Acknowledgments

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- The many students from Homestead High School and Whitnall High School, who helped shape the ideas as they were being developed
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About *Data-Driven Mathematics*

Historically, the purposes of secondary-school mathematics have been to provide students with opportunities to acquire the mathematical knowledge needed for daily life and effective citizenship, to prepare students for the workforce, and to prepare students for postsecondary education. In order to accomplish these purposes today, students must be able to analyze, interpret, and communicate information from data.

*Data-Driven Mathematics* is a series of modules meant to complement a mathematics curriculum in the process of reform. The modules offer materials that integrate data analysis with secondary mathematics courses. Using these materials helps teachers motivate, develop, and reinforce concepts taught in current texts. The materials incorporate the major concepts from data analysis to provide realistic situations for the development of mathematical knowledge and realistic opportunities for practice. The extensive use of real data provides opportunities for students to engage in meaningful mathematics. The use of real-world examples increases student motivation and provides opportunities to apply the mathematics taught in secondary school.

The project, funded by the National Science Foundation, included writing and field testing the modules, and holding conferences for teachers to introduce them to the materials and to seek their input on the form and direction of the modules. The modules are the result of a collaboration between statisticians and teachers who have agreed on the statistical concepts most important for students to know and the relationship of these concepts to the secondary mathematics curriculum.

A diagram of the modules and possible relationships to the curriculum is on the back cover of each Teacher's Edition of the modules.
Using This Module

Why the Content Is Important

Many problems in the world require looking for relationships between two events that are changing at the same time. For example, when will the time for women running the 1600 meters be the same as the time for men, or when did the circulation of morning newspapers exceed the circulation of evening papers? Solving systems of equations and inequalities are important tools in answering questions similar to these questions. Solving systems enables us to find the intersection of two lines on a graph and analyze patterns and relationships.

This module will extend the concept of rate of change and graphing linear equations to graphing two equations or inequalities on the same coordinate system. In Unit I, students will investigate algebraic techniques that can be used to find the point of intersection of two lines. These techniques will to used to graph and analyze inequalities in Unit II. People often use inequalities to determine when a quantity is greater than or less than some given standard. Does the food at a fast-food restaurant exceed nutritionists' guidelines? Will increasing the price of a chocolate-chip cookie decrease sales?

Content

Mathematics content: Students will be able to
- Find and interpret slope.
- Graph a system of equations.
- Identify parallel lines.
- Solve a system of equations using the substitution method.
- Graph a system of inequalities.
- Solve a system of inequalities.

Statistics content: Students will be able to
- Make and interpret plots over time.
- Make and interpret scatter plots.
- Find a line of best fit for a set of data.
Instructional Model

The instructional emphasis *Exploring Systems of Equations and Inequalities*, as in all of the modules in *Data-Driven Mathematics*, is on discourse and student involvement. Each lesson is designed around a problem or mathematical situation and begins with a series of introductory questions or scenarios that can prompt discussion and raise issues about that problem. These questions can involve students in thinking about the problem and help them understand why such a problem might be of interest to someone in the world outside the classroom. The questions can be used in whole-class discussion or in student groups. In some cases, the questions are appropriate to assign as homework to be done with input from families or from others not a part of the school environment.

These opening questions are followed by discussion issues that clarify the initial questions and begin to shape the direction of the lesson. Once the stage has been set for the problem, students begin to investigate the situation mathematically. As students work their way through the investigations, it is important that they have the opportunity to share their thinking with others and to discuss their solutions in small groups and with the entire class. Many of the exercises are designed for groups in which each member does one part of the problem and the results are compiled for final analysis and solution. Multiple solutions and solution strategies are also possible, and it is important for students to recognize these situations and to discuss the reasoning behind different approaches. This will provide each student with a wide variety of ways to build his or her own understanding of the mathematics.

In many cases, students are expected to construct their own understanding by thinking about the problem from several perspectives. They do, however, need validation of their thinking and confirmation that they are on the right track, which is why discourse among students, and between students and teacher, is critical. In addition, an important part of the teacher's role is to help students link the ideas within an investigation and to provide an overview of the "big picture" of the mathematics within the investigation. To facilitate this, a review of the mathematