Exploring Systems of Inequalities

GAIL F. BURRILL AND PATRICK W. HOPFENSPERGER

DATA - DRIVEN MATHEMATICS

Exploring Systems of Equations and Inequalities

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This book is published by Dale Seymour Publications®, an imprint of Addison Wesley Longman, Inc.

Dale Seymour Publications
10 Bank Street
White Plains, NY 10602
Customer Service: 800-872-1100

Printed in the United States of America.

Order number DS21845

ISBN 1-57232-539-9

1 2 3 4 5 6 7 8 9 10-ML-03 02 01 00 99 98

This Book Is Printed
On Recycled Paper

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Acknowledgments

The authors thank the following people for their assistance during the preparation of this module:

- The many teachers who reviewed drafts and participated in the field tests of the manuscripts

- The members of the *Data-Driven Mathematics* leadership team, the consultants, and the writers

- Homestead High School students, John Atwater, Sarah Finney, and Irena Goldberg, for their comments and helpful critique

- The many students from Homestead High School and Whitnall High School, who helped shape the ideas as they were being developed

Table of Contents

About *Data-Driven Mathematics*

Historically, the purposes of secondary-school mathematics have been to provide students with opportunities to acquire the mathematical knowledge needed for daily life and effective citizenship, to prepare students for the workforce, and to prepare students for postsecondary education. In order to accomplish these purposes today, students must be able to analyze, interpret, and communicate information from data.

Data-Driven Mathematics is a series of modules meant to complement a mathematics curriculum in the process of reform. The modules offer materials that integrate data analysis with secondary mathematics courses. Using these materials will help teachers motivate, develop, and reinforce concepts taught in current texts. The materials incorporate major concepts from data analysis to provide realistic situations for the development of mathematical knowledge and realistic opportunities for practice. The extensive use of real data provides opportunities for students to engage in meaningful mathematics. The use of real-world examples increases student motivation and provides opportunities to apply the mathematics taught in secondary school.

The project, funded by the National Science Foundation, included writing and field testing the modules, and holding conferences for teachers to introduce them to the materials and to seek their input on the form and direction of the modules. The modules are the result of a collaboration between statisticians and teachers who have agreed on statistical concepts most important for students to know and the relationship of these concepts to the secondary mathematics curriculum.

Using This Module

Why the Content Is Important

Many problems in the world require looking for relationships between two events that are changing at the same time. For example, when will women's times for running a 800-meter race be the same as the men's time, and when did the circulation of morning newspapers exceed the circulation of evening papers? Solving systems of equations and inequalities are important tools in answering questions similar to these. Solving systems enables us to find the intersection of two lines on a graph and analyze patterns and relationships.

This module will extend the concepts of rate of change and graphing linear equations to graphing two equations or inequalities on the same coordinate system. In Unit I, you will investigate algebraic techniques that can be used to find the point of intersection of two lines. These techniques will be used to graph and analyze inequalities in Unit II. People often use inequalities to determine when a quantity is greater than or less than some given standard. Does the food at a fast-food restaurant exceed nutritionists' guidelines? Will increasing the price of a chocolate-chip cookie decrease sales?

Content

Mathematics content: You will be able to

- Find and interpret slope.
- Graph a system of equations.
- Identify parallel lines.
- Solve a system of equations using the substitution method.
- Graph a system of inequalities.
- Solve a system of inequalities.

Statistics content: You will be able to

- Make and interpret plots over time.
- Make and interpret scatter plots.
- Find a line of best fit for a set of data.

Solving Systems of Equations

Men's and Women's Olympic Times

Do you think that women's times will ever catch up to men's times in Olympic races?

"Women may outrun men, researchers suggest."

Analyzing trends in records for sporting events is of great interest to many people. Often these trends can be determinded by graphing. In this unit, you will learn how to find relationships between two sets of data by studying their graphs.

EXPLORE

Within a few generations, female runners may start beating male runners in world-class competitions. An analysis of world records for a variety of distances found that women's rates have been improving about twice as quickly as men's rates.

If this trend continues, the top female and male runners may start performing equally well between the years 2015 and 2055 in the 200-, 400-, 800-, and 1500-meter events. These findings were reported in a letter in the journal *Nature*.

> "None of the current women's world-record holders at these events could even meet the men's qualifying standard to compete in the 1992 Olympic games," researchers Brian Wipp and Susan Ward wrote. "However, it is the rates of improvement that are so strikingly different—the gap is progressively closing." But other researchers said they doubted the projections because they believed women's rate of improvement would decrease.
>
> Source: Staff and Wire Services, *Milwaukee Journal*, January, 1992

1. What data could you collect that would help you answer the question on page 3?

2. If you found that the men's times in an event were decreasing 0.08 second per year, by what amount must the women's times decrease to prove the researchers' claim?

The graph below shows men's and women's times for the 800-meter race.

Men's and Women's 800-Meter Olympic Times

□ Men's times
• Women's times
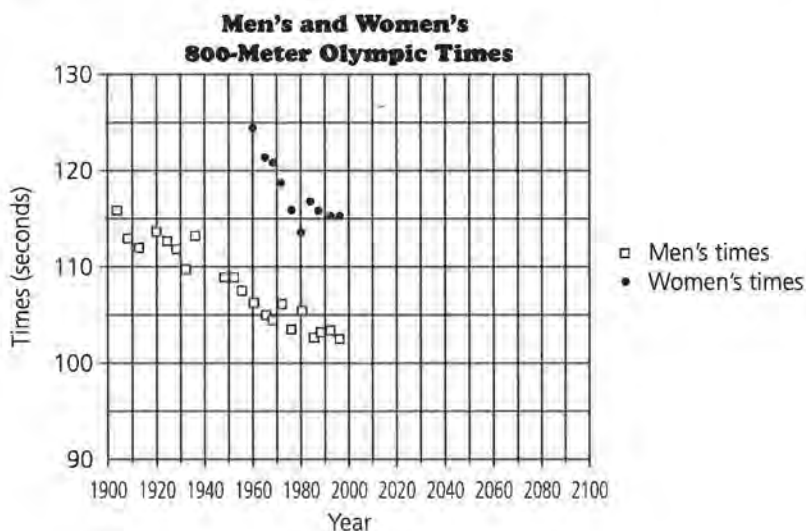
3. On the graph on *Activity Sheet 1*, draw a line that you think represents the trend for the men's times.

4. On the same graph, draw a line that you think represents the trend for the women's times.

5. Extend the two lines. Estimate their point of intersection.

6. What does this point represent in terms of the data?

Systems of Equations

Does your family subscribe to a daily paper?

If your family subscribes, who reads the paper?

What sections do people read the most often?

We often want to observe trends in related events. In this lesson, you will investigate the relationships between morning and evening newspaper circulation, as well as several other sets of data.

INVESTIGATE

Trends in circulation of daily newspapers have been changing in recent years. Some cities in the United States have both an evening and a morning paper while others have one daily newspaper. A recent trend for evening newspapers has been to either stop publishing or to convert to a morning paper. The graph on page 6 shows the total circulation in millions of morning and evening newspapers across the United States.

OBJECTIVES

Find the point of intersection of two lines on a graph.

Solve a system of equations.

**Circulation of Morning
and Evening Newspapers**
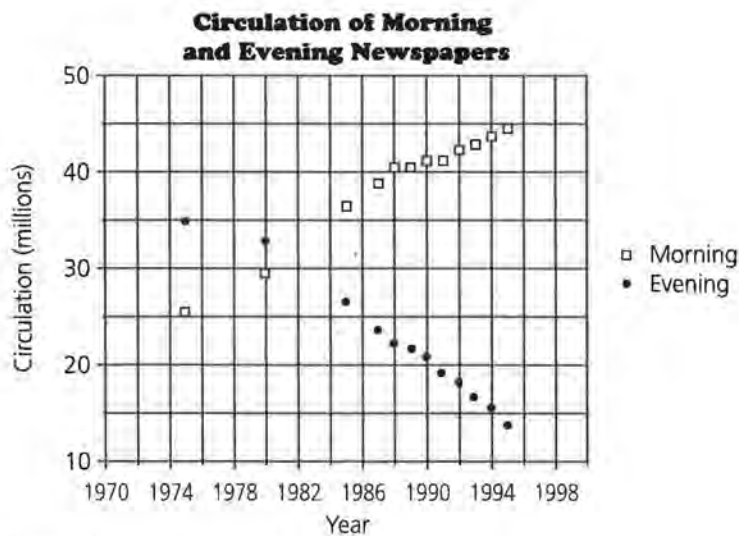
Source: *Statistical Abstract of the United States,* 1995

Discussion and Practice

1. Refer to the graph above.

   a. What trends do you observe in the plots?

   b. Describe what you think will happen in the next few years if the trends continue. This is often called *extrapolation,* or predicting beyond the given information.

   c. Who might be interested in these trends? Why?

2. Use the graph to find the total combined circulation of morning and evening newspapers for the years 1975, 1980, 1985, 1990, and 1995. Has the total circulation remained about the same since 1975?

3. On the graph on *Activity Sheet* 2, draw a line that you think best represents each set of data.

   a. Find the slope of each line. Describe the trends you observe in terms of the slopes.

   b. Write an equation of each line.

   c. Predict the circulation for evening and morning newspapers in 2000. How did you make your prediction?

   d. Tanya claims that in 1982 both morning and evening newspapers had a circulation of about 32 million. How can you evaluate her claim?

   e. When do you estimate that the circulation was the same for morning and evening newspapers? Approximately how large a circulation did each have when they were equal? Explain how you found your answer.

A set of two or more equations in the same variables is a *system of equations*. Finding the point where two or more lines intersect, or cross, is called *solving a system of equations*. A solution of a system of equations in two variables is a set of values that makes all the equations in the system true.

To solve some systems of equations, you can graph each equation and find the point where the graphs intersect.

Sometimes you can find the point where the lines intersect by constructing a table of values. Tanya drew lines on her graph and found the following equations: $Y1 = 0.97X - 1893$ and $Y2 = -1.1X + 2210$. She used her calculator to make the following table of values.

X	$Y_1 = 0.97X - 1893$	$Y_2 = -1.1X + 2210$
1985	32.45	26.5
1986	33.42	25.4
1987	34.39	24.3
1988	35.36	23.2
1989	36.33	22.1
1990	37.3	21
1991	38.27	19.9
X = 1991		

4. Construct a table of values for the years from 1975 to 1985 for each of your equations.

   a. Look at a table of values for each equation. Is any ordered pair the same for the two equations?

   b. How can you determine from the table between what two years the ordered pairs will be the same? List the years.

   c. Change the increment for your table to 0.1. Are any of the ordered pairs the same? If not, continue to change the increment until you find the value for X for which the values in Y1 and Y2 are the same. What does the ordered pair tell you?

You can also find the solution to the system algebraically. Let C represent the circulation and T the year. Rewrite the equations for this system as shown, letting C_m represent morning newspaper circulation and C_e represent evening newspaper circulation.

$$C_m = 0.97T - 1893 \qquad C_e = -1.1T + 2210$$

You would like to know when $C_m = C_e$.

Since C_m equals $0.97T - 1893$, you can substitute $0.97T - 1893$ for C_e in the second equation and solve for T.

$0.97T - 1893 = -1.1T + 2210$	The two equations are equal.
$\underline{+\,1.1T \qquad\qquad +\,1.1T}$	Add $1.1T$ to both sides.
$2.07T - 1893 = \qquad 2210$	
$\underline{+\,1893 \qquad\qquad +1893}$	Add 1893 to both sides.
$2.07T \qquad = \qquad 4103$	Divide both sides by 2.07.
$T = 1982.1$	

To find C_m, substitute 1982.1 for T in the equation.

$$C_m = 0.97(1982.1) - 1893$$
$$C_m = 29.64$$

The solution is $(1982.1, 29.64)$.

The solution $(1982.1, 29.64)$ means that at the beginning of 1982 the circulation of both morning and evening newspapers was about 29.6 million.

5. Use 1982.1 for T in the equation for the C_e and solve for C_e. How does your answer compare to the solution $(1982.1, 29.64)$?

Summary

Some problems have relationships that can be described by a set of two equations. To find values that make both equations true, you have to solve the two equations at the same time. You can do this

- by studying the graph to see where the lines intersect,

- by looking at a table of values for each equation to see where they match, or

- by using algebra and substituting from one equation into the other.

Practice and Applications

6. Consider an equation with a variable on both sides such as $6x + 51 = 2x + 175$.

    a. You already know how to solve an equation with a variable listed only once. Solve $6x + 51 = 105$.

    b. It seems reasonable to try to turn an equation with a variable on both sides into one you know how to solve. Eliminate the term $2x$ in $6x + 51 = 2x + 175$ by subtracting $2x$ from both sides of the equation. Write the new equation.

    c. Solve your new equation.

    d. Graph $y = 6x + 51$, $y = 2x + 175$, and $y = 237$ on the same set of axes. What observation can you make?

7. Consider the equations $y_1 = 150 + 3x$ and $y_2 = 75 - 2(x - 100)$.

    a. Graph the equations on the same grid. Estimate the point of intersection of the two graphs.

    b. Use your calculator to make a table of values (x, y) for each equation. Are any of the ordered pairs in the table the same? If not, continue to change the ΔTbl until you find an ordered pair that is the same for each equation.

    c. Solve the system using algebra. For what x-value will $y_1 = y_2$? What are the values of y_1 and y_2 for this x-value?

8. According to the American Music Conference, the sales of electronic pianos and keyboards have nearly overtaken sales of conventional pianos. In 1980, approximately 45,000 electronic pianos were sold; and in 1990, approximately 115,000 electronic pianos were sold. In 1980, approximately 240,000 conventional pianos were sold; and in 1990, approximately 150,000 conventional pianos were sold.

    a. Assume the trends for both sales are linear. Make a plot of the data and estimate when the sales of electronic keyboards will or did overtake those of conventional pianos.

    b. Find an equation for each line, and then solve the system algebraically. How does your solution compare to the estimate you made in part a?

    c. Find the x-intercept, or the zero, for the equation for conventional pianos. What does this value indicate?

9. Does elevation, or the height above sea level, have any effect on the weather? Are the numbers of clear days and cloudy days affected by the elevation of a city? The table below contains the elevation and the numbers of clear and cloudy days per year for selected cities in the United States.

City	Elevation (feet)	Number of Clear Days per Year	Number of Cloudy Days per Year
Albany, NY	275	67	190
Albuquerque, NM	5311	144	120
Anchorage, AK	114	64	238
Boise, ID	2838	111	153
Boston, MA	15	90	179
Burlington, VT	332	68	207
Helena, MT	3828	94	153
Lander, WY	5557	117	133
Milwaukee, WI	672	77	201
Minneapolis, MN	834	78	192
Mobile, AL	211	92	157
Nashville, TN	590	88	161
Newark, NJ	7	84	188
New Orleans, LA	4	83	162
Portland, OR	21	73	189
Raleigh, NC	434	93	167
Rapid City, SD	3162	123	130
Scotts Bluff, NB	3957	115	133
St. Louis, MO	535	69	202
Salt Lake City, UT	4221	114	134
San Antonio, TX	788	91	152
Springfield, MO	1268	103	178
Spokane, WA	2356	82	192
Tampa, FL	19	93	111

Source: *Statistical Abstract of the United States*, 1995

a. What would it mean if a city had an elevation of zero? Which city has the highest elevation?

b. Which city has the greatest number of cloudy days? The least?

c. On the first graph on *Activity Sheet 3*, draw a line that best fits the data for (elevation, cloudy days).

d. Write an equation for the line you drew in part c. This equation expresses the relationship between elevation and number of cloudy days per year.

e. On the second graph on *Activity Sheet 3*, draw a line that best fits the data for (elevation, clear days).

f. Write an equation for the line you drew in part e. This equation expresses the relationship between elevation and number of clear days per year.

g. For what elevation will the expected number of cloudy days and the number of clear days be the same? Explain how you got your answer.

10. Find the intersection point for each system of equations both by graphing and by finding the solution algebraically.

a. $y = 12 + 2(x - 3)$
$y = 16 + 4(x - 5)$

b. $y = x$
$y = -5x - 15$

c. $y = 2x - 8$
$y = 0.5x + 4$

d. $y = 12 + 2(x - 3)$
$y = 12 + 4(x - 3)$

11. When you solve a system of equations algebraically, what are you finding?

12. Below is a list of selected United States cities, their latitudes, and their average January high temperatures and average July low temperatures in degrees Fahrenheit (°F).

City	Latitude (°N)	January (°F)	July (°F)
Atlanta, GA	33	51	69
Baltimore, MD	39	41	67
Jackson, MI	30	57	68
Bismarck, ND	46	18	56
Boise, ID	43	37	59
Cleveland, OH	40	33	61
Chicago, IL	41	29	63
Detroit, MI	42	31	61
Houston, TX	29	62	73
Key West, FL	27	72	80
New York, NY	40	37	69
Oklahoma City, OK	35	47	77
Portland, ME	44	31	57
Portland, OR	44	44	56
Las Vegas, NV	36	55	76
Los Angeles, CA	34	55	76
Tacoma, WA	47	44	54

Source: *World Almanac and Book of Facts,* 1992

a. What is *latitude?* If you don't know, refer to a dictionary for the definition.

b. Make a scatter plot of (latitude, average January high temperature). On the same graph, plot (latitude, average July low temperatures). Look at the plot, and then decide if the following statement is true: "The greater the degree of latitude, the warmer the city is in January and the colder in July."

c. Draw lines that you think best fit the data for (latitude, January temperature) and (latitude, July temperature). Extend each line and estimate their point of intersection from the graph.

d. Write an equation for each line.

e. For each 10° that the latitude increases, how does the average July low temperature change?

f. Use algebra to find the point of intersection of the two lines.

g. What does the point of intersection tell you about the relationship between temperature and latitude?

13. In 1975 there were 75 Division 1A field-goal kickers in football who kicked soccer style, while in 1990 there were 135. In 1975 there were 120 field-goal kickers who kicked straight on, and in 1990 there were only 2. When do you think the number of soccer-style kickers overtook the number of straight-on kickers? Explain how you made your decision.

14. The table below gives the estimated circulation figures for the magazines *Time, Newsweek,* and *U.S. News & World Report* in 1985 and 1991.

Magazine	1985	1991
Time	4.7 million	4.1 million
Newsweek	3.1	3.2
U.S. News & World Report	2.0	2.2

Source: *Magazine Publishers of America*

a. Assume that the rate of change for the three magazines is constant. Find the rate of change for each magazine.

b. Will the circulation for *Newsweek* overtake that of *Time?* If so, when? Create equations from the information to help you answer the question.

c. Answer the same question about *U.S. News & World Report* and *Newsweek.*

d. Do you think it is safe to assume the rate of change is constant? What are some factors that might affect this assumption?

15. The following tables contain the times for the top female and male runners in the Olympic events referred to in the Introductory Activity of this unit. In groups, select an event and plot the male and female times over the given years.

a. Draw a line to represent the male data and a line to represent the female data.

b. Write an equation for each line.

c. On page 3, the authors stated that women have been improving about twice as quickly as men. Do you agree with this statement? Explain your reasoning.

d. Will the top male and female runners be performing equally well in each event between the years 2015 and 2055? Show plots and use algebra to justify your claim.

Times for the Olympic 200-Meter Run (seconds)

	Male		Female	
1900	Walter Tewksbury, US	22.2s		
1904	Archie Hahn, US	21.6s		
1908	Robert Kerr, Canada	22.6s		
1912	Ralph Craig, US	21.7s		
1920	Allan Woodring, US	22.0s		
1924	Jackson Scholz, US	21.6s		
1928	Percy Williams, Canada	21.8s		
1932	Eddie Tolan, US	21.2s		
1936	Jesse Owens, US	20.7s		
1948	Mel Patton, US	21.1s	Francina Blankers-Koen, Netherlands	24.4s
1952	Andrew Stanfield, US	20.7s	Marjorie Jackson, Australia	23.7s
1956	Bobby Morrow, US	20.6s	Betty Cuthbert, Australia	23.4s
1960	Livio Berruti, Italy	20.5s	Wilma Rudolph, US	24.0s
1964	Harry Car, US	20.3s	Edith McGuire, US	23.0s
1968	Tommie Smith, US	19.83s	Irena Szewinska, Poland	22.5s
1972	Valeri Borzov, USSR	20.00s	Renate Stecher, E. Germany	22.40s
1976	Donald Quarrie, Jamaica	20.23s	Barbel Eckert, E. Germany	22.37s
1980	Pietro Mennea, Italy	20.19s	Barbel Wockel, E. Germany	22.03s
1984	Carl Lewis, US	19.80s	Valerie Brisco-Hooks, US	21.81s
1988	Joe Deloach, US	19.75s	Florence Griffith-Joyner, US	21.34s
1992	Mike Marsh, US	20.01s	Gwinn Torrence, US	21.81s
1996	Michael Johnson, US	19.32s	Marie-Jose Perec, France	22.12s

Times for the Olympic 400-Meter Run (seconds)

	Male		Female	
1896	Thomas Burke, US	54.2s		
1900	Maxey Long, US	49.4s		
1904	Harry Hillman, US	49.2s		
1908	Wyndham Haiswelle, GB	50.0s		
1912	Charles Reidpath, US	48.2s		
1920	Bevil Rudd, S. Africa	49.6s		
1924	Eric Liddell, GB	47.6s		
1928	Ray Barbuti, US	47.8s		
1932	William Carr, US	46.2s		
1936	Archie Williams, US	46.5s		
1948	Arthur Wint, Jamaica, BWI	46.2s		
1952	George Rhoden, Jamaica, BWI	45.9s		
1956	Charles Jenkins, US	46.7s		
1960	Otis Davis, US	44.9s		
1964	Michael Larrabee, US	45.1s	Betty Cuthbert, Australia	52.0s
1968	Lee Evans, US	43.86s	Colette Besson, France	52.0s
1972	Vincent Matthews, US	44.66s	Monika Zehrt, E. Germany	51.08s
1976	Alberto Juantorena, Cuba	44.26s	Irena Szewinska, Poland	49.29s
1980	Viktor Markin, USSR	44.60s	Marita Koch, E. Germany	48.88s
1984	Alonzo Babers, US	44.27s	Valerie Brisco-Hooks, US	48.83s
1988	Steven Lewis, US	43.87s	Olga Bryzgina, USSR	48.65s
1992	Quincy Watts, US	43.50s	Marie-Jose Perec, France	48.83s
1996	Michael Johnson, US	43.49s	Marie-Jose Perec, France	48.25s

Times for the Olympic 800-Meter Run (seconds)

	Male		Female	
1896	Edwin Flack, Australia	2m11s		
1900	Alfred Tysoe, GB	2m1.2s		
1904	James Lightbody, US	1m56s		
1908	Mel Sheppard, US	1m52.8s		
1912	James Meredith, US	1m51.9s		
1920	Albert Hill, GB	1m53.4s		
1924	Douglas Lowe, GB	1m52.4s		
1928	Douglas Lowe, GB	1m51.8s	Lina Radke, Germany	2m16.8s
1932	Thomas Hampson, GB	1m49.8s		
1936	John Woodruff, US	1m52.9s		
1948	Mal Whitfield, US	1m49.2s		
1952	Mal Whitfield, US	1m49.2s		
1956	Thomas Courtney, US	1m47.7s		
1960	Peter Snell, New Zealand	1m46.3s	Lyudmila Shevtsova, USSR	2m4.3s
1964	Peter Snell, New Zealand	1m45.1s	Ann Packer, GB	2m1.1s
1968	Ralph Doubell, Australia	1m44.3s	Madeline Manning, US	2m0.9s
1972	Dave Wottle, US	1m45.9s	Hildegard Falck, W. Germany	1m58.55s
1976	Alberto Juantorena, Cuba	1m43.50s	Tatyana Kazankina, USSR	1m54.94s
1980	Steve Ovett, GB	1m45.40s	Nadezhda Olizayrenko, USSR	1m53.42s
1984	Joaquim Cruz, Brazil	1m43.00s	Doina Melinte, Romania	1m57.6s
1988	Paul Ereng, Kenya	1m43.45s	Sigrun Wodars, E. Germany	1m56.10s
1992	William Tanui, Kenya	1m43.66s	Ellen Van Langen, Netherlands	1m55.54s
1996	Vebjoern Rodal, Norway	1m42.58s	Svetlana Masterkova, Russia	1m55.54s

Times for the Olympic 1500-Meter Run (minutes, seconds)

	Male		Female	
1896	Edwin Flack, Australia	4m33.2s		
1900	Charles Bennet, GB	4m6.2s		
1904	James Lightbody, US	4m5.4s		
1908	Mel Sheppard, US	4m3.4s		
1912	Arnold Jackson, GB	3m56.8s		
1920	Albert Hill, GB	4m1.8s		
1924	Paavo Nurmi, Finland	3m53.6s		
1928	Harry Larva, Finland	3m53.2s		
1932	Luigi Beccali, Italy	3m51.2s		
1936	Jack Lovelock, New Zealand	3m47.8s		
1948	Henri Eriksson, Sweden	3m49.8s		
1952	Joseph Barthel, Luxembourg	3m45.2s		
1956	Ron Delany, Ireland	3m41.2s		
1960	Herb Elliott, Australia	3m35.6s		
1964	Peter Snell, New Zealand	3m38.1s		
1968	Kipchoge Keino, Kenya	3m34.9s		
1972	Pekka Vasala, Finland	3m36.3s	Lyudmila Bragina, USSR	4m01.4s
1976	John Walker, New Zealand	3m39.17s	Tatyana Kazankina, USSR	4m05.48s
1980	Sebastian Coe, GB	3m38.4s	Tatyana Kazankina, USSR	3m56.6s
1984	Sebastian Coe, GB	3m32.53s	Gabriella Dorio, Italy	4m03.25s
1988	Peter Rono, Kenya	3m35.96s	Paula Ivan, Romania	3m53.96s
1992	Fermin Casho Ruiz, Spain	3m40.12s	Hassiba Boulmerka, Algeria	3m55.30s
1996	Noureddine Morceli, Algeria	3m35.78s	Svetlana Masterkova, Russia	4m00.83s

16. The researchers Brian Whipp and Susan Ward wrote,
"None of the current women's world-record holders at
these events could even meet the men's qualifying standard
to compete in the 1992 Olympic games. However, it is the
rates of improvement that are so strikingly different—the
gap is progressively closing."

a. What numerical information were Whipp and Ward
using to make their claim?

b. Other researchers doubted the projections. List some of
the reasons they might have given for their skepticism.

Lines with the Same Slope

Do two lines always have to intersect? How can you tell?

**If two lines do not intersect, what characteristics will their graphs have? What is true about their equations?**

In this lesson, you will learn about parallel lines and their equations. You will also see they can help you answer the questions above.

INVESTIGATE

The scatter plot below shows the percent of homes that are wired for cable and the percent of homes subscribing to a cable TV service for the years 1980 to 1990.

Cable TV

□ Percent wired for cable
• Percent subscribing to cable
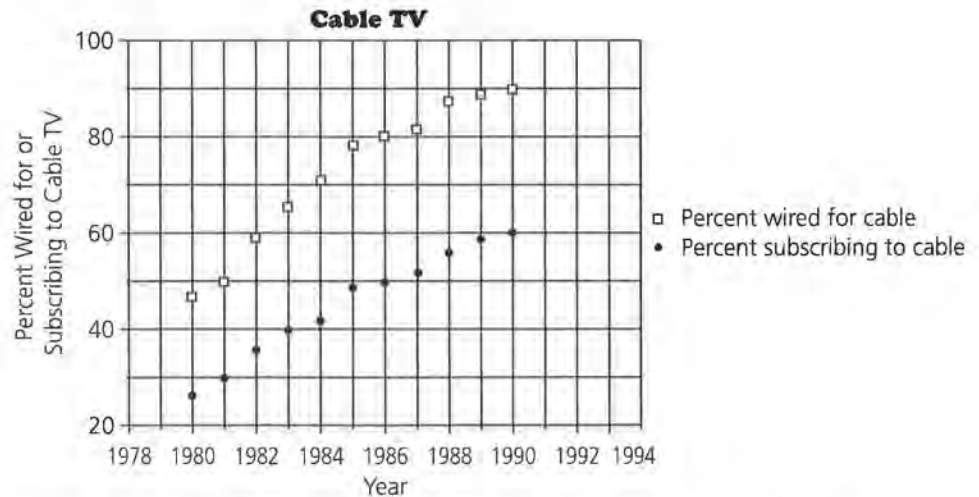
Source: "Cable TV." Copyright 1991 by Consumers Union of U.S., Inc., Yonkers, NY 10703-1057
Reprinted by permission from *Consumer Reports*, September, 1991

Discussion and Practice

1. Refer to the graph on page 16.

   a. What trends do you observe in the graph?

   b. Use the graph to complete a table like the following.

Year	Percent Wired	Percent Subscribing	Difference Between Percent Wired and Percent Subscribing
1980	——————	——————	——————
1981	——————	——————	——————
1982	——————	——————	——————
1983	——————	——————	——————
1984	——————	——————	——————
1985	——————	——————	——————
1986	——————	——————	——————
1987	——————	——————	——————
1988	——————	——————	——————
1989	——————	——————	——————
1990	——————	——————	——————

   c. Describe how the difference between the percent of homes wired for cable and the percent of homes subscribing to cable has changed over the years from 1980 to 1990.

