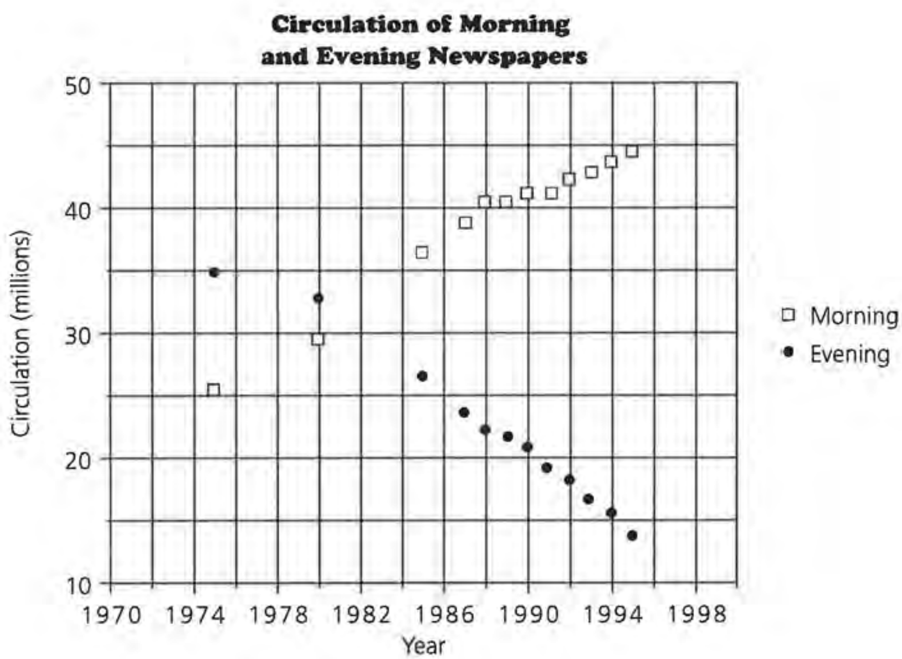
Introductory Activity for Unit I, Problems 3–6