2. Use the first graph on *Activity Sheet 4* for this problem.

   a. Draw a line for the percent wired for cable as a function of the year and one for the percent subscribing to cable as a function of the year.

   b. Estimate when the percent of homes wired for cable will be the same as the percent subscribing to cable.

   c. It appears that the two lines have approximately the same slope. What does this tell you about the two lines?

   d. For each of the lines you drew, find the rate of change, or *slope*. Are the lines parallel? Why or why not?

3. The scatter plot below shows the average weight of American men by height and age for men 20–24 years old and 60–69 years old.

Average Height and Weight of American Men by Age
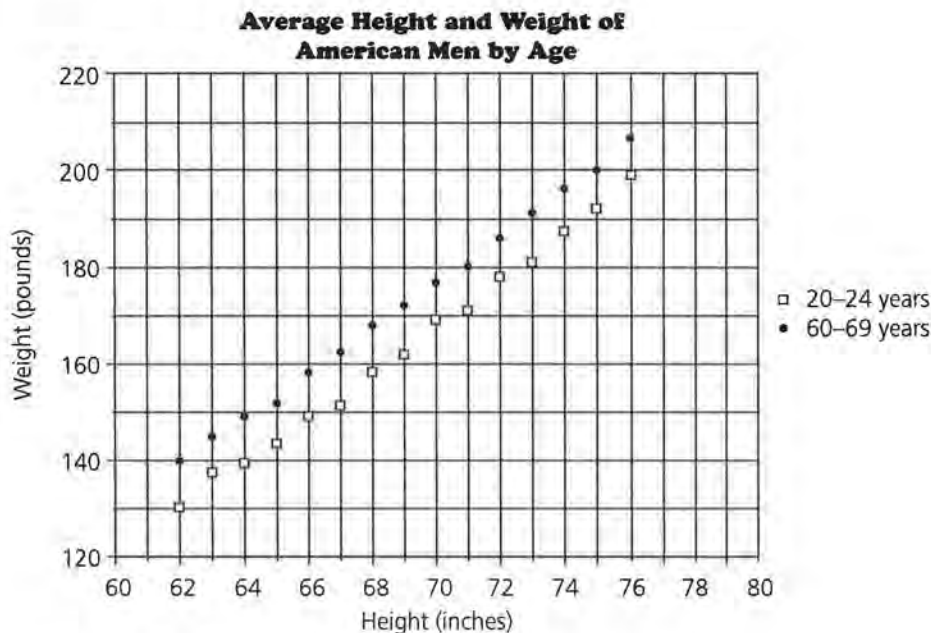
Source: *The World Almanac and Book of Facts,* 1992

a. For each height, find the average difference in weight between a 20–24-year-old and a 60–69-year-old.

b. For each 1-inch increase in height, what is the increase in weight for a 20–24-year-old and a 60–69-year-old?

c. What do the values you found in part b tell you about the rate of change in weight for the two age groups?

4. Sam wanted to find where the two lines $y_1 = 2x - 10$ and $y_2 = 15 + 2(x - 4)$ intersect. His solution is shown here.

$$y_1 = 2x - 10$$
$$y_2 = 15 + 2(x - 4)$$
$$2x - 10 = 15 + 2(x - 4)$$
$$2x - 10 = 15 + 2x - 8$$
$$2x - 10 = 7 + 2x$$
$$\underline{-2x \qquad\quad -2x}$$
$$-10 = 7$$

a. Did he make a mistake? If so, where?

b. Describe the graphs of the two lines. How does the graph help you understand the problem?

c. Can Sam conclude that $x = -10$ and $y = 7$? Explain.

Summary

If two lines have the same slope, they are parallel and will not have a point of intersection. If two equations have the same slope, you have to determine whether they represent the same line or parallel lines. You can do this by investigating the points that work in the equations or the graphs of the lines they represent. Or you can find the solution algebraically.

- If two distinct lines have the same slope, or rate of change, then they are parallel and have no points in common.

- If two distinct lines intersect, their slopes are different, and they have one point in common.

Practice and Applications

5. Make a conjecture about the relation between the graphs of the lines represented by the pairs of equations in each problem. Verify your conjecture in some way.

    a. $C = 250 + 5(x - 1980)$ **b.** $r = 18 + -2(x - 4)$
        $C = 120 + 5(x - 1965)$ $r = 18 + 2(x - 4)$

6. Below are the list prices in dollars for new cars for the years 1971–1997. On the same set of axes, make a plot of years and prices for the Honda Civic, the Toyota Celica, and the Ford Mustang. Then fit a line to each set of data.

Year	Honda Civic	Chevrolet Camaro	Toyota Celica	Ford Mustang	Mercury Cougar	BMW	Corvette
1971	1395	3790	2847	3783	4069	5845	6327
1973	2150	3829	3159	3723	4045	8230	7007
1975	2799	4739	3694	4906	6121	10,605	9424
1977	2849	5423	5252	4814	6225	14,840	11,508
1979	3649	6021	6904	5339	6423	20,185	12,550
1981	5199	8142	7974	7581	8762	24,605	16,141
1983	4899	9862	8824	8466	10,725	24,760	19,368
1985	6479	10,273	9549	8441	11,825	20,970	26,901
1987	7968	11,674	12,608	9948	14,062	24,070	28,874
1989	9140	13,199	13,528	11,145	15,903	25,620	32,445
1991	9405	13,454	14,658	11,873	16,890	26,700	33,410
1993	8730	15,379	15,983	12,847	17,833	32,205	36,230
1995	10,130	17,536	19,410	17,550	18,960	34,220	37,955
1997	12,449	18,930	20,825	18,525	19,150	35,060	38,160

Source: *Kelley Blue Book*

a. Are the prices for the Honda Civic, the Toyota Celica, and the Ford Mustang increasing at approximately the same rate?

b. How do the slopes of the lines compare?

c. If the rates of change stay approximately the same, predict the relation between the car prices in the future.

7. The scatter plot below shows the attendance (in millions of people) for Disney World and Universal Studios Florida.
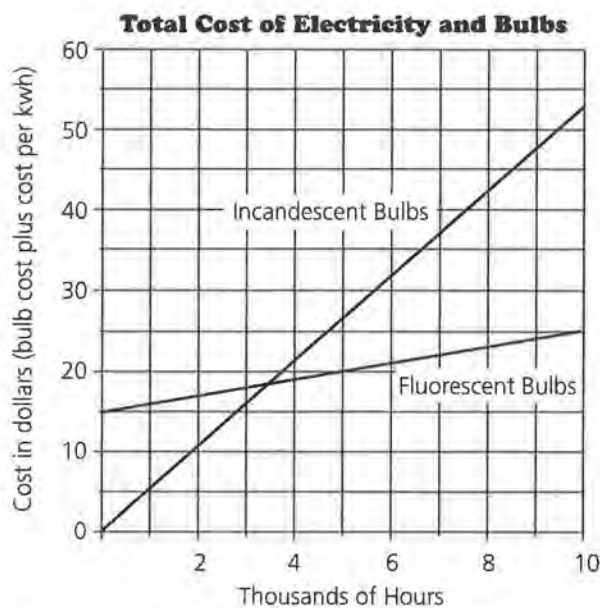
Attendance at Disney World and Universal Studios
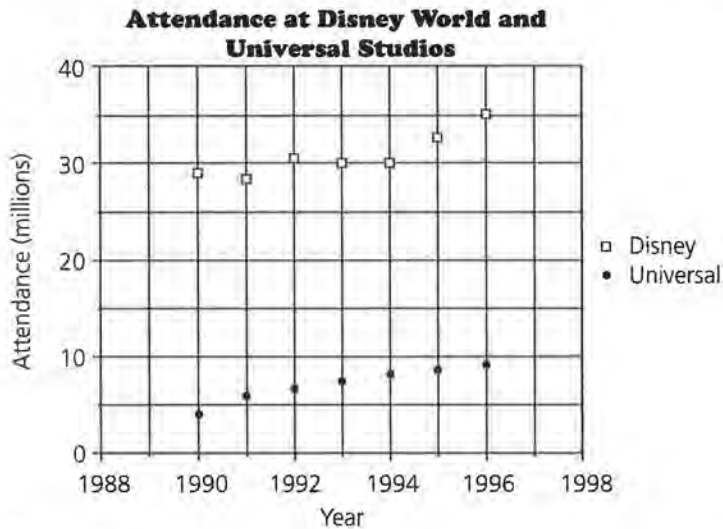
Source: *Amusement Business Magazine*

a. On the second graph on *Activity Sheet 4*, fit a line to each set of data. Will the attendance at Disney World and Universal Studios ever be the same if the trend continues? How did you decide?

b. Describe the slope for each line in terms of the data.

c. If the rate of change for each line remains the same, predict the attendance for Disney World and Universal Studios Florida in the year 2000.

Assessment for Unit I

1. Probably most of the light bulbs in your house are *incandescent* bulbs. These bulbs use electricity to superheat a filament. You may also have some *fluorescent* light bulbs. These bulbs use a small electric charge to ionize mercury vapor, which then gives off light. Below is a graph comparing the costs of incandescent light bulbs and fluorescent light bulbs. Incandescent bulbs cost about $1 each and fluorescent bulbs cost about $15 each. One thousand hours of wattage is denoted by "kwh."

Total Cost of Electricity and Bulbs

(graph: Cost in dollars (bulb cost plus cost per kwh) vs. Thousands of Hours, showing lines for Incandescent Bulbs and Fluorescent Bulbs)

Source: *Milwaukee Journal Sentinel*, February 4, 1996

a. Find the slope of each line. Describe the slope in terms of the data.

b. Write an equation for each line. Describe the *y*-intercept of each line in terms of the data.

c. Solve the system of equations. Then interpret your answer.

2. The graph below shows the costs of lumber at a local lumber store. The prices are for 2-inch-by-4-inch and 2-inch-by-8-inch boards of various lengths.

Cost of Construction Lumber

- □ 2-inch-by-4-inch
- • 2-inch-by-8-inch
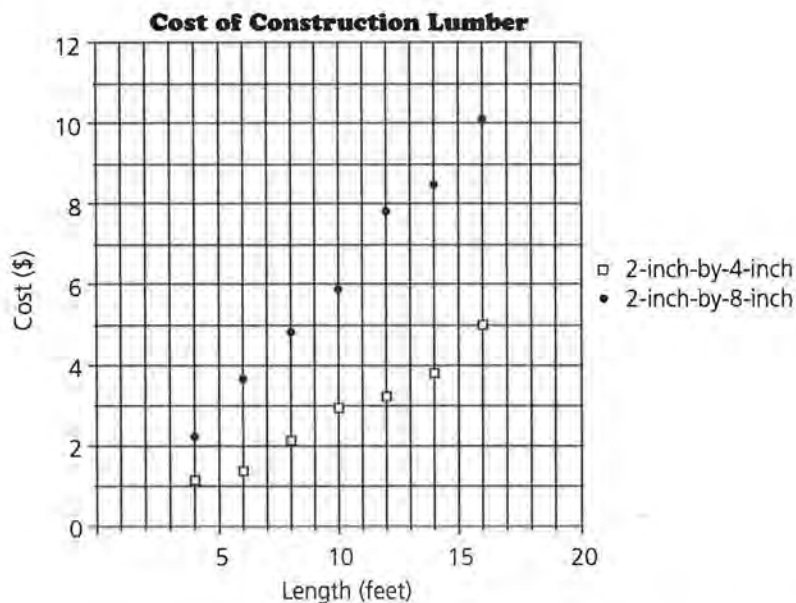
a. On the graph on *Activity Sheet 5,* draw a line for each set of data and find an equation for each line.

b. Describe the slope of each line in terms of the data.

c. Use your equation to predict the cost of a 2-inch-by-4-inch board that is 7 feet long.

d. Solve the system of equations. Interpret your answer.

3. The life expectancy of males born in certain years is given in the table below.

Year	Life Expectancy (years)		Year	Life Expectancy (years)	
	White Male	Black Male		White Male	Black Male
1920	54.4	45.5	1980	70.7	65.3
1930	59.7	47.3	1985	71.9	64.8
1940	62.1	51.5	1986	72.0	66.8
1950	66.5	59.1	1987	72.2	65.0
1960	67.4	61.1	1988	72.3	66.7
1965	67.6	61.1	1989	72.5	66.7
1970	68.0	61.3	1990	72.9	67.0
1975	69.5	63.7	1992	73.2	67.8

Source: *World Almanac and Book of Facts,* 1994

a. Plot the year and the life expectancy for white males and the year and life expectancy for black males and find equations for the lines that best fit the data.

b. Find the slope of each line and write a paragraph comparing the life expectancies of white and black males. Indicate what might be predicted for future life expectancies for each group.

4. Solve each system of equations.

a. $y = -2x + 5$

$y = 4x - 6$

b. $y = \frac{1}{2}x$

$y = -\frac{3}{4}x + 12$

c. $y = 6 + 0.35x$

$y = 0.2 + 3.5x$

Graphing Inequalities

Estimating the Number of Raisins

How many raisins do you suppose there are in a small box?

_____

How did you make your estimate?

_____

Do you think your estimate is greater than or less than the actual number of raisins in the box?

_____

In this unit, you will investigate data that are less than or greater than a given standard, such as the weight or number of raisins in a box of a given size.

EXPLORE

Each of you should have your own $\frac{1}{2}$-ounce box of raisins.

Look carefully at your box of raisins without opening it. Then write your estimate of the number of raisins contained in the box. Now open your box and count the raisins. How close were you to the actual number? Was your estimate high or low?

1. Collect from each student in class his or her estimate for the number of raisins and the actual number. Record the data in a table similar to the one below.

Student	Estimate	Actual Number
1	_____	_____
2	_____	_____
3	_____	_____
.	_____	_____
.	_____	_____
.	_____	_____

2. Make a scatter plot of the ordered pairs (estimate, actual number) for the class data.

3. Where will the point lie if your estimate is correct? Draw the line that represents 100% accurate estimates. Write an equation for the line.

4. What does it mean if a point is above this line?

5. Describe, in terms of estimating raisins, a point found below the line.

Shading a Region

How good were you at estimating the number of raisins in a $\frac{1}{2}$-ounce box?

How good do you think you are at estimating the number of calories in certain fast-food items?

When comparing prices between two stores, how can you tell whether one store is generally higher priced or lower priced than the other?

In this lesson, you will use inequalities to investigate these questions.

INVESTIGATE

Many fast-food chains base their advertising on the quality and prices of their food or on claims that their food is nutritious and tasty.

Discussion and Practice

A single hamburger at McDonald's has 270 calories. On page 30 is a list of some other items from McDonald's.

Item	Estimated Calories	Actual Calories
Small Bag of French Fries	———	———
Filet o' Fish	———	———
6 Chicken McNuggets	———	———
Chicken Sandwich	———	———
Big Mac	———	———
Small Chocolate Shake	———	———
Small Diet Coke	———	———
Apple Pie	———	———
Cheeseburger	———	———
Quarter Pounder	———	———

1. Copy the table above and estimate the number of calories to the nearest 10 calories in each item. Then your teacher will give you the correct number of calories to the nearest 10 calories.

2. To help you determine how well you estimated the number of calories, complete the following.

   a. Make a scatter plot of the ordered pairs (estimated calories, actual calories).

   b. Where would the point lie if your estimate were correct? Draw the line that represents 100% accurate estimates. Write an equation for that line.

   c. What does it mean if a point is above this line? Shade the region above the line. Make a list of five ordered pairs in the shaded region. What is true about the relationship between the actual calories (y-value) and the estimated calories (x-value) in each ordered pair you wrote?

   d. A point in the region shaded above the line can be described with the inequality $y > x$. Verify that the ordered pairs you listed above satisfy this inequality.

   e. Describe a point found in the region below the line in words. Write a symbolic description with an inequality.

3. Listed below are randomly selected items and their prices from two grocery stores.

Item	Store A	Store B
Soda (12-pack)	$3.48	$3.68
Head of lettuce	$0.98	$1.29
Salad dressing (24 oz)	$2.38	$2.69
Punch (liter)	$0.88	$0.98
Catsup (28 oz)	$1.58	$2.25
Peanut butter (28 oz)	$5.38	$5.68
1 package of Kool Aid	$0.19	$0.22
1 box of Jell-O	$0.47	$0.48
Can of corn (11oz)	$0.49	$0.59
Box of rice	$1.28	$1.35
Pizza sauce (32 oz)	$1.88	$1.98
Baked beans (21 oz)	$0.78	$1.09
Mushroom soup (10.5 oz)	$0.49	$0.55
Breakfast cereal (21.3 oz)	$2.18	$2.39
Coffee (39 oz)	$5.98	$6.55

a. Make a scatter plot of the data with the prices from Store A on the x-axis and prices from Store B on the y-axis.

b. Where will the point for items lie if the prices at the two stores are the same? Draw a line representing these prices.

c. Describe the region containing most of the ordered pairs.

d. Which store do you think has the cheaper prices? Why?

4. Refer to the graph below.

a. Write an equation for the line drawn on the graph.

b. Use x and y in an inequality that describes the shaded region.
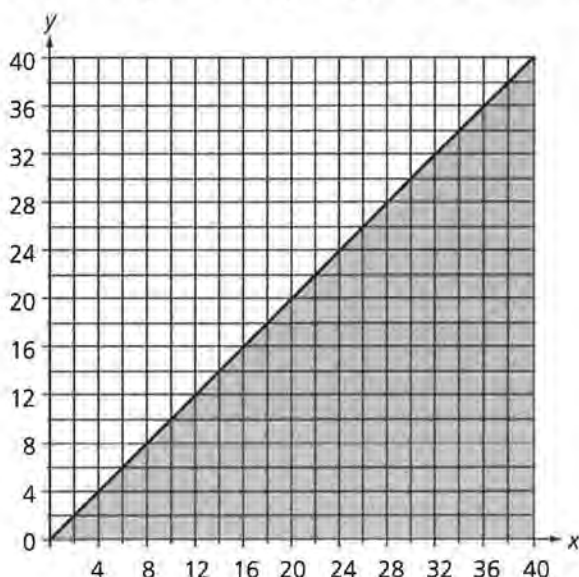
Summary

To compare two quantities with the same units, you can draw the line $y = x$. This line divides the plane into two regions. One region contains points where y is greater than x, and the other region contains points where y is less than x. When graphing the inequality $y > x$ or the inequality $y < x$, the line $y = x$ is usually shown as a dashed line to indicate that the points on the line do not satisfy the inequality.

Practice and Applications

5. In the graph that follows, the prices from Store C are plotted against the prices from store B.
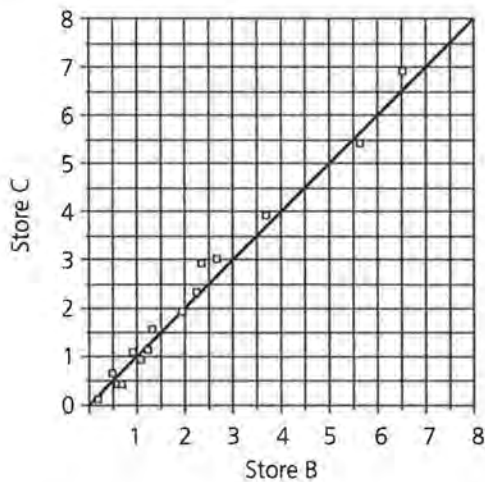
   a. Identify a point where Store B has lower prices than Store C does. Describe the costs for that item.

   b. Identify a point where Store B is more expensive than Store C. Describe the costs for that item.

   c. Make a sketch of the plot and shade in the area where Store B has lower prices than Store C. In general, which store has lower prices? How can you tell?

6. The following data are the percents of teens and children who watched America's favorite prime time network television programs in 1992–93, according to the Nielsen Media Research.

Rank	Program	%Teens	%Children
1	60 Minutes	2.8	2.9
2	Roseanne	15.3	12.2
3	Home Improvement	13.4	11.8
4	Murphy Brown	6.6	4.9
5	Murder, She Wrote	2.6	2.4
6	Coach	10.4	8.4
7	NFL Monday Night Football	7.5	4.1
8	CBS Sunday Night Movie	3.9	2.6
9	Cheers	7.0	5.1
10	Full House	13.0	16.9
11	Northern Exposure	3.9	2.0
12	Rescue 911	6.4	7.2
12	20/20	4.6	4.8
14	CBS Tuesday Night Movie	5.9	4.2
15	Love and War	4.5	3.1
16	Fresh Prince of Bel Air	17.1	12.4
16	Hangin' With Mr. Cooper	13.2	14.0
16	Jackie Thomas Show	10.3	7.0
19	Evening Shade	3.6	3.7
20	Hearts Afire	3.8	3.6
20	Unsolved Mysteries	4.7	4.5
22	PrimeTIME LIVE	2.5	1.6
23	NBC Monday Night Movie	7.6	4.6
24	Dr. Quinn, Medicine Woman	5.3	5.0
25	Seinfeld	5.0	3.2
26	Blossom	17.1	11.5
26	48 Hours	3.0	2.0
28	ABC Sunday Night Movie	6.9	4.9
29	Matlock	2.5	2.2
30	The Simpsons	15.5	16.2

Source: *The World Almanac and Book of Facts,* 1994

A scatter plot of the percent of children and the percent of teens who watched the top thirty programs is given below and on *Activity Sheet 6*.

Percent of Teens and Children Watching Top 30 TV Shows in 1993
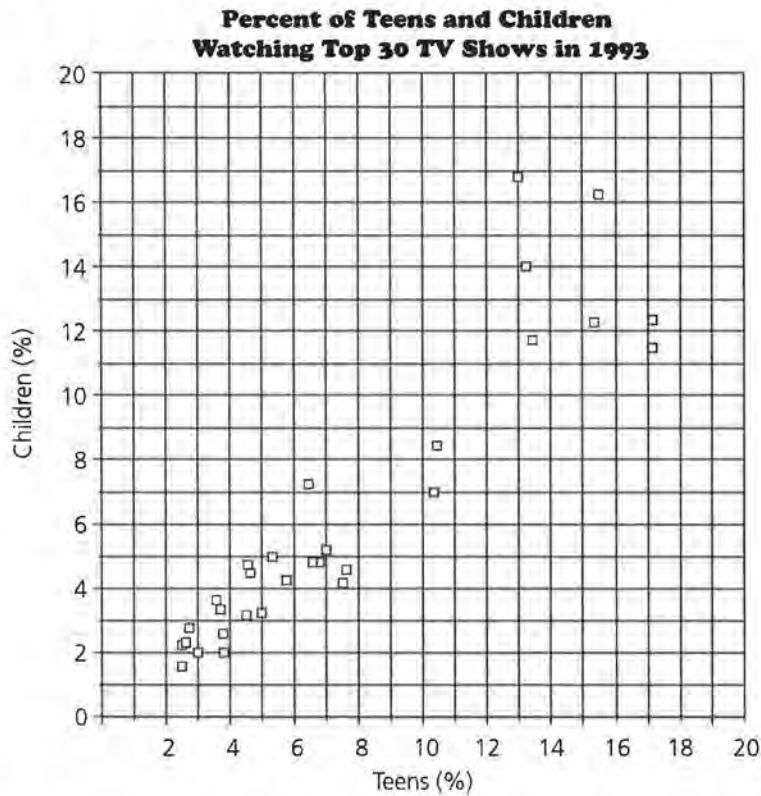
a. On the graph on *Activity Sheet 6*, draw a line to help determine where the percent of teens watching is greater than the percent of children watching. Indicate the region by shading on the plot.

b. Write an inequality to describe this region.

c. For which program is the difference between the percent of children and the percent of teens who watch the greatest? How can you tell from the graph?

7. Consider the equations $y = x$ and $y = -x$.

a. Describe the graph of $y < x$ in words and with a picture.

b. Graph the line $y = -x$. Shade the region that describes the set of all points that satisfy the inequality $y < -x$.

c. Graph the inequalities $y > x$ and $y \geq x$. How can you show that the two inequalities are different?

8. Graph each inequality.

a. $x < 5$ b. $y \geq -3$ c. $x \geq 2$ d. $y < 0$

Graphing Inequalities

How old do you have to be to drive a car?

How many points do you have to earn in track to qualify for a letter?

Many situations involve inequalities. You will get a grade of B if you have more than a certain number of points; renting a car can be more economical if you drive more than a certain number of miles; selecting the best telephone company and package depends on the range of calls you make per month. In this lesson, you will learn to find a region that contains a set of points that satisfy a statement of inequality.

OBJECTIVE

Graph and interpret a linear inequality.

INVESTIGATE

Each year, students are encouraged to participate in the National Physical-Fitness Tests to demonstrate how physically fit they are and to qualify for a national award. The qualifying standards for boys for the Presidential Award are given below.

Physical-Fitness Qualifying Standards for Boys, The Presidential Award

| Ages	Curl-Ups (1 minute)	Shuttle Run (seconds)	Sit and Reach (inches)	1-Mile Run (min:sec)	Pull-Ups
6	33	12.1	3.5	10:15	2
7	36	11.5	3.5	9:22	4
8	40	11.1	3.0	8:48	5
9	41	10.9	3.0	8:31	5
10	45	10.3	4.0	7:57	6
11	47	10.0	4.0	7:32	6
12	50	9.8	4.0	7:11	7
13	53	9.5	3.5	6:50	7
14	56	9.1	4.5	6:26	10
15	57	9.0	5.0	6:20	11
16	56	8.7	6.0	6:08	11
17	55	8.7	7.0	6:06	13

Source: *The Orlando Sentinel*, November 17, 1991

Discussion and Practice

1. Below is a scatter plot of the physical-fitness qualifying standards for boys aged 6 to 17 to earn a Presidential Award in the 1-mile run.

Presidential Award in 1-Mile Run
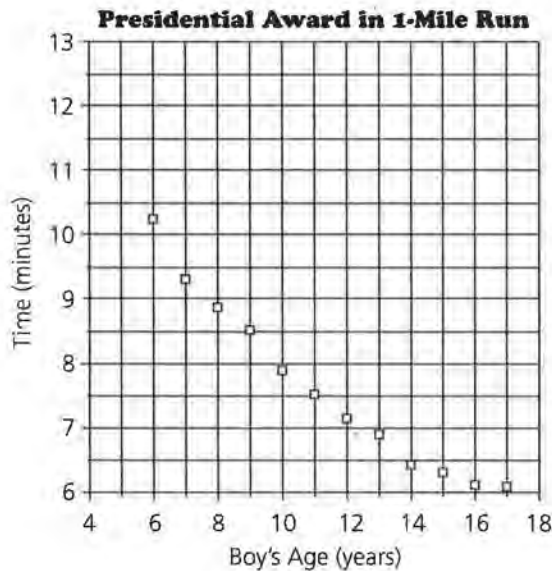
a. What trend do you see in the plot?

b. Would the line $y = x$ make sense in this plot? Explain.

c. Suppose that the equation of a line that seems to fit the data is $y = -0.4x + 11.9$. Graph this line on the graph on *Activity Sheet* 7. Describe how you graphed the line.

d. How can you determine the qualifying standard for an 8-year-old boy using the equation? How does this value compare with the time listed in the table?

2. If an 8-year-old boy ran the mile in 8 minutes and 30 seconds (8.5 min), he would have exceeded the Presidential standards. That is, he would have run the mile in a time less than or equal to the established standard.

a. On the graph for Problem 1c, plot the point representing the boy's time. Shade the region of the graph that contains this point. What does this region represent?

b. List three ordered pairs that lie in the shaded region.

c. Write an inequality for the shaded region. Show how each ordered pair you listed in part b satisfies the inequality.

d. If a 15-year-old boy ran the mile in 7 minutes and 10 seconds, would he qualify for a Presidential Award? How can you use your graph to determine your answer?

3. Shown below is the graph of boy's age in years and time in seconds necessary to earn a Presidential Award in the shuttle run.
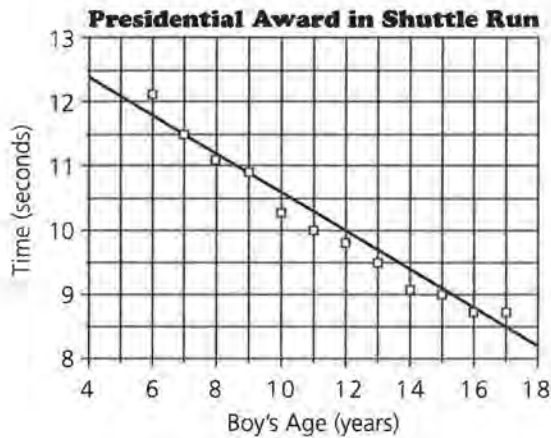
a. Write an equation for the line drawn on the plot.

b. Sketch the plot and the line, and then shade the region that represents the times for boys who would not earn an award for the shuttle run.

c. Write an inequality that represents the shaded region drawn in part b.

Summary

- If two variables have the same unit, you can plot them and use the graph of $y = x$ to determine whether the x-variable is greater than or less than the y-variable. The region above the graph for $y = x$ can be represented by the inequality $y > x$. The regions below the graph for $y = x$ can be represented by the inequality $y < x$.