NAME ______________________________



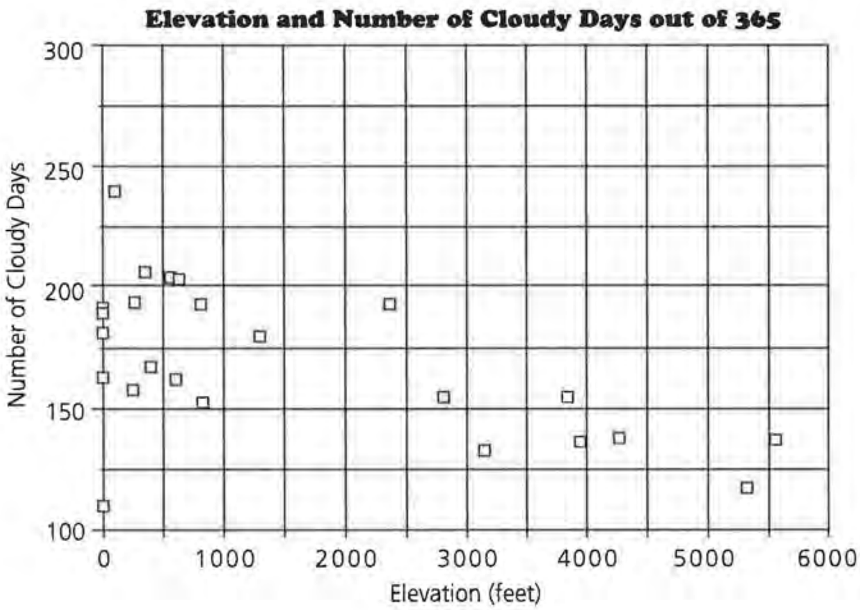
Lesson 1, Problem 3

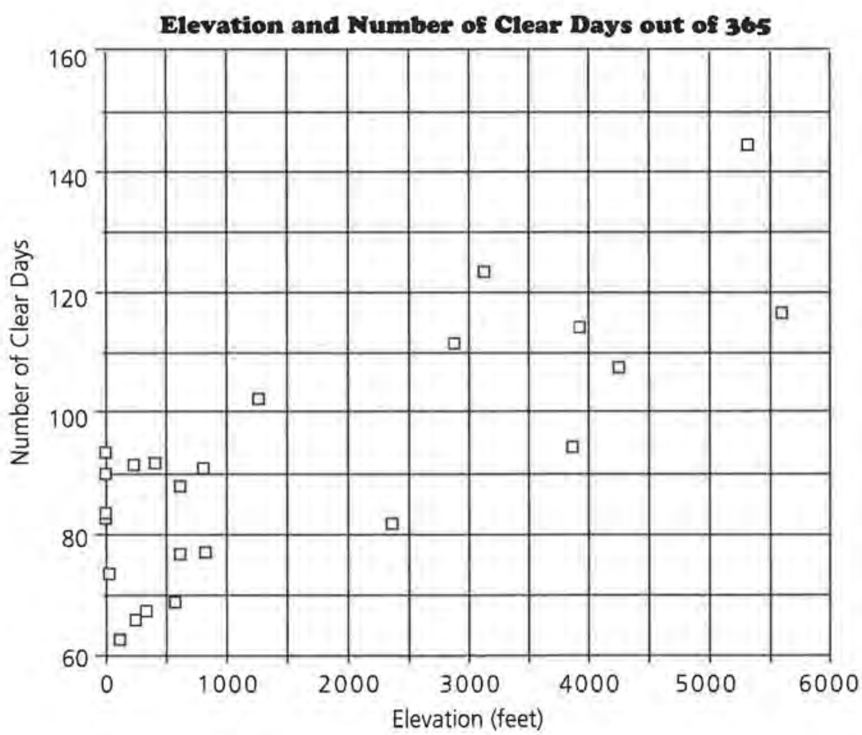
NAME ____________________



Lesson 1, Problem 9

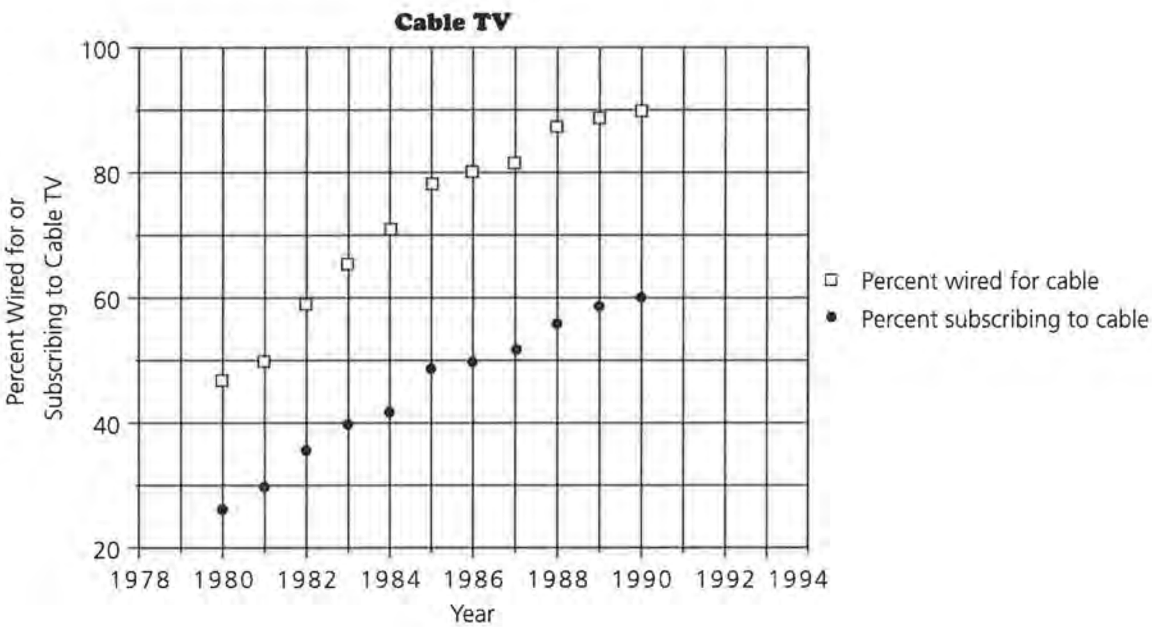
NAME ______________________________

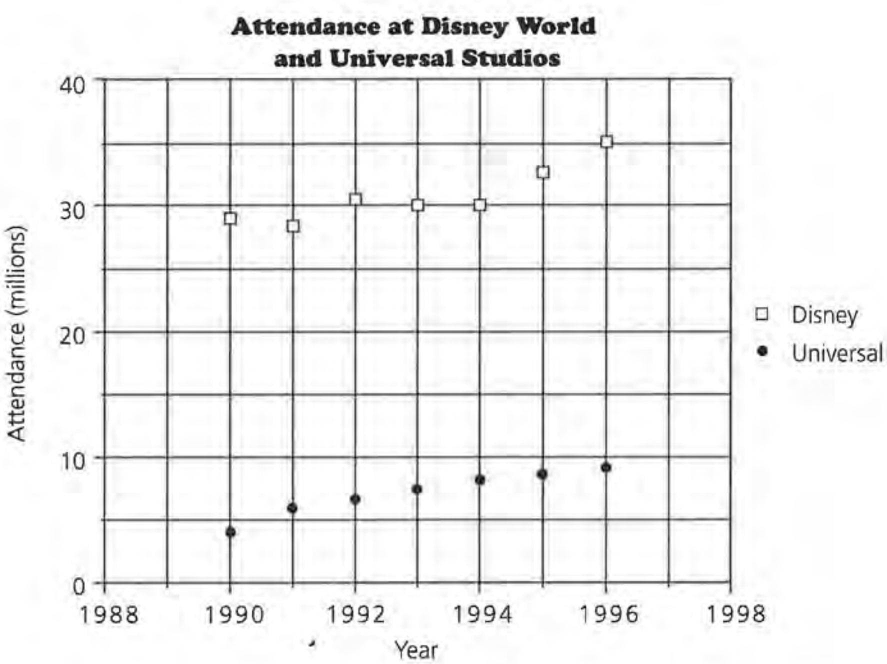




Lesson 2, Problems 2 and 7

NAME ______________________________

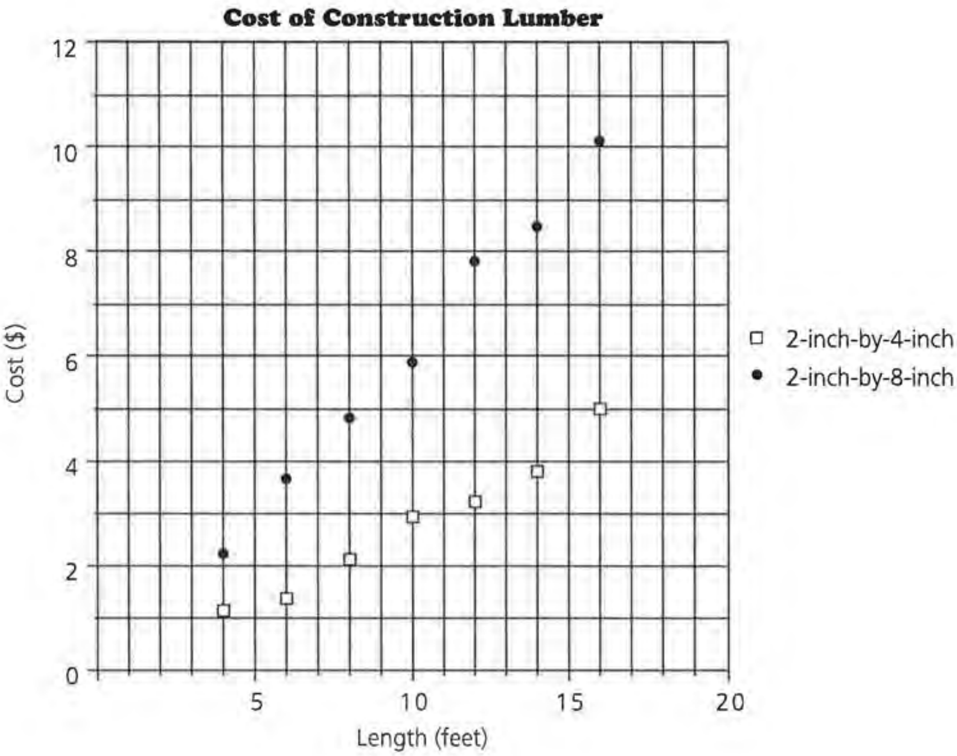




ACTIVITY SHEET 5

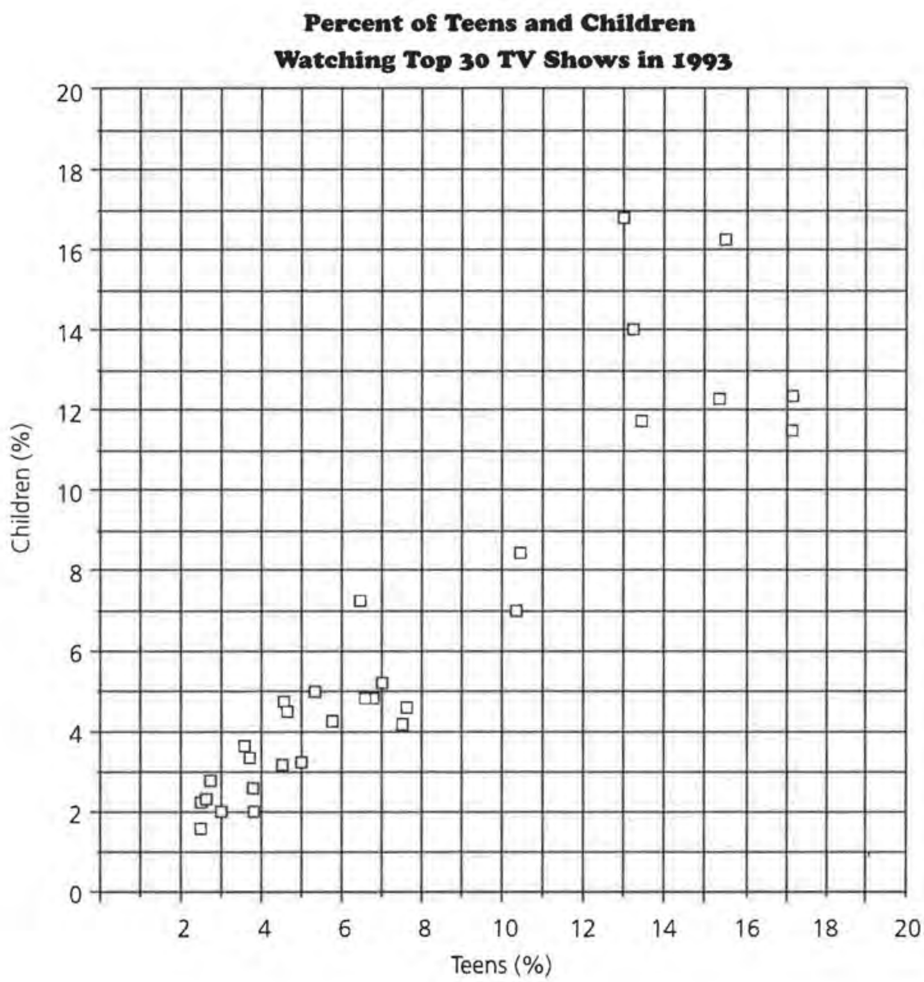
Assessment for Unit I, Problem 2

NAME ______________________________



Lesson 3, Problem 6

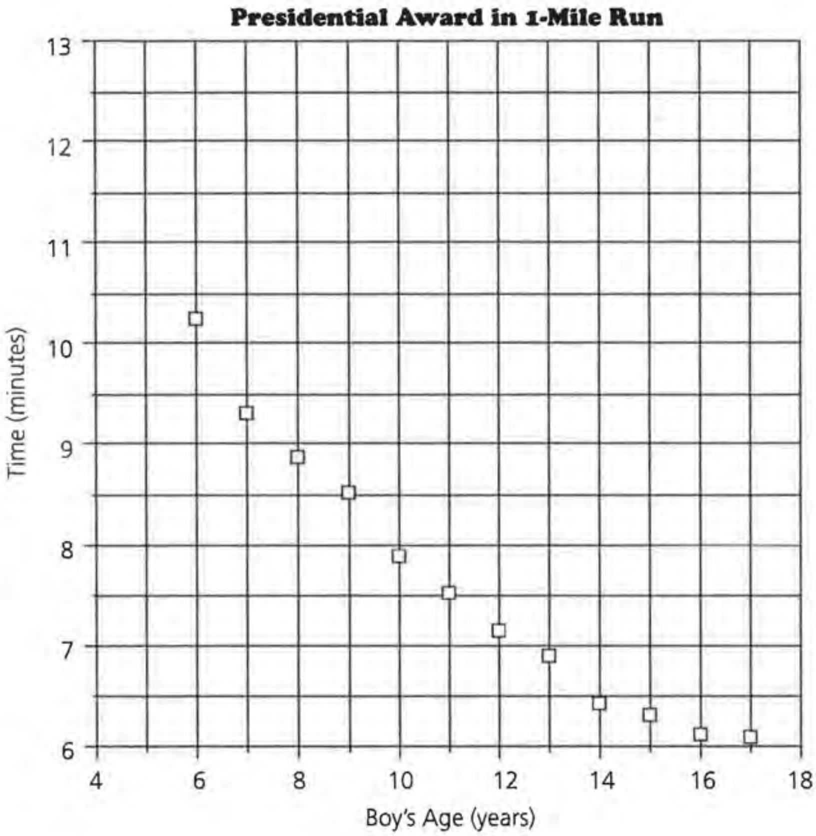
NAME ______________________________



ACTIVITY SHEET 7

Lesson 4, Problems 1 and 2

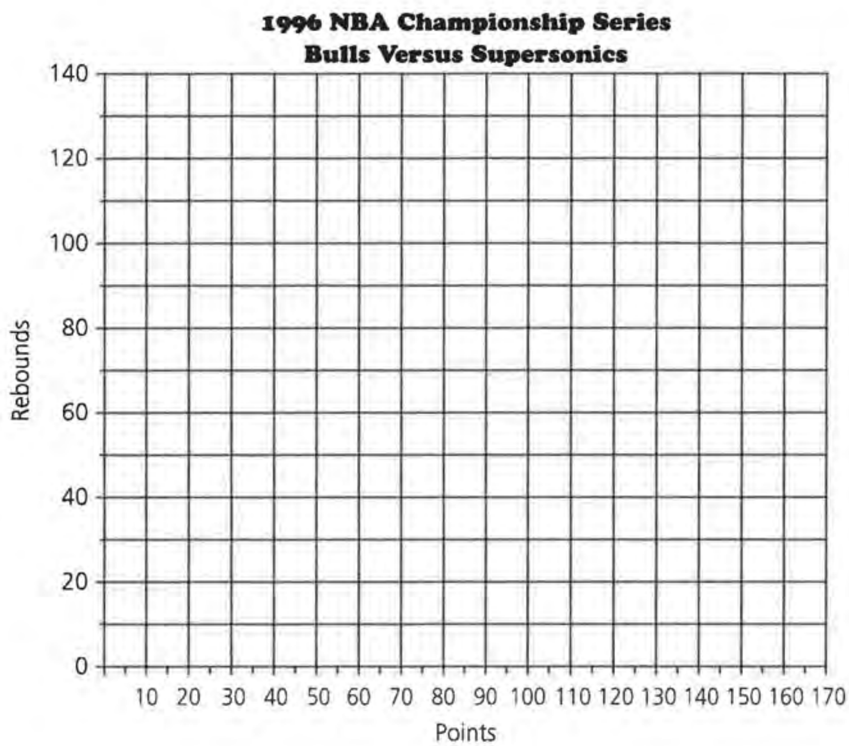
NAME ____

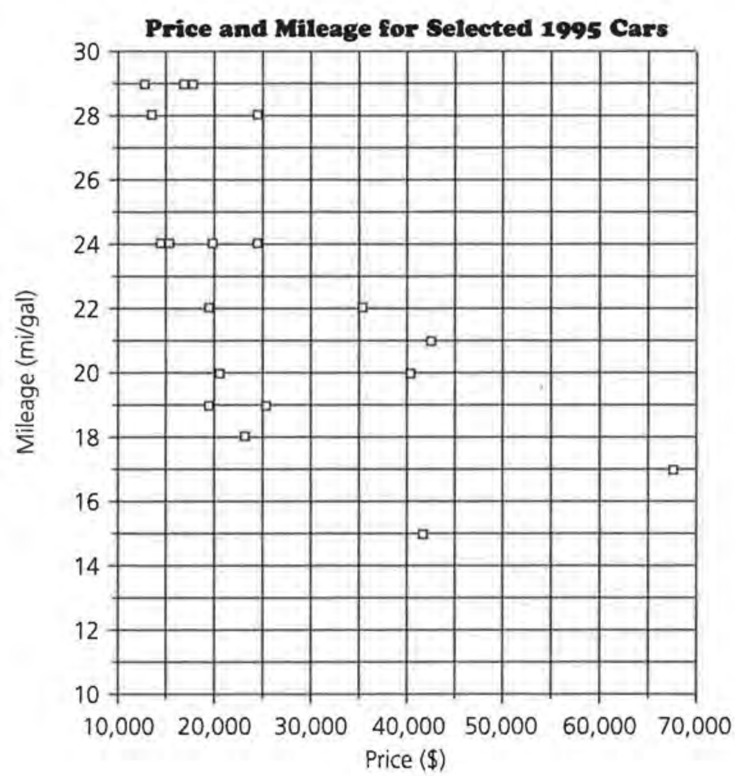


ACTIVITY SHEET 8

Lesson 5, Problems 1–7, 11, and 12

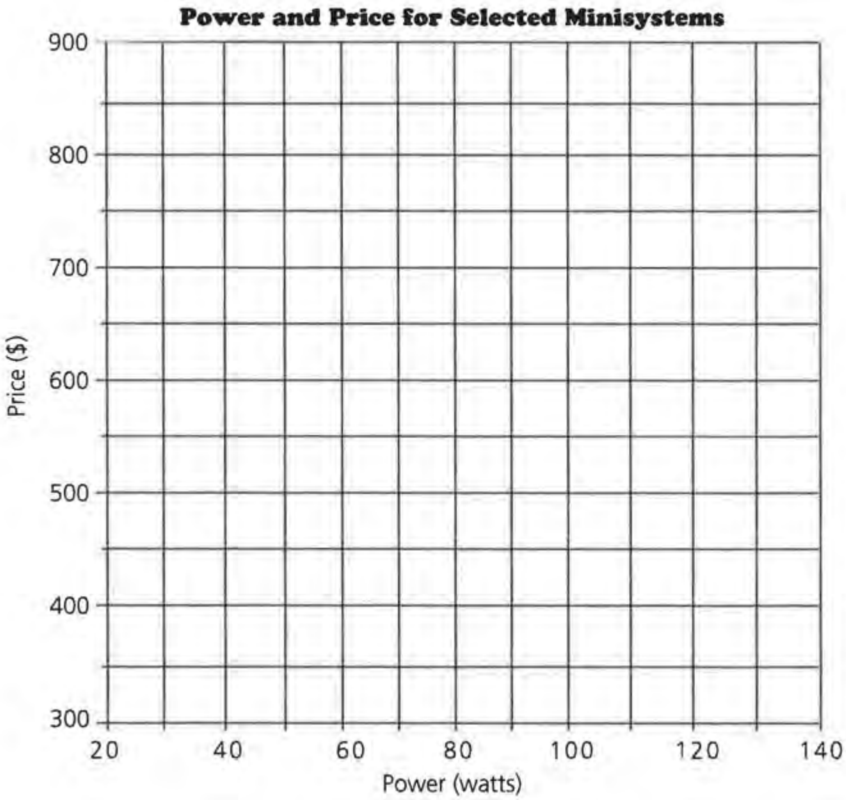
NAME ____





Lesson 5, Problem 15

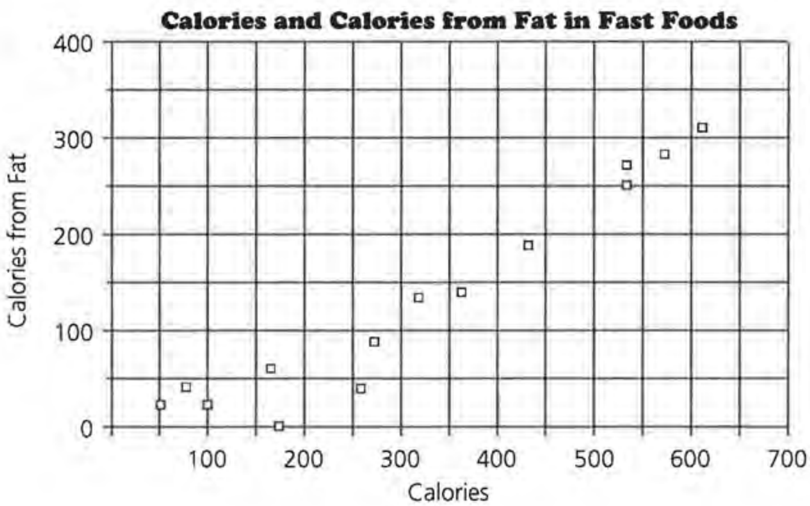
NAME ______________________________

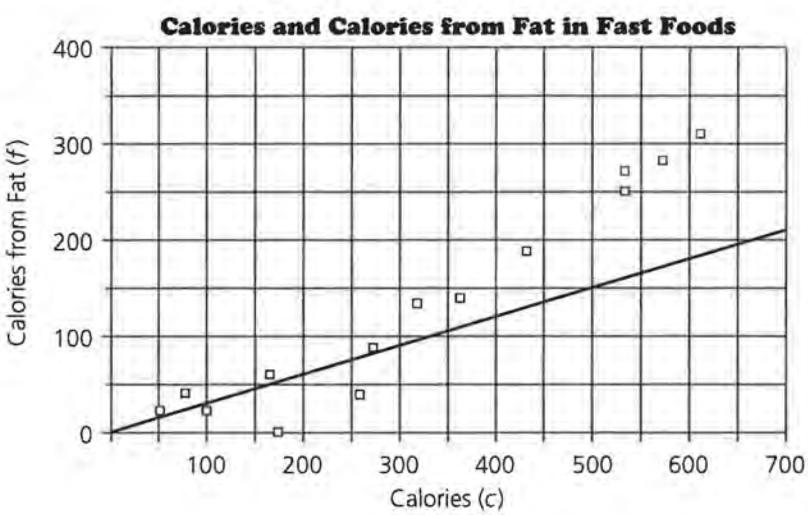


ACTIVITY SHEET 10

Lesson 6, Problems 2–6 and 8

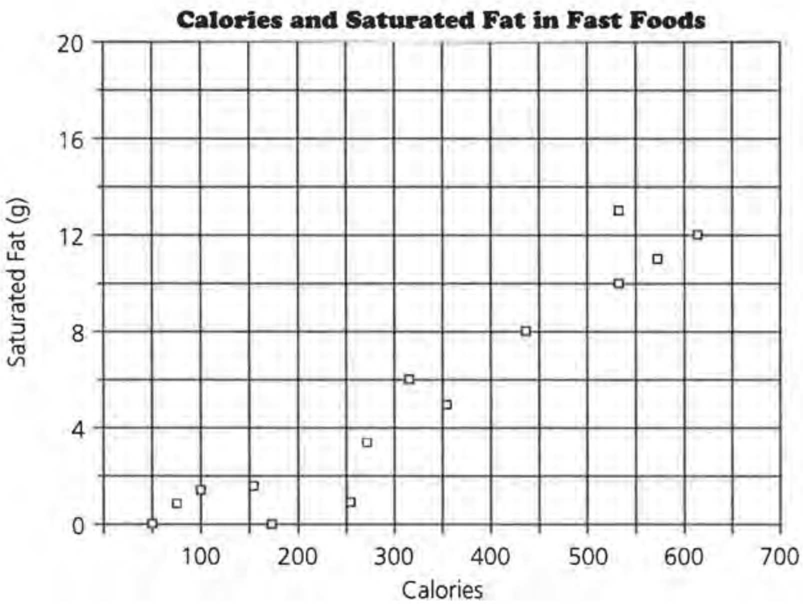
NAME ______________________________





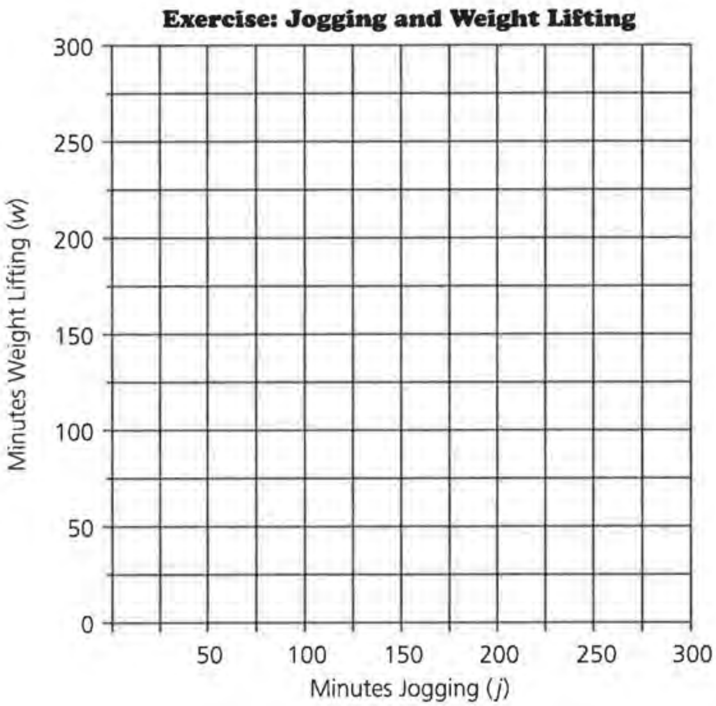
Lesson 6, Problems 11–13 and 18

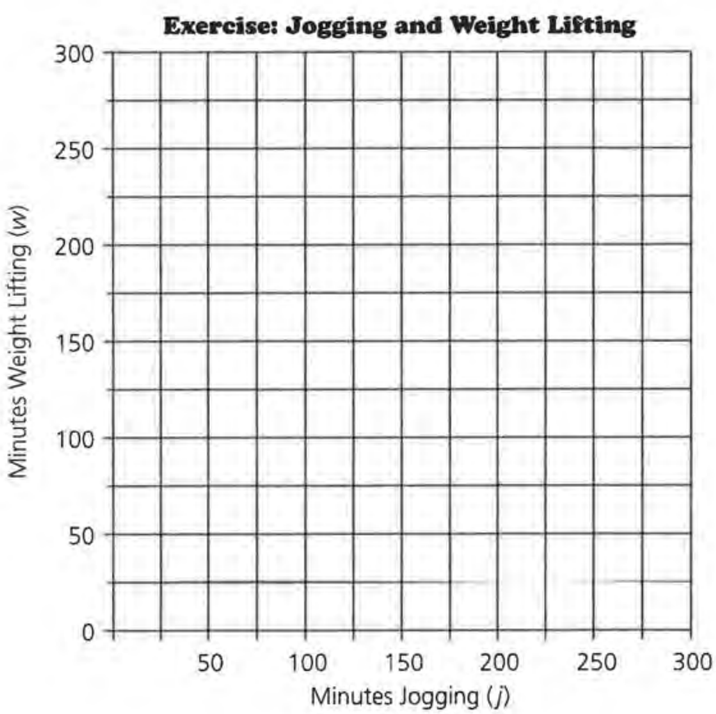
NAME ____________________



Lesson 7, Problems 2, 3, and 7

NAME ______________________________

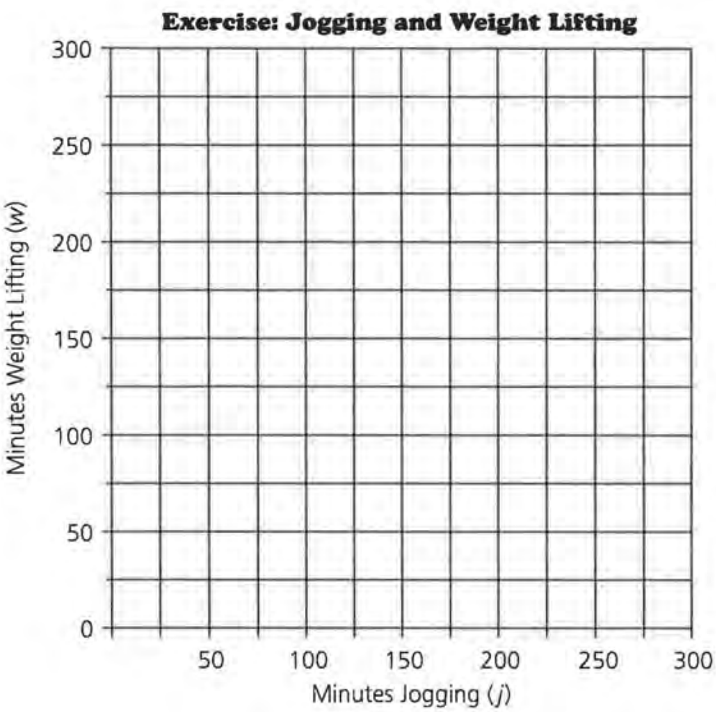