- If two variables do not have the same unit but seem to have a linear relationship, you can draw a line summarizing the relationship. This line can be used to determine an inequality for the relationship. The area on one side of the line is greater than the relationship; the area on the other side is less. To find which side is greater than the relationship, you must investigate what the ordered pairs on each side represent and whether or not they satisfy the inequality. The inequality $y > ax + b$ represents the region above the graph of $y = ax + b$, and the inequality $y < ax + b$ represents the region below the graph of $y = ax + b$.

Practice and Applications

4. If the equation of a line is $y = sx + d$, describe each region.

    a. $y > sx + d$

    b. $y < sx + d$

    c. $y \leq sx + d$

5. The data for the physical fitness qualifying standards for the Presidential Award for girls are in the table below.

Physical-Fitness Qualifying Standards for Girls, The Presidential Award

Ages	Curl-Ups (1 minute)	Shuttle Run (seconds)	Sit and Reach (inches)	1-Mile Run (min:sec)	Pull-Ups
6	32	12.4	5.5	11:20	2
7	34	12.1	5.0	10:36	2
8	38	11.8	4.5	10:02	2
9	39	11.1	5.5	9:30	2
10	40	10.8	6.0	9:19	3
11	42	10.5	6.5	9:02	3
12	45	10.4	7.0	8:23	2
13	46	10.2	7.0	8:13	2
14	47	10.1	8.0	7:59	2
15	48	10.0	8.0	9:08	2
16	45	10.1	9.0	8:23	1
17	44	10.0	8.0	8:15	1

Source: *The Orlando Sentinel,* November 17, 1991

   a. Look carefully at the data for age and time for the shuttle run. Describe the relationship between age and time.

   b. Make a scatter plot of the girls' Presidential Award qualifying times for the shuttle run.

   c. Draw a line to fit the data. Find an equation for the line.

   d. Write an inequality that represents the region that describes those girls who would qualify for an award.

6. Compare the times for boys and the times for girls to receive a Presidential Award for the shuttle run. What are your conclusions?

7. Walking is one of the most effective ways to exercise. Here is a basic 12-week plan to get started on a walking program. Walk three days a week for the amount of time shown on the chart. At the end of 12 weeks, maintain the time shown for the twelfth week.

Minutes per Day Walking

Week	Day 1	Day 2	Day 3
1	10	10	10
2	12	12	16
3	15	15	20
4	15	20	25
5	20	25	35
6	30	25	45
7	35	35	50
8	40	40	60
9	45	45	60
10	45	45	70
11	45	45	80
12	45	45	90

Source: *Men's Health Magazine*

   a. Make a scatter plot of (week number, minutes walking) for day 3. Draw a line to summarize the relationship. Describe any trends that you see in the graph.

   b. Find an equation for the line. What relationship does the line describe?

   c. What does the region below the line represent? Write an inequality that describes this region.

8. Graph each inequality.

   a. $y \geq 2x - 3$ **b.** $y < \frac{1}{2}x + 4$

   c. $y \leq 5 - 0.5(x - 2)$ **d.** $y > 30 + 4(x - 2)$

9. Write an inequality to describe each graph.
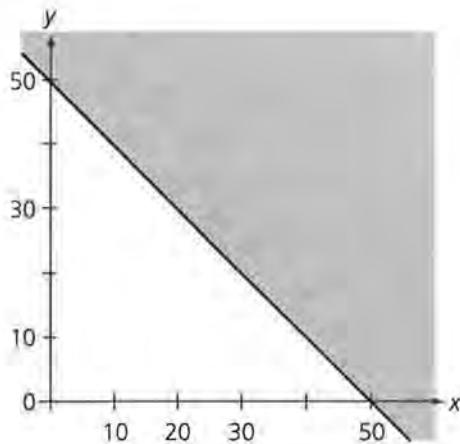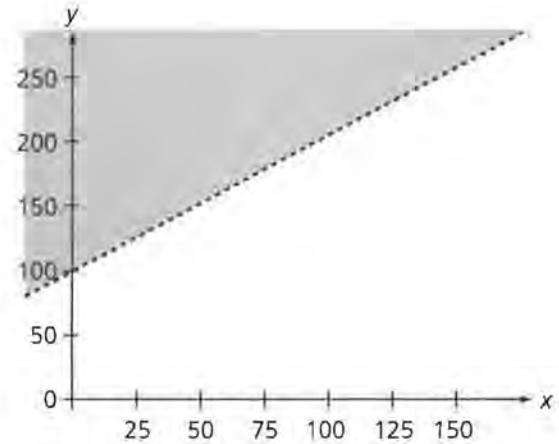
a.

b.

Assessment for Unit II

1. Age groups in the United States can be ranked according to their percent of the total population. The 1970 and 1995 ranks for each age group used in the United States Census are given in this table.

Age Group	1970 Rank	1995 Rank
5 and younger	4	4
5–9	2	5
10–14	1	7
15–19	3	9
20–24	5	10
25–29	6	6
30–34	10	2
35–39	11	1
40–44	9	3
45–49	7	11
50–54	12	12
55–59	13	14
60–64	14	15
65–74	8	8
75–84	15	13
85 and older	16	16

Source: *Statistical Abstract of the United States*, 1996

a. Make a scatter plot of (1970 rank, 1995 rank).

b. Draw the line showing all the points where the rank in 1970 is the same as in 1995. Write an equation for the line.

c. Where on the scatter plot are the age groups whose rank in 1995 is higher than the rank in 1970? "Higher" means that the rank moves closer to 1. How many of the age groups are there in this region?

d. Where on the scatter plot are the age groups whose rank in 1995 is lower than the rank in 1970? "Lower" means that the rank moves toward closer to 16. How many of the age groups are there in this region?

e. Which age group had the greatest change in rank? Describe this change.

2. Listed below are the 15 highest-ranked national universities among 204 schools that are research-oriented. The ranks were determined by the *U.S. News and World Report* magazine. The chart gives the enrollment and the number of faculty for each school for the 1992–93 school year.

Rank	College	Enrollment	Faculty
1	Harvard University	18,273	2,278
2	Princeton University	6,438	935
3	Yale University	11,129	712
4	Massachusetts Institute of Technology	9,798	975
5	California Institute of Technology	2,009	270
6	Stanford University	13,893	1,408
7	Duke University	11,426	2,304
8	Dartmouth College	5,475	468
9	University of Chicago	10,231	1,843
10	Cornell University	18,450	1,593
11	Columbia University School of Engineering and Applied Sciences	1,832	101
12	Brown University	7,593	663
13	Northwestern University	12,032	1,079
14	Rice University	4,033	542
15	Johns Hopkins University	4,613	473

Source: *US News and World Report, 1994 College Guide*

a. Would you expect to see some relation between the number of faculty members and the enrollment? Why or why not?

b. Make a scatter plot of (enrollment, number of faculty). Describe any pattern you see.

c. Draw a line that seems to fit the data. Write an equation of this line that relates the number of faculty members to the enrollment.

d. Shade the region where the number of faculty members is less than expected. Write an inequality to represent this region.

e. The University of Texas at Austin has an enrollment of 49,253 and 2,358 faculty members. Is the relationship better or worse than you expected? Tell how you made your decision.
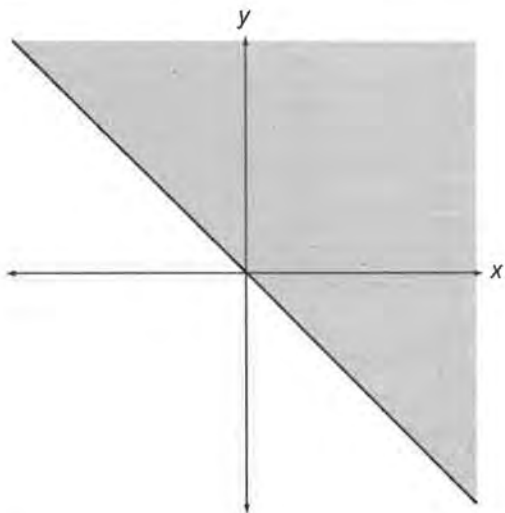
3. Graph each inequality
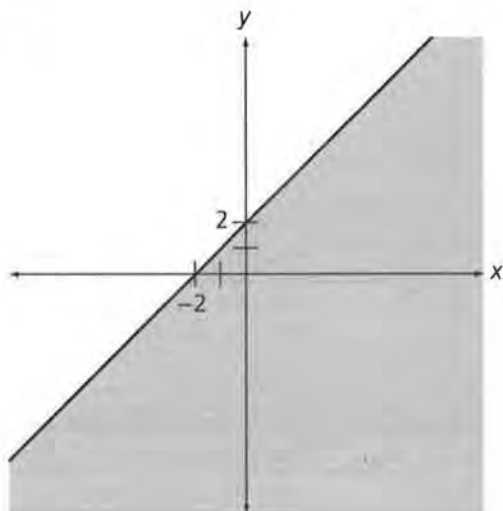
   a. $y \le x$

   b. $y \ge x$

   c. $y \le 2x + 6$

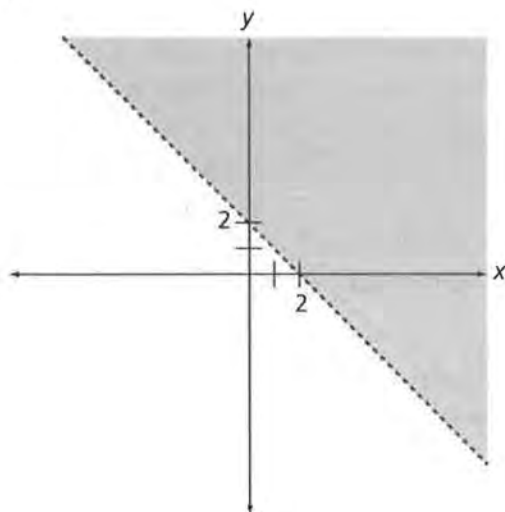   d. $y \ge -2x - 7$

4. Write an inequality to describe each graph.

   a.

   b.

   c.

Solving Systems of Inequalities

Fast Foods

Do you ever think about how many calories are in the food you eat?

Are you concerned about the amount of fat and salt in your diet?

Many people need to be aware of the amount of cholesterol and sodium in the food that they eat, because high cholesterol levels can lead to heart attacks and high sodium levels may be a cause of high blood pressure. If your daily dietary intake is about 2,500 calories a day, then your daily diet should include less than 300 mg of cholesterol and less than 2,400 mg of sodium.

EXPLORE

The amounts of cholesterol and sodium in a chicken sandwich and in 1 cup (8 fluid ounces) of 1% lowfat milk at a fast-food restaurant are listed below.

	Chicken Sandwich	1 Cup of 1% Milk
Cholesterol (mg)	50	10
Sodium (mg)	820	115

1. Suppose you ate 2 chicken sandwiches and drank 2 cups of 1% milk.

   a. Would you stay under the dietary levels of 300 mg of cholesterol? Show how you determined your answer.

   b. Would you stay under the dietary levels of 2,400 mg of sodium? Show how you determined your answer.

2. If you ordered just chicken sandwiches and no milk,

    a. how many sandwiches could you eat and stay under the dietary levels of 300 mg of cholesterol?

    b. how many sandwiches could you eat and stay under the dietary levels of 2,400 mg of sodium?

3. If you just ordered just 1% milk and no chicken sandwiches,

    a. how many cups of milk could you drink and stay under the dietary levels of 300 mg of cholesterol?

    b. how many cups of milk could you drink and stay under the dietary levels of 2,400 mg of sodium?

4. Find an ordered pair that represents the *greatest* number of chicken sandwiches and cups of 1% milk that you could consume and stay under dietary levels for cholesterol and sodium. Do you think one person actually might eat the combination that you found?

Graphing Conditions

Do you watch professional basketball or other sports on television?

Who is your favorite player?

L ed by Michael Jordan, the Chicago Bulls won the 1996 NBA Championship. They defeated the Seattle Supersonics, led by Shawn Kemp, in six games. Jordan was voted the most valuable player by the media covering the game. If you had been allowed to vote for MVP, would you have voted for Jordan?

OBJECTIVE
Graph and interpret systems of inequalities in the form $x < a$ and $y < b$.

INVESTIGATE

The following tables contain the statistics for the 1996 Championship series. Only those players who played a total of at least 40 minutes in the series are listed.

1996 NBA Championship Series, Seattle

Player	Min	FG-A	FT-A	RB	AST	PF	PT
Kemp	242	49-89	42-49	86	13	28	140
Schrempf	238	35-80	21-24	39	16	18	98
Payton	274	40-90	19-26	44	44	22	108
Hawkins	230	25-55	24-26	28	6	21	80
Perkins	190	23-61	17-21	37	12	13	67
Wingate	43	5-10	4-4	3	0	13	15
McMillan	51	3-7	2-3	11	6	6	11
Brickowski	68	2-9	0-0	14	3	16	5

Min (minutes played), FG-A (field goals made-field goals attempted), FT-A (Free throws made-free throws attempted), RB (rebounds), AST (assists), PF (personal fouls), PT (total points)

Source: *Chicago Tribune*

1996 NBA Championship Series, Chicago

Player	Min	FG-A	FT-A	RB	AST	PF	PT
Pippen	248	34-99	17-24	69	32	22	94
Rodman	225	17-35	11-19	129	15	27	45
Longley	170	27-47	16-22	31	13	23	70
Harper	115	12-32	11-12	17	10	8	39
Jordan	252	51-123	56-67	32	20	17	164
Kukoc	176	30-71	8-10	41	21	14	78
Brown	49	6-12	2-4	3	5	2	17
Wedington	42	7-10	1-2	5	1	7	14

Min (minutes played), FG-A (field goals made-field goals attempted), FT-A (Free throws made-free throws attempted), RB (rebounds), AST (assists), PF (personal fouls), PT (total points)

Source: *Chicago Tribune*

Discussion and Practice

Most Valuable Player

Two categories that are important considerations in determining the MVP are total points and rebounds.

Use the first grid on *Activity Sheet 8* for Problems 1–7.

1. Make a scatter plot of the ordered pairs (total points, rebounds) of all the players on both teams.

2. What observations can you make from the scatter plot?

3. Do you think there is an association between total points and rebounds? Explain your answer.

4. Suppose you felt that the most valuable player should have scored at least 80 points in the six games. Draw a line on your scatter plot showing all of the ordered pairs whose *x*-coordinate, or total points, is 80.

5. The line that you have drawn separates the scatter plot into two regions. Shade the region that shows the location of all the ordered pairs for total points greater than or equal to 80. Identify the players that are in this region.

6. Suppose you also felt that the most valuable player is one that had 40 or more rebounds. Draw a line on your scatter plot showing all the ordered pairs whose *y*-coordinate, or total rebounds, is 40.

7. Shade the region that shows the location of all the ordered pairs with at least 40 rebounds. List all the players who had more than 40 rebounds.

Your scatter plot should be separated into four nonoverlapping regions.

1996 NBA Championship Series
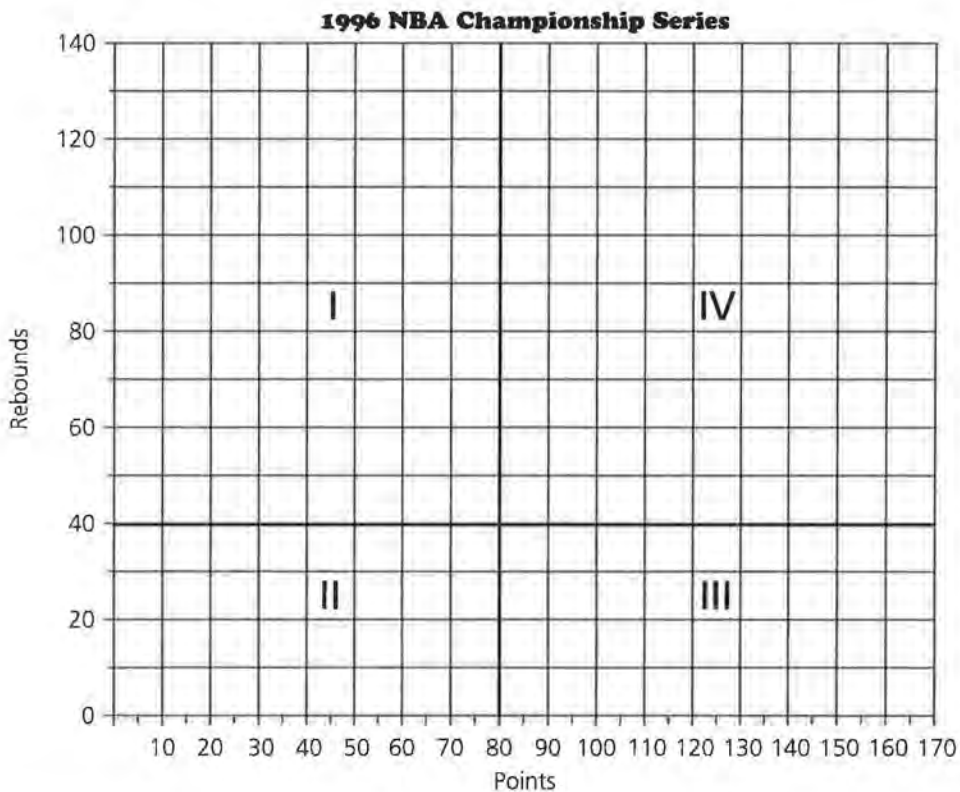
The points in region I are those whose x-coordinate is less than 80 and whose y-coordinate is greater than 40.

8. Describe the ordered pairs that are in each other region.

   a. Region II   **b.** Region III   **c.** Region IV

If you let t represent the total points and let r represent the total rebounds, then region II could be represented by the inequalities $t < 80$ and $r < 40$.

9. Write a pair of inequalities to represent each other region.

   a. Region I   **b.** Region III   **c.** Region IV

10. Consider the region that satisfies the conditions that the most valuable player should have at least 80 points and at least 40 rebounds.

   a. Which region satisfies these conditions?

   b. Identify the players in the region.

   c. Based on these two conditions, do you agree that Michael Jordan should have been voted the MVP of the Championship series? Why or why not?

Buying a Car

The scatter plot below shows the prices and the miles per gallon for a sample of 1995 cars. A car buyer decides that the car she will purchase must cost no more than $25,000 and get at least 25 miles per gallon.

Price and Mileage for Selected 1995 Cars

Mileage (mi/gal) vs *Price ($)*
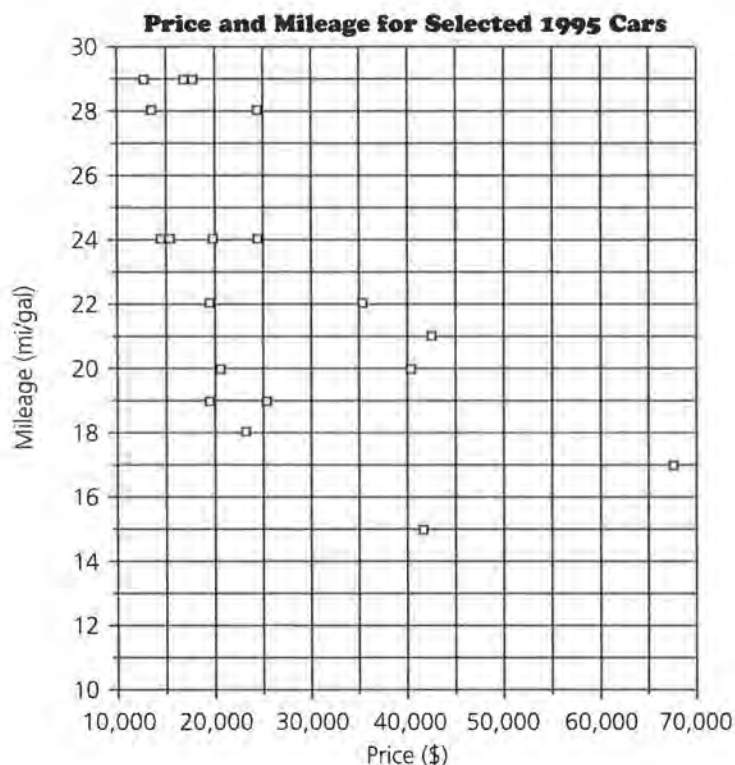
11. Use the second grid on *Activity Sheet 8* for this problem and the next.

   a. Shade the region of the graph that represents all the cars that cost no more than $25,000.

   b. Write an inequality to represent this region.

   c. Shade the region of the graph that represents all the cars that get at least 25 miles per gallon.

   d. Write an inequality to represent this region.

   e. How many cars satisfy both conditions?

12. Write a pair of inequalities and sketch a region that represents each set of conditions.

   a. Costs at least $25,000 and gets at least 25 miles per gallon

   b. Costs no more than $25,000 and gets no more than 25 miles per gallon

Summary

The graph of the the conditions
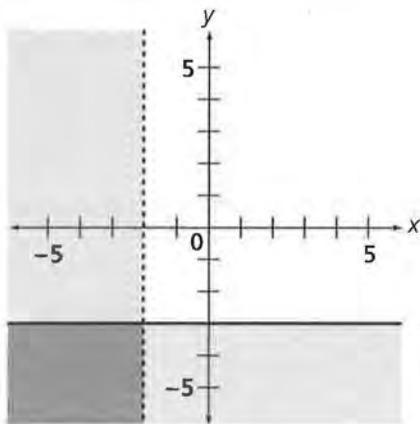$x \le a$ and $y \le b$ is the doubly-
shaded region shown at the right.
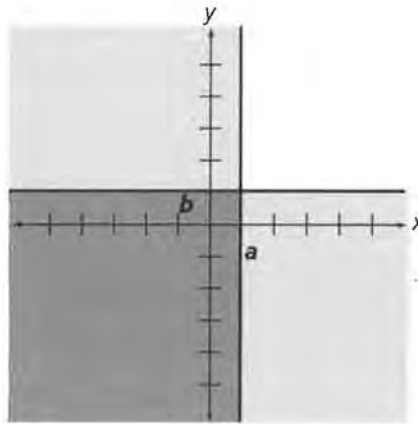
Practice and Applications

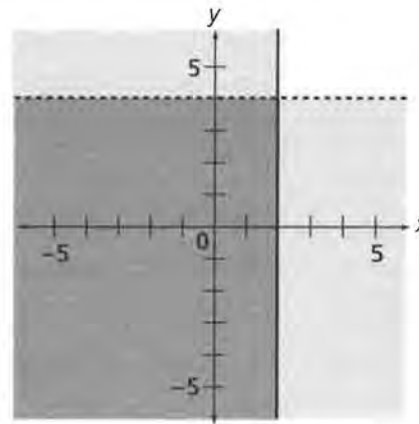**13.** Graph each pair of inequalities.

    a. $x < 5$ **b.** $x > 1$ **c.** $x \ge 0$
      $y > 2$ $y < 3$ $y \ge 0$

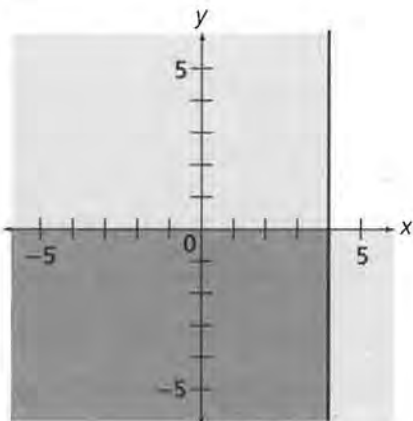**14.** Write a pair of inequalities to represent the doubly-shaded region of each graph.
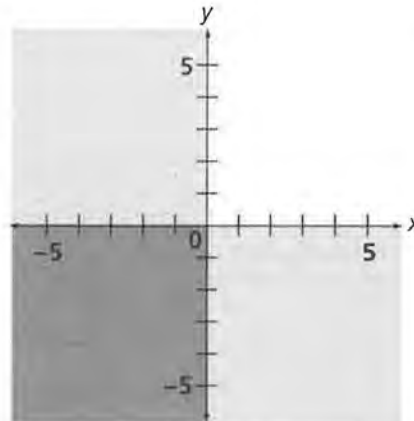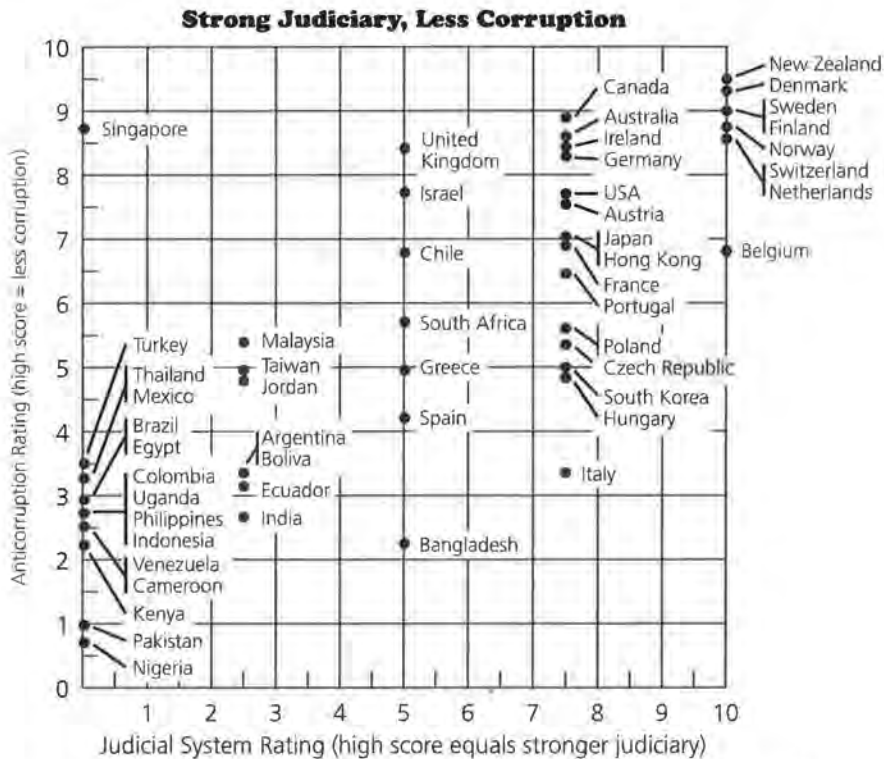
a.

b.

c.

d.

15. The table below contains the watts per channel and suggested retail price in 1996 for selected stereo minisystems. A consumer wants a stereo minisystem with at least 50 watts per channel and wishes to pay no more than $500.

Model	Power (watts)	Price ($)
Yamaha GX-5	35	550
JVC MX-C550	40	530
AIWA NSC-V51G	40	480
AIWA NSX-999	120	600
Onkyo PCS-207	25	580
JVC MX-C330	30	440
Denon D-500	45	800
Sony MHC-C605SS	100	590
AIWA NSX-V90	50	690
Pioneer CCS-404	100	650
Sony MHC-405SS	100	490
Fisher DCS-M37	50	450
Pioneer CCS-204	33	400
Kenwood UD-303	53	470
Kenwood UD-403	55	730

Source: *Consumer Reports*, Febuary, 1996

a. On the grid on *Activity Sheet* 9, make a scatter plot of the ordered pairs (power, price).

b. Shade the region of the graph that represents all minisystems that have 50 or more watts per channel. Write an inequality to represent this region.

c. On the scatter plot, shade the region of the graph that represents all mininsystems that cost $500 or less. Write an inequality to represent this region.

d. Which minisystems satisfy both conditions?

e. Write a pair of inequalities to represent the region where cost is more than $500 and watts per channel is greater than 50.

16. The scatter plot below shows the relationship between a country's judicial-system rating and anticorruption rating.

Strong Judiciary, Less Corruption

Anticorruption Rating (high score = less corruption)

Singapore, Turkey, Thailand, Mexico, Brazil, Egypt, Colombia, Uganda, Philippines, Indonesia, Venezuela, Cameroon, Kenya, Pakistan, Nigeria, Malaysia, Taiwan, Jordan, Argentina, Bolivia, Ecuador, India, United Kingdom, Israel, Chile, South Africa, Greece, Spain, Bangladesh, Canada, Australia, Ireland, Germany, USA, Austria, Japan, Hong Kong, France, Portugal, Poland, Czech Republic, South Korea, Hungary, Italy, New Zealand, Denmark, Sweden, Finland, Norway, Switzerland, Netherlands, Belgium

Judicial System Rating (high score equals stronger judiciary)

Source: *Economic Freedom of the World 1975–95; Transparency International 1996*

a. Identify the countries that have a rating of 8 or higher on both scales.

b. Write a pair of inequalities to represent the region of the graph where countries have a rating of 8 or higher on each scale.

c. Write a pair of inequalities to represent the region of the graph where countries have a judicial-system rating of 4 or lower and an anticorruption rating of 6 or higher. Identify the countries in this region.

Extension

17. Use the categories of total minutes played and field goals made from the 1996 NBA Championship series data presented in this section to decide which players are the most valuable. Your work should include a scatter plot and the conditions, written as inequalities, used to make your decisions.

Systems of Inequalities

How often do you eat breakfast, lunch, or dinner at a fast-food restaurant?

How healthy do you think the food is at these restaurants?