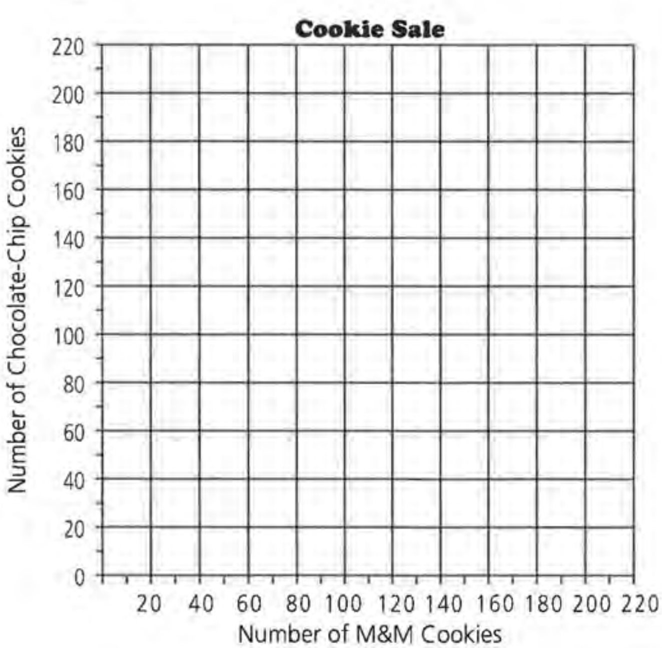


ACTIVITY SHEET 13

Lesson 7, Problems 9, 14, and 18

NAME ______________________________

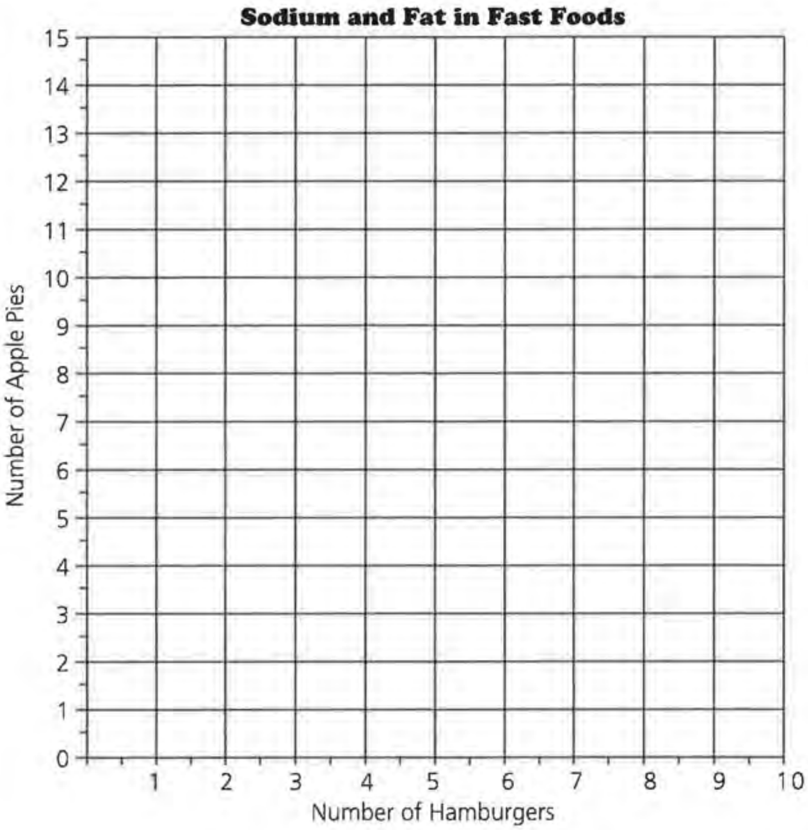




ACTIVITY SHEET 14

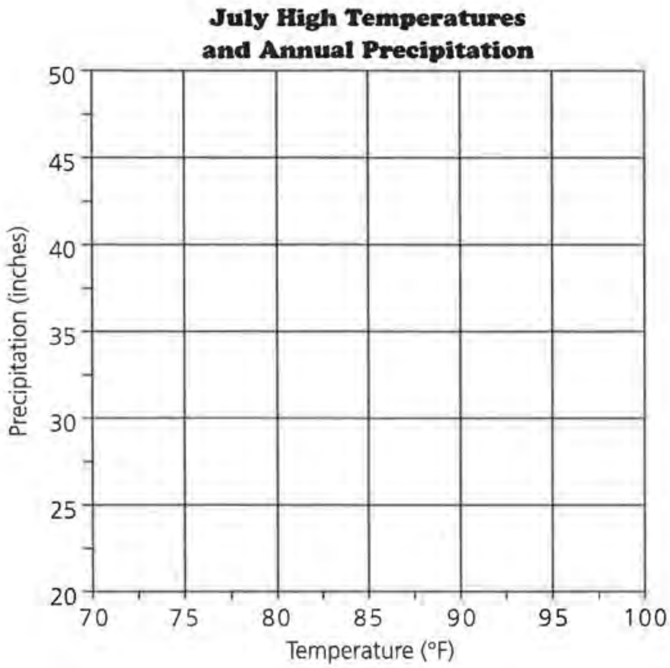
Lesson 7, Problems 27 and 28

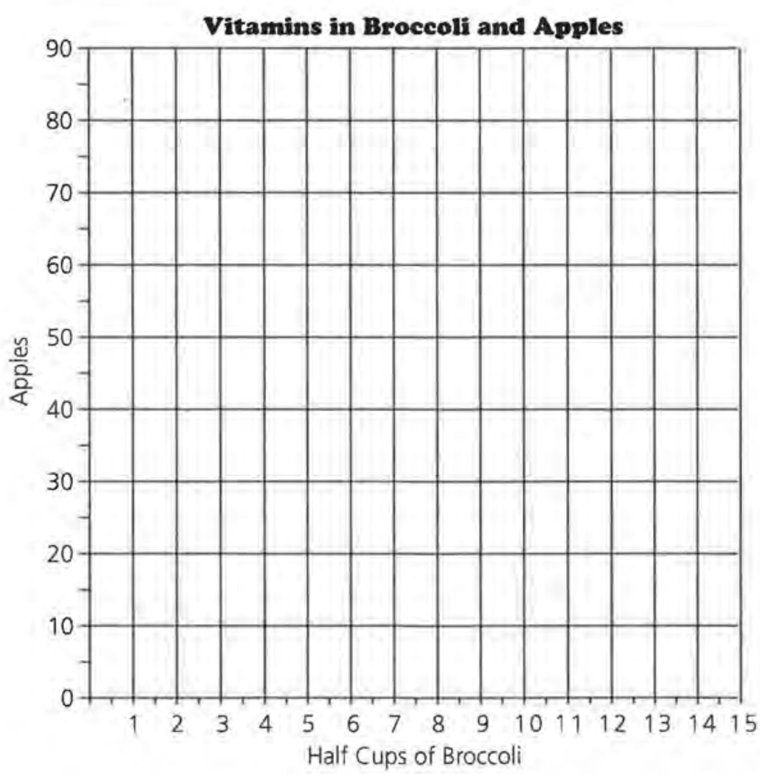
NAME ________________________________



Assessment for Unit III, Problems 1 and 2

NAME ______________________________





Data-Driven Mathematics
Procedures for Using the TI-83

I. Clear menus

ENTER will execute any command or selection. Before beginning a new problem, previous instructions or data should be cleared. Press **ENTER** after each step below.

1. To clear the function menu, **Y=**, place the cursor anyplace in each expression, **CLEAR**
2. To clear the list menu, **2nd MEM ClrAllLists**
3. To clear the draw menu, **2nd DRAW ClrDraw**
4. To turn off any statistics plots, **2nd STATPLOT PlotsOff**
5. To remove user-created lists from the Editor, **STAT SetUpEditor**

II. Basic information

1. A rule is active if there is a dark rectangle over the option. See Figure 1.

```
Plot1 Plot2 Plot3
\Y1 = 170 + 7(X–72)
\Y2 = –.35X – 22
\Y3 =
\Y4 =
\Y5 =
\Y6 =
\Y7 =
```

FIGURE 1

On the screen above, Y1 and Plot1 are active; Y2 is not. You may toggle Y1 or Y2 from active to inactive by putting the cursor over the = and pressing **ENTER**. Arrow up to **Plot1** and press **ENTER** to turn it off; arrow right to **Plot2** and press **ENTER** to turn it on, etc.

2. The Home Screen (Figure 2) is available when the blinking cursor is on the left as in the diagram below. There may be other writing on the screen. To get to the Home Screen, press **2nd QUIT**. You may also clear the screen completely by pressing **CLEAR**.

```
7→X
                 7
Y1
                11
█
```

FIGURE 2

3. The variable x is accessed by the **X, T, Θ, *n*** key.
4. Replay option: **2nd ENTER** allows you to back up to an earlier command. Repeated use of **2nd ENTER** continues to replay earlier commands.
5. Under **MATH**, the **MATH** menu has options for fractions to decimals and decimals to fractions, for taking *n*th roots, and for other mathematical operations. **NUM** contains the absolute value function as well as other numerical operations. (Figure 3)

```
MATH NUM CPX PRB
1: abs(
2: round(
3: iPart(
4: fPart(
5: int(
6: min(
7↓max(
```

FIGURE 3

III. The STAT Menus

1. There are three basic menus under the **STAT** key: **EDIT**, **CALC**, and **TESTS**. Data are entered and modified in the **EDIT** mode; all numerical calculations are made in the **CALC** mode; statistical tests are run in the **TEST** mode.
2. **Lists and Data Entry**
Data is entered and stored in Lists (Figure 4). Data will remain in a list until the list is cleared. Data can be cleared using **CLEAR** L_i or (List name), or by placing the cursor over the List heading and selecting **CLEAR ENTER**. To enter data, select **STAT EDIT**, and with the arrow keys move the cursor to the list you want to use.

40	14	
35	12	
30	9	
25	4	
L5 (7) = 4		

FIGURE 5

Naming Lists

Six lists are supplied to begin with. L1, L2, L3, L4, L5, and L6 can be accessed also as **2nd** $\mathbf{L_i}$. Other lists can be named using words as follows. Put the cursor at the top of one of the lists. Press **2nd INS** and the screen will look like that in Figure 6.

and the screen will say "error: data type.") The newly named list and the data will remain until you go to Memory and delete the list from the memory. To access the list for later use, press **2nd LIST** and use the arrow key to locate the list you want under the **NAMES** menu. You can accelerate the process by typing **ALPHA P** (for price). (Figure 8) Remember, to delete all but the standard set of lists from the editor, use SetUp Editor from the **STAT** menu.

```
NAMES OPS MATH
↑PRICE
:RATIO
:RECT
:RED
:RESID
:SATM
↓SATV
```

FIGURE 8

3 Type in a numerical value and press **ENTER.** Note that the bottom of the screen indicates the List you are in and the list element you have highlighted. 275 is the first entry in L1. (It is sometimes easier to enter a complete list before beginning another.)

L1	L2	L3
275	67	190
5311	144	120
114	64	238
2838	111	153