OBJECTIVE

Graph and interpret systems of inequalities in the form $y < mx$.

Many people think that the food at fast-food restaurants is high in calories and saturated fats. In this lesson, you will investigate how to use inequalities to evaluate how the number of calories from fast food compares to suggested dietary recommendations.

INVESTIGATE
Calories from Fat

With the increase in spending at fast-food restaurants comes a greater awareness of dietary concerns. In 1989, the National Research Council published Recommended Daily Allowances stating that no more than 30% of a person's daily caloric intake should come from fats. The table on page 55 shows the calories and calories from fat for selected items from McDonald's in a recent year.

Item	Calories	Calories from Fat
Hamburger	270	90
Cheeseburger	320	130
Quarter Pounder	430	190
Quarter Pounder with Cheese	530	270
Arch Deluxe	570	280
Arch Deluxe with Bacon	610	310
Big Mac	530	250
McGrilled Chicken Classic	260	35
Garden Salad	80	35
Fajita Chicken Salad	160	60
Lite Vinaigrette Dressing	50	20
Apple Bran Muffin	170	0
Apple Danish	360	140
1% Lowfat Milk	100	20

Source: *McDonald's Nutrition Facts*

Below is a scatter plot of the ordered pairs (calories, calories from fat).

Calories and Calories from Fat in Fast Foods
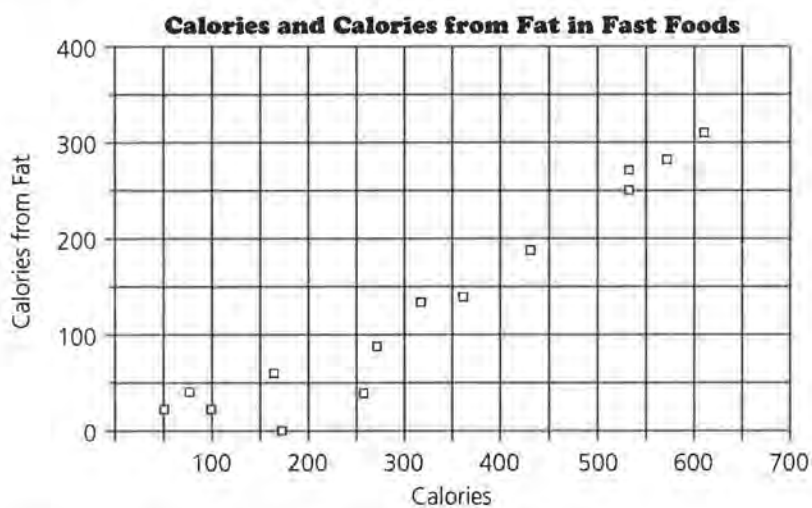
Discussion and Practice

1. What patterns do you observe in the scatter plot above?

2. Refer to the scatter plot above.

   a. What does the point (0, 0) mean in terms of the data?

   b. On the first graph on *Activity Sheet 10*, draw a line that passes through the point (0, 0) that you feel summarizes the data.

3. Refer to Problem 2b.

   a. Write an equation for the line you have drawn.

   b. Explain the meaning of the slope of this line in terms of the data.

4. The number of calories in a small bag of French fries is 220.

   a. Use your line and predict the number of calories from fat for a small bag of French fries.

   b. If the actual number of calories from fat is 110, how accurate was your prediction?

5. The following items were not shown on the scatter plot above. Use the equation for the line you drew and predict the number of calories from fat for each of the given number of calories.

Item	Calories	Predicted Calories from Fat
Filet o' Fish	360	————
McChicken Sandwich	510	————
Egg McMuffin	290	————

6. The actual numbers of calories and calories from fat are listed in the table below.

Item	Calories	Actual Calories from Fat
Filet o' Fish	360	150
McChicken Sandwich	510	270
Egg McMuffin	290	110

   a. On the graph for Problem 2, add the ordered pairs (calories, calories from fat) for the three items listed in the table above.

   b. Are the ordered pairs above, below, or on the line that you have drawn?

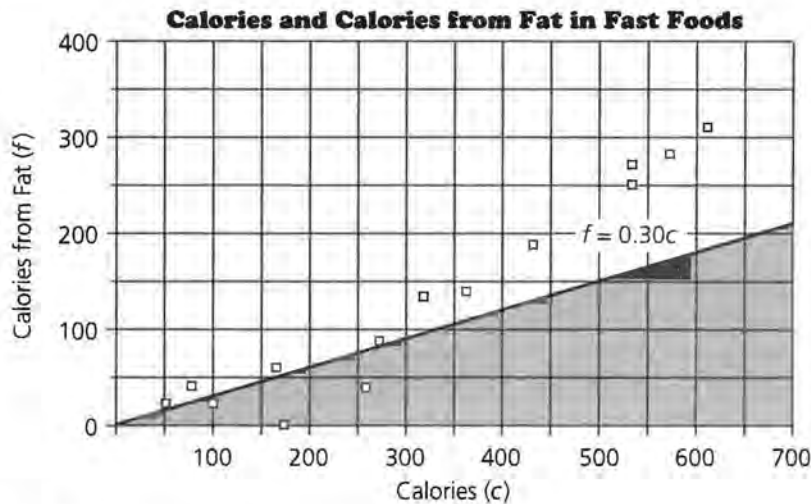   c. Explain what it means if a point is above the line.

The National Research Council suggests that we should get no more than 30% of our calories from fat. To express this recommendation as an inequality,

let c = number of calories and
let f = number of calories from fat.
Then, $f \le 0.30c$.

The graph of the inequality is shown below.

Calories and Calories from Fat in Fast Foods

7. Refer to the graph above.

   a. Identify the items that are in the shaded region.

   b. What do the items in this region represent?

   c. Could you make a meal out of these items?

8. Draw the line you drew in Problem 2 on the plot of the inequality given on the second graph on *Activity Sheet 10*.

9. Refer to the line in Problem 8.

   a. Identify the items that lie in the unshaded region but below the original line that you drew.

   b. What do the items in this region represent?

   c. Write a pair of inequalities to describe this region.

Saturated Fats

The following table contains the number of calories and the amount of saturated fat in selected items from McDonald's.

Item	Calories	Saturated Fat (g)
Hamburger	270	3.5
Cheeseburger	320	6
Quarter Pounder	430	8
Quarter Pounder with Cheese	530	13
Arch Deluxe	570	11
Arch Deluxe with Bacon	610	12
Big Mac	530	10
McGrilled Chicken Classic	260	1
Garden Salad	80	1
Fajita Chicken Salad	160	1.5
Lite Vinaigrette Dressing	50	0
Apple Bran Muffin	170	0
Apple Danish	360	5
1% Lowfat Milk	100	1.5

Source: *McDonald's Nutrition Facts*

Below is a scatter plot of the ordered pairs (calories, saturated fat) of the items in this table.

Calories and Saturated Fat in Fast Foods
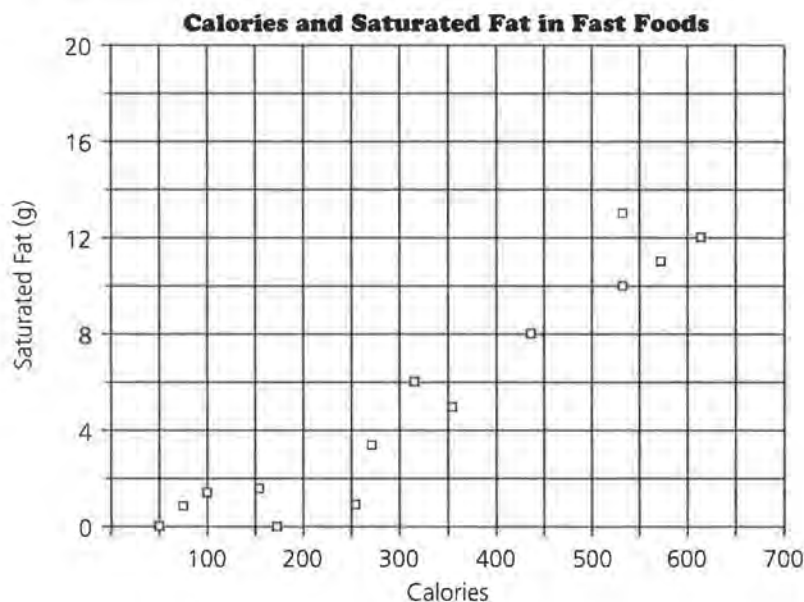
10. Describe any trends that you observe.

11. On the first graph on *Activity Sheet 11*, draw a line that passes through the origin and summarizes the data.

12. Consider the line you drew in Problem 11.

    a. Write an equation for the line.

    b. Describe the region below the line.

    c. Write an inequality to represent this region.

13. The Research Council suggests that no more than 10% of our calories should come from saturated fats.

    a. Write an inequality to describe this relationship.

    b. On the graph for Problem 11, graph the inequality from part a.

    c. Identify the items that are in the shaded region.

14. The plot from Problem 13b is divided into three regions.

    a. Describe each region.

    b. Write a set of inequalities for each region.

Summary

An inequality in the form $y \leq ax$ separates the coordinate system into two regions. In the graph below, region I represents the ordered pairs that satisfy the inequality and region II represents the ordered pairs that do not satisfy the inequality.
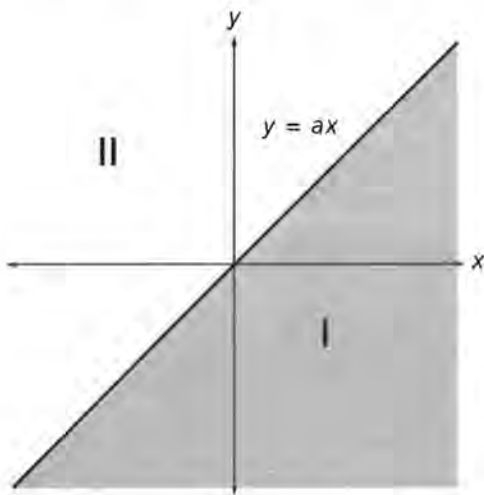
Given a system of two inequalities in the form $y \le ax$ and $y \le bx$, the coordinate system can be divided into four regions. In the graph below, region I represents the ordered pairs that satisfy both inequalities. Region II satisfies the inequality $y \le ax$ but not the inequality $y \le bx$. Region III satisfies the inequality $y \le bx$ but not the inequality $y \le ax$. Region IV represents the ordered pairs that satisfy neither inequality.
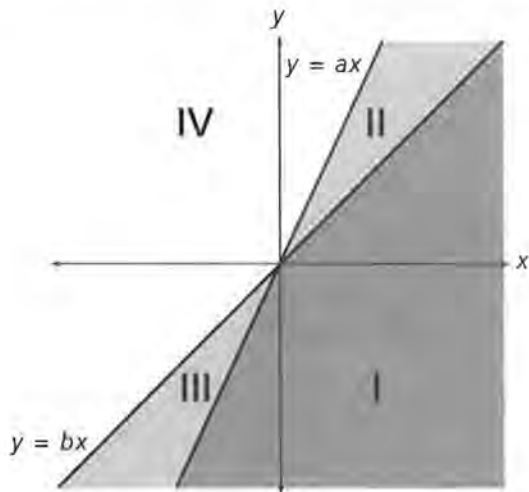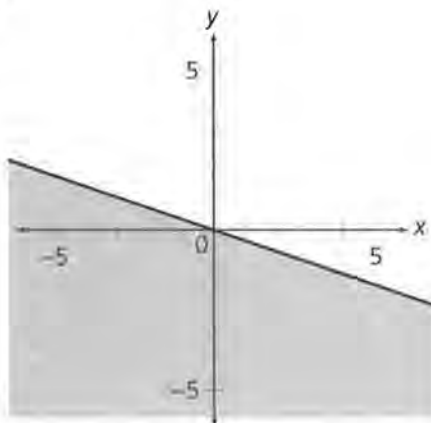

Practice and Applications

15. Graph each inequality.

   a. $y \le 2x$ **b.** $y \ge \frac{1}{2}x$ **c.** $y \ge -3x$

16. Write an inequality to represent the shaded region in each graph.

  a.

  b.
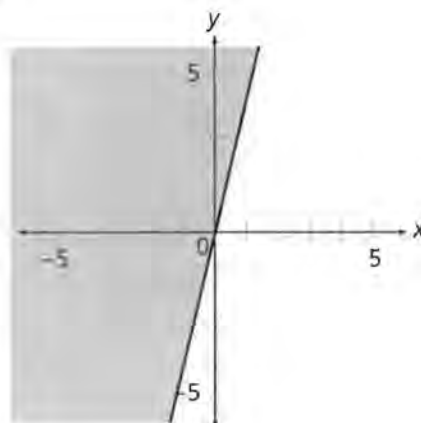
17. Graph each system of inequalities.

  a. $y \le \frac{2}{3}x$ **b.** $y \ge 1x$

       $y \le 2x$ $y \le 1.5x$

18. Many financial advisors suggest that a single person should spend no more than 20% of his or her gross monthly income on housing costs. These costs include rent or house payment and utility payments. The scatter plot below shows the relationship between gross monthly income and housing costs for a random sample of 15 single people.

Gross Monthly Income and Housing Costs

a. On the second graph on *Activity Sheet 11*, draw a line that summarizes the data.

b. Find an equation for the line. Describe the line in terms of the data.

c. What does it mean if an ordered pair is above the line that you have drawn?

d. Write an inequality to represent the recommendation that a single person's housing cost should be no more than 20% of his or her gross monthly income.

e. On *Activity Sheet 11*, graph the inequality on the scatter plot.

f. What do the ordered pairs that lie in the shaded region represent?

g. Write a pair of inequalities to describe the ordered pairs not in the shaded region but below the original line that you drew.

Applying Systems of Inequalities

Do you ever have to change your plans because you don't have enough money or enough time?

Suppose you want to buy some concert tickets but you have a limit on how much money you can spend. Or suppose you want to start an exercise program but have a limited amount of time you can exercise. When you have to make a decision, you are often faced with limits or constraints. In this lesson, you will investigate the use of inequalities to find the best solution to a problem when several conditions or constraints must be met. This process is known as *linear programming*.

INVESTIGATE

Linear programming was developed after World War II by mathematicians and economists to help find the best solutions to problems in industry, such as the question of which combination of products will produce the maximum profit.

Discussion and Practice

Burning Calories

Josh, a high-school sophomore, wants to begin an exercise program that combines jogging and weight lifting. Suppose Josh's coach wants him to exercise at least 180 minutes a week.

1. Let j = number of minutes jogging and w = number of minutes weight lifting. Write an inequality that shows the relationship between minutes jogging and minutes weight lifting and the condition of at least 180 minutes of exercise a week.