15	90	179
332	68	207
3828	94	153
L1 (1) = 275		

FIGURE 4

For data with varying frequencies, one list can be used for the data, and a second for the frequency of the data. In Figure 5 below, the L5(7) can be used to indicate that the seventh element in list 5 is 4, and that 25 is a value that occurs 4 times.

	L1	L2 1
	-------	-------
Name =		

FIGURE 6

The alpha key is on, so type in the name (up to five characters) and press **ENTER.** (Figure 7)

PRICE	L1	L2 2
	-------	-------

4. Graphing Statistical Data

General Comments

- Any graphing uses the **GRAPH** key.
- Any function entered in Y1 will be graphed if it is active. The graph will be visible only if a suitable viewing window is selected.
- The appropriate *x*- and *y*-scales can be selected in **WINDOW**. Be sure to select a scale that is suitable to the range of the variables.

Statistical Graphs

To make a statistical plot, select **2nd Y=** for the **STAT PLOT** option. It is possible to make three plots concurrently if the viewing windows are identical. In Figure 9, Plots 2 and 3 are off, Plot 1 is a scatter plot of the data (Costs, Seats), Plot 2 is a scatter plot of (L3,L4), and Plot 3 is a box plot of the data in L3.

```
STAT PLOTS
1: Plot1...On
     COST SEATS  □
2: Plot2...Off
     L3   L4     +
3: Plot3...Off
     L3   1
4↓ PlotsOff
```

FIGURE 9

Activate one of the plots by selecting that plot and pressing **ENTER**.

Choose **ON**, then use the arrow keys to select the type of plot (scatter, line, histogram, box plot with outliers, box plot, or normal probability plot). (In a line plot, the points are connected by segments in the order in which they are entered. It is best used with data over time.) Choose the lists you wish to use for the plot. In the window below, a scatter plot has been selected with the *x*-coordinate data from COSTS, and the *y*-coordinate data from SEATS. (Figure 10) (When pasting in list names, press **2nd LIST**, press **ENTER** to activate the name, and press **ENTER** again to locate the name in that position.)

```
Plot1 Plot2 Plot3
On Off
Type:
Xlist: COSTS
Ylist: SEATS
Mark:  +  •
```

FIGURE 10

For a histogram or box plot, you will need to select the list containing the data and indicate whether you used another list for the frequency or are using 1 for the frequency of each value. The *x*-scale selected under **WINDOW** determines the width of the bars in the histogram. It is important to specify a scale that makes sense with the data being plotted.

5. Statistical Calculations

One-variable calculations such as mean, median, maximum value of the list, standard deviation, and quartiles can be found by selecting **STAT CALC 1-Var Stats** followed by the list in which you are interested. Use the arrow to continue reading the statistics. (Figures 11, 12, 13)

```
EDIT CALC TESTS
1: 1–Var Stats
2: 2–Var Stats
3: Med–Med
4: LinReg(ax + b)
5: QuadReg
6: CubicReg
7↓QuartReg
```

FIGURE 11

```
1–Var Stats L1
```

FIGURE 12

```
1–Var Stats
x̄ = 1556.20833
Σx = 37349
Σx² = 135261515
Sx = 1831.353621
σx = 1792.79449
↓n = 24
```

FIGURE 13

Calculations of numerical statistics for bivariate data can be made by selecting two variable statistics. Specific lists must be selected after choosing the **2-Var Stats** option. (Figure 14)

```
2–Var Stats L1, L
2
```

FIGURE 14

Individual statistics for one- or two-data sets can be obtained by selecting **VARS Statistics,** but you must first have calculated either 1-Var or 2-Var Statistics. (Figure 15)

```
XY Σ EQ TEST PTS
1: n
2: x̄
3: Sx
4: σx
5: ȳ
6: Sy
7↓σy
```

FIGURE 15

6. **Fitting Lines and Drawing Their Graphs**
Calculations for fitting lines can be made by selecting the appropriate model under **STAT CALC: Med-Med** gives the median fit regression, **LinReg** the least-squares linear regression, and so on. (Note the only difference between **LinReg (ax+b)** and **LinReg (a+bx)** is the assignment of the letters a and b.) Be sure to specify the appropriate lists for *x* and *y*. (Figure 16)

```
Med–Med LCal, LFA
CAL
```

FIGURE 16

To graph a regression line on a scatter plot, follow the steps below:

- Enter your data into the Lists.
- Select an appropriate viewing window and set up the plot of the data as above.
- Select a regression line followed by the lists for *x* and *y*, **VARS Y-VARS Function** (Figures 17, 18) and the Y_i you want to use for the equation, followed by **ENTER.**

```
VARS Y-VARS
1: Function...
2: Parametric...
3: Polar...
4: On/Off...
```

FIGURE 17

```
Med–Med _CAL, LFA
CAL, Y1
```

FIGURE 18

The result will be the regression equation pasted into the function Y1. Press **GRAPH** and both the scatter plot and the regression line will appear in the viewing window. (Figures 19, 20)

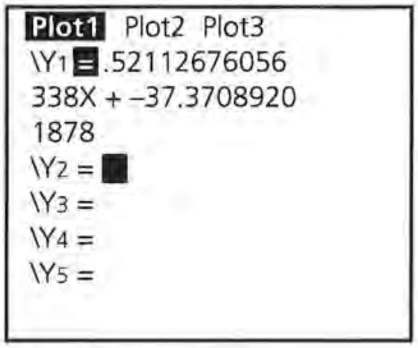


FIGURE 19

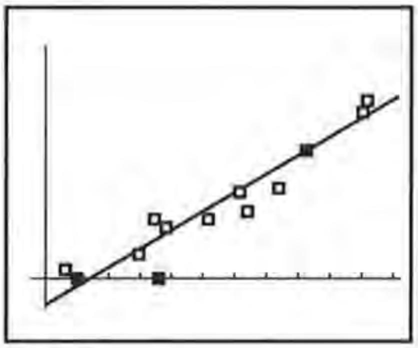


FIGURE 20

- There are two cursors that can be used in the graphing screen.

TRACE activates a cursor that moves along either the data (Figure 21) or the function entered in the Y-variable menu (Figure 22). The coordinates of the point located by the cursor are given at the bottom of the screen.

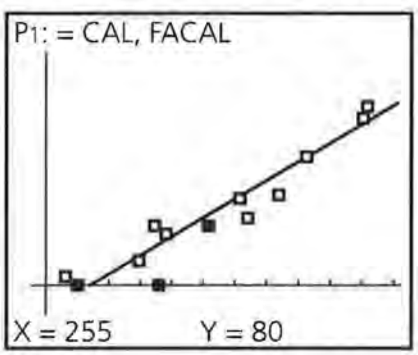


FIGURE 21

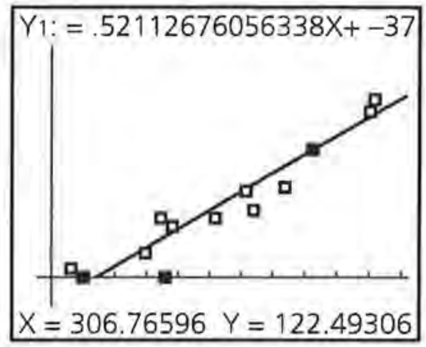


FIGURE 22

Pressing **GRAPH** returns the screen to the original plot. The up arrow key activates a cross cursor that can be moved freely about the screen using the arrow keys. See Figure 23.

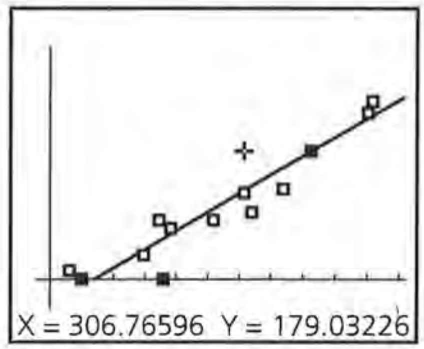


FIGURE 23

Exact values can be obtained from the plot by selecting **2nd CALC** Value. Select **2nd CALC Value ENTER**. Type in the value of x you would like to use, and the exact ordered pair will appear on the screen with the cursor located at that point on the line. (Figure 24)

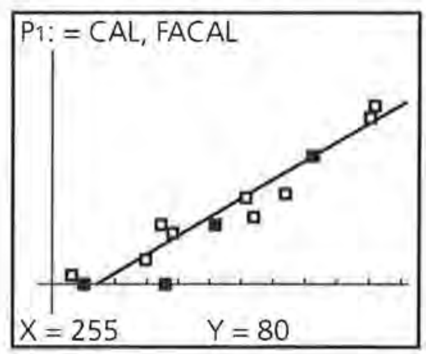


FIGURE 24

IV. Evaluating an expression

To evaluate $y = .225x - 15.6$ for $x = 17$, 20, and 24, you can:

1. Type the expression in Y1, return to the home screen, 17 **STO X,T,Θ,n ENTER, VARS Y-VARS Function Y1 ENTER ENTER.** (Figure 25)

```
17→X
                    17
Y1
           -11.775
█
```

FIGURE 25

Repeat the process for $x = 20$ and 24.