2. On *Activity Sheet 12*, graph your inequality from Problem 1 on the first grid.

3. Find an ordered pair in the shaded region. Describe what the ordered pair represents.

Josh knows that for his weight and size jogging burns about 9 calories per minute and weight lifting burns approximately 4.5 calories per minute.

4. How many calories will Josh burn if he jogs

   a. for 90 minutes and lifts weights for 90 minutes each week?

   b. for 240 minutes and lifts weights for 30 minutes each week?

   c. for 60 minutes and lifts weights for 250 minutes each week?

5. If Josh wants to burn at least 1200 calories per week, could he reach this goal by jogging

   a. for 150 minutes and lifting weights for 100 minutes?

   b. for 100 minutes and lifting weights for 90 minutes?

   c. for 50 minutes and lifting weights for 150 minutes?

6. What do you think is the minimum number of minutes that Josh can jog and lift weights and still reach his goal of burning 1200 calories in a week? Remember he must exercise a total of at least 180 minutes a week.

You can use the following equation to find the total number of calories burned:

   $9j + 4.5w = c$, where j = number of minutes jogging, w = number of minutes weight lifting, and c = number of calories burned

Since Josh wants to burn at least 1200 calories per week, you can use the following inequality:

   $9j + 4.5w \geq 1200$

7. On *Activity Sheet 12*, graph this inequality on the second grid.

8. Describe all the ordered pairs in the shaded region.

Josh wants to exercise at least 180 minutes and burn at least 1200 calories in a week. In order for him to determine the minimum number of minutes that he must jog and lift weights to reach this goal, he needs to consider the two inequalities together.

9. On *Activity Sheet 13*, graph the two inequalities $j + w \geq 180$ and $9j + 4.5w \geq 1200$ on the first grid.

The region that is represented by the overlap of the two inequalities is called the *feasible region*. This region represents all the possible combinations of minutes jogging and weight lifting that meet the constraints of at least 180 minutes exercising and at least 1200 calories burned. Consider this graph.

Exercise: Jogging and Weight Lifting
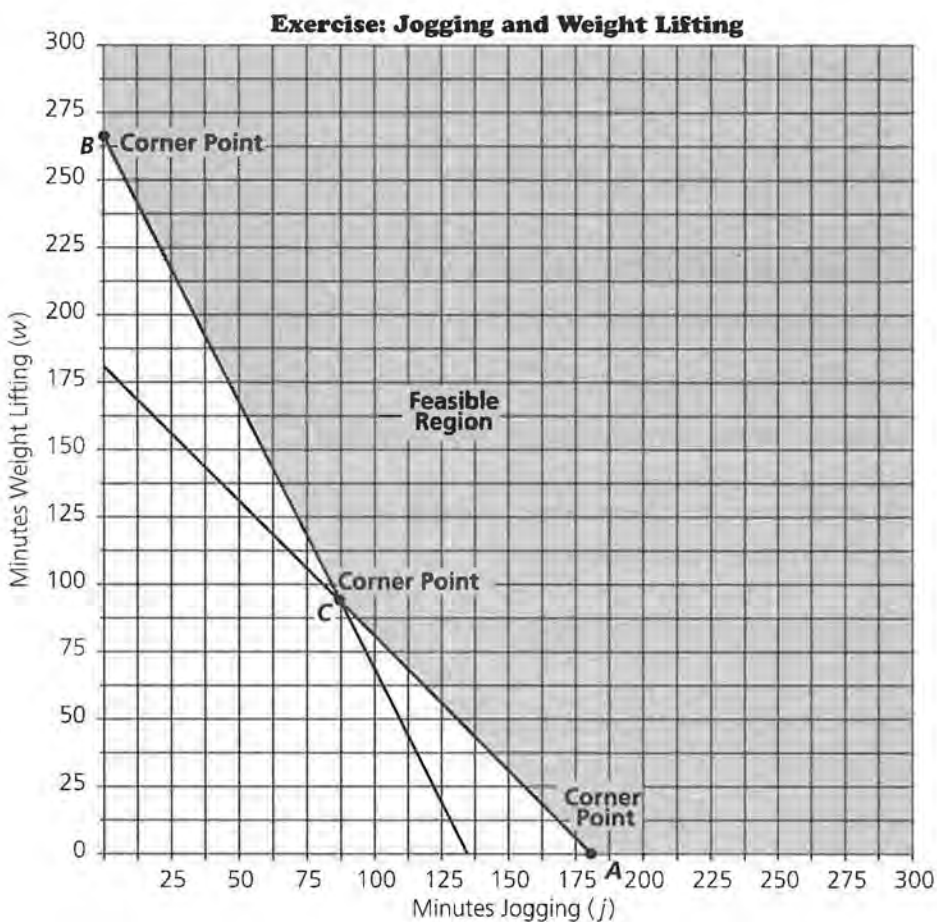
Points *A*, *B*, and *C* are known as *corner points* of the feasible region. Point *A* is the *x*-intercept of the equation $j + w = 180$. Point *B* is the *y*-intercept of the equation $9j + 4.5w = 1200$. Point *C* is the point whose ordered pair satisfies both equations.

10. Refer to the graph on page 64.

   a. The shaded region is the feasible region. Explain why this is the feasible region.

   b. Find the ordered pairs for points *A*, *B*, and *C*. List them in a table like the one below. Show your work.

Point	Ordered Pair	Total Minutes Exercising	Total Calories Burned
A	_____	_____	_____
B	_____	_____	_____
C	_____	_____	_____

   c. Find the total number of minutes exercising and the total calories burned for each corner point. List the results in your table.

   d. Of the three corner points, which one gives the greatest calorie burn for the fewest minutes exercising?

11. In your table, list three more ordered pairs that are in the feasible region. Find the total number of minutes exercised and the total calories burned for each ordered pair.

12. Suppose Josh wants to exercise for 180 minutes to stay in shape but does not want to lose weight. Of all the points listed in the table, which point gives the least number of calories burned for 180 minutes exercised? Compare answers within your group.

Cookie Sale

The Conservation Club at a high school decides to sell cookies to earn some money to buy trees to plant around school. They decide to make no more than 12 dozen, or 144, cookies. They also decide to make only two types of cookies, one with M&Ms® and the other with chocolate chips.

13. Let m = number of M&M cookies and let c = number of chocolate-chip cookies. Write an inequality that shows the relationship between the number of each type of cookie under the condition that club members are planning to make no more than 144 cookies.

M&Ms is a registered trademark of M&M/Mars, a division of Mars, Inc.

14. On *Activity Sheet 13*, graph your inequality from Problem 13 on the second grid.

15. List an ordered pair in the shaded region of your graph in Problem 14. Describe what this ordered pair represents.

The Conservation Club members know that each M&M cookie will cost about $0.15 to make and each chocolate-chip cookie will cost about $0.10 to make.

16. About how much will it cost to make

    a. 72 cookies of each kind?

    b. 44 M&M cookies and 100 chocolate-chip cookies?

    c. 100 M&M cookies and 44 chocolate-chip cookies?

17. Use *m* to represent the number of M&M cookies made, *c* the number of chocolate-chip cookies made, and *t* the total cost to write an equation that can be used to find the total cost of making all the M&M and chocolate-chip cookies.

18. The advisor of the club said that the club members can spend no more than $20.00 on supplies for the cookies.

    a. Express the total cost as an inequality that shows the condition that the total cost cannot exceed $20.00.

    b. Graph the inequality on the same grid you used for Problem 14.

    c. In a table like the one below, list three ordered pairs in the feasible region. Describe what each point represents.

Ordered Pair	Number of M&M Cookies	Number of Chocolate-Chip Cookies	Total Cost
____	____	____	____
____	____	____	____
____	____	____	____
____	____	____	____
____	____	____	____
____	____	____	____

    d. Find the total cost for each ordered pair and list it in your table.

    e. Find the corner points of the feasible region. Add these points to your table and find the total cost.

19. The Club decides to sell M&M cookies for $0.60 each and chocolate-chip cookies for $0.50 each. This means a profit of $0.45 for each M&M cookie and $0.40 for each chocolate-chip cookie.

    a. Use m to represent the number of M&M cookies made, c the number of chocolate-chip cookies made, and p the total profit to write an equation that could be used to find the total profit.

    b. Find the profit for each ordered pair listed in your table in Problem 18.

    c. Of the points listed in the table, which point gives the maximum profit?

    d. Compare your answer with those of other members in your group. Which point in the feasible region gave the maximum profit?

Summary

Linear programming involves writing a system of inequalities based on various constraints. The graph of the system of inequalities determines the *feasible region*. This region represents all of the ordered pairs that satisfy the system of inequalities. The *corner points* are those points at which various quantities are maximized or minimized.
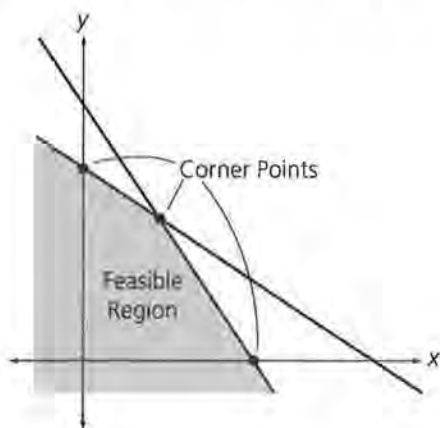
Practice and Applications

20. Graph the feasible region for each system of inequalities.

    a. $y \le -2x + 8$ **b.** $y \ge 5 - x$ **c.** $y \ge 2$
         $y \le -0.5x + 4$ $y \ge 1.5x + 1$ $y \le 8$
         $y \ge 0$ $y \ge 0$ $y \ge 2x - 5$
         $x \ge 0$ $x \ge 0$ $x \ge 0$

21. Suppose the Conservation Club decides to sell each cookie for $0.60.

    a. Write an equation for the total profit that the club could make from the sale of M&M and chocolate-chip cookies.

    b. Find the profit for each ordered pair that you listed in Problem 18.

    c. Of the points listed, which one gives the maximum profit?

The National Research Council suggests that we have at most 2400 mg of sodium and at most 65 g of fat each day. If you were going to eat at a fast-food restaurant, how many hamburgers and apple pies could you eat and stay within these dietary conditions?

22. A hamburger from McDonald's contains 530 mg of sodium and 10 g of fat. Apple pie contains 200 mg of sodium and 13 g of fat.

    a. How many milligrams of sodium are contained in 2 hamburgers and 1 apple pie?

    b. How many grams of fat are contained in 2 hamburgers and 1 apple pie?

    c. How many milligrams of sodium are contained in 3 hamburgers and 2 apple pies?

    d. How many grams of fat are contained in 3 hamburgers and 2 apple pies?

Could you eat 6 hamburgers and no apple pies and stay within the dietary conditions concerning sodium and total fat? What do you think is the maximum number of hamburgers and apple pies you could order and eat and still stay under the dietary conditions?

Let h = the number of hamburgers eaten,
let a = the number of apple pies eaten,
let s = amount of sodium in mg, and
let f = amount of fat in g.

23. Write an equation that can be used to find the total amount of sodium for a given number of hamburgers and apple pies.

24. Write an equation that can be used to find the total amount of fat for a given number of hamburgers and apple pies.

25. The dietary conditions state that the amount of sodium should be less than 2400 mg per day. Write an inequality that shows the relationship between the amount of sodium from a given number of hamburgers and apple pies and the limit of 2400 mg of sodium per day.

26. The dietary conditions state that the amount of fat should be less than 65 grams each day. Write an inequality that shows the amount of fat less than or equal to 65 grams.

27. On the grid on *Activity Sheet 14*, graph your inequalities from Problems 25 and 26. Clearly show the feasible region.

28. Refer to your graph from Problem 27.

    a. Describe the ordered pairs in the feasible region.

    b. Find the corner points of the feasible region.

    c. For each corner point, find the milligrams of sodium and the grams of fat.

    d. What is the maximum number of hamburgers and apple pies that you could eat and stay under the dietary constraints of sodium and fat?

Assessment for Unit III

1. The following table contains the normal July high temperature (°F) and the normal annual precipitation (inches) for selected cities in the central United States.

City	High Temperature (°F)	Precipitation (inches)
Milwaukee, WI	80	30.9
Nashville, TN	90	48.5
Cleveland, OH	82	35.4
Cincinnati, OH	86	40.1
Omaha, NE	89	30.3
St. Louis, MO	89	33.9
Minneapolis, MN	83	26.4
Sault Ste. Marie, MI	75	33.5
Detroit, MI	83	31.0
Louisville, KY	88	43.6
Wichita, KS	93	28.6
Des Moines, IA	86	30.8
Indianapolis, IN	85	39.1
Chicago, IL	83	33.3

Source: *The World Almanac and Book of Facts,* 1994

a. On *Activity Sheet 15,* use the first grid to make a scatter plot of the ordered pairs (temperature, precipitation).

b. Shade the region of the graph that represents the cities that have a normal high temperature in July of less than 85°F. Write an inequality to represent this region.

c. Shade the region of the graph that represents the cities that have an annual precipitation of more than 40 inches. Write an inequality to represent this region.

d. Identify the cities that satisfy both conditions.

e. Write a pair of inequalities that represent the region where July temperature is higher than 85°F and annual precipitation is more than 40 inches.

2. Vitamins are an important part of everyone's diet. Vitamins B6 and riboflavin work together with the other B vitamins to help cells absorb and burn energy. A diet deficient in B vitamins often results in muscle weakness and in psychiatric problems. The U.S. Recommended Daily Allowance of vitamin B6 is 2 mg and of riboflavin is 1.7 mg. Listed below are the amounts of B6 and riboflavin in a half cup of broccoli and in 1 apple.

Food	B6 (mg)	Riboflavin (mg)
Broccoli (half cup)	0.154	0.166
Apple (1)	0.066	0.019

a. How many milligrams of riboflavin are contained in 4 cups of broccoli and 3 apples?

b. How many milligrams of vitamin B6 are contained in 4 cups of broccoli and 3 apples?

c. Write an expression that can be used to find the total number of milligrams of B6 for a given number of apples and number of half cups of broccoli.

d. Write an expression that can be used to find the total number of milligrams of riboflavin for a given number of apples and number of half cups of broccoli.

e. Use your expressions from parts c and d to write two inequalities that show that the recommended daily allowance of B6 is at least 2 mg and the recommended daily allowance of riboflavin is at least 1.7 mg.

f. On the second grid on *Activity Sheet 15*, graph your inequalities from part e and clearly show the feasible region.

g. Describe the ordered pairs in the feasible region.

h. Find the corner points of the feasible region.

i. For each corner point, find the number of milligrams of vitamin B6 and riboflavin.

j. What is the least number of apples and half cups of broccoli a person could eat to reach the recommended daily allowance for B6 and riboflavin?