2. Type $17^2 - 4$ for $x = 17$, **ENTER** (Figure 26). Then use **2nd ENTRY** to return to the arithmetic line. Use the arrows to return to the value 17 and type over to enter 20.

```
.225*17 - 15.6
                 -11.775
.225*2█ - 15.6
```

FIGURE 26

You can also find the value of x by using the table command. Select **2nd TblSet** (Figure 27). (Y1 must be turned on.) Let **TBlStart** = 17, and the increment **ΔTbl** = 1.

```
TABLE SETUP
  TblStart = 17
  ΔTbl = █
Indpnt:  Auto  Ask
Depend:  Auto  Ask
```

FIGURE 27

Select **2nd TABLE** and the values of x and y generated by the equation in Y1 will be displayed. (Figure 28)

X	Y1	
17	-11.78	
18	-11.55	
19	-11.33	
20	-11.1	
21	-10.88	
22	-10.65	
23	-10.43	
X = 17		

FIGURE 28

V. Operating with Lists

1. A list can be treated as a function and defined by placing the cursor on the label above the list entries. List 2 can be defined as L1 + 5. (Figure 29)

L1	L2	L3
275	--------	190
5311		120
114		238
2838		153
15		179
332		207
3828		153
L2 = L1 + 5		

FIGURE 29

Pressing **ENTER** will fill List 2 with the values defined by L1+5. (Figure 30)

L1	L2	L3
275	280	190
5311	5316	120
114	119	238
2838	2843	153
15	20	179
332	337	207
3828	3833	153

L2(1) = 280

FIGURE 30

2. List entries can be cleared by putting the cursor on the heading above the list, and selecting **CLEAR** and **ENTER.**

3. A list can be generated by an equation from Y= over a domain specified by the values in L1 by putting the cursor on the heading above the list entries. Select **VARS Y-VARS Function Y1 ENTER (L1) ENTER.** (Figure 31)

L1	L2	L3
120	12	-------
110	14	
110	12	
110	11	
100	7	
100	6	
120	9	

L3 = Y1(L1)

FIGURE 31

4. The rule for generating a list can be attached to the list and retrieved by using quotation marks (**ALPHA** +) around the rule. (Figure 32) Any change in the rule (Y1 in the illustration) will result in a change in the values for L1. To delete the rule, put the cursor on the heading at the top of the list, press **ENTER**, and then use the delete key. (Because L1 is defined in terms of **CAL**, if you delete **CAL** without deleting the rule for L1 you will cause an error.)

CAL	FACAL	L1	5
255	80	-------	
305	120		
410	180		
510	250		
320	90		
370	125		
500	235		

L1 = "Y1(LCAL)"

FIGURE 32

VI. Using the DRAW Command

To draw line segments, start from the graph of a plot, press **2ND DRAW**, and select **Line(**. (Figure 33)

DRAW POINTS STO
1: ClrDraw
2: Line(
3: Horizontal
4: Vertical
5: Tangent(
6: DrawF
7↓Shade(

FIGURE 33

This will activate a cursor that can be used to mark the beginning and ending of a line segment. Move the cursor to the beginning point and press **ENTER**; use the cursor to mark the end of the segment, and press **ENTER** again. To draw a second segment, repeat the process. (Figure 34)

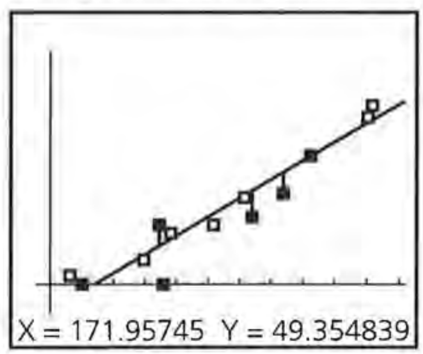


FIGURE 34

VII. Random Numbers

To generate random numbers, press **MATH** and **PRB**. This will give you access to a random number function, **rand**, that will generate random numbers between 0 and 1 or **randInt(** that will generate random numbers from a beginning integer to an ending integer for a specified set of numbers. (Figure 35) In Figure 36, five random numbers from 1 to 6 were generated.

```
MATH NUM CPX PRB
1: rand
2: nPr
3: nCr
4: !
5: randInt(
6: randNorm(
7: randBin(
```

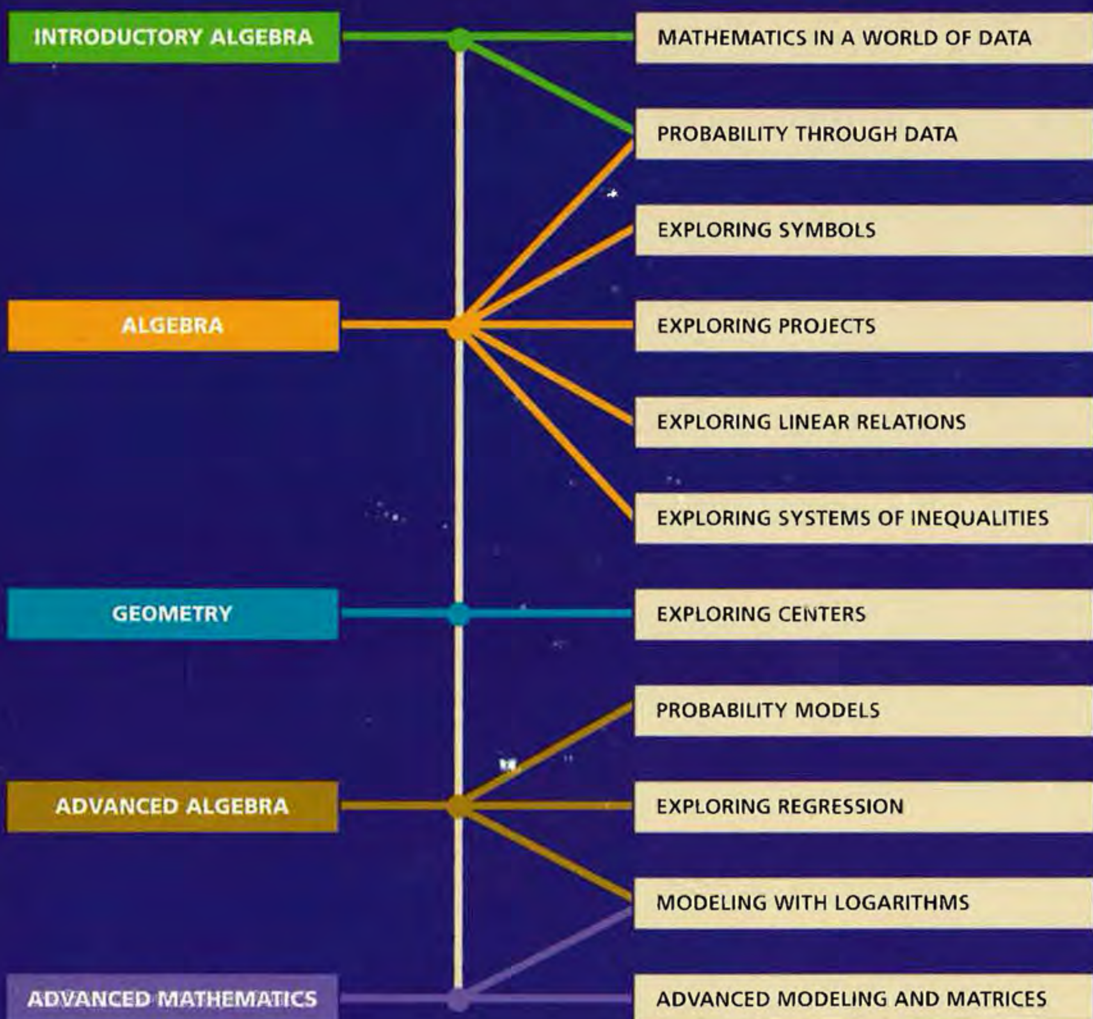
FIGURE 35

```
randInt(1, 6, 5,)
     (2 4 2 5 3)
```

FIGURE 36

Pressing **ENTER** will generate a second set of random numbers.

Data-Driven Mathematics is a series of modules written by teachers and statisticians that focuses on the use of real data and statistics to motivate traditional mathematics topics. This chart suggests which modules could be used to supplement specific middle-school and high-school mathematics courses.



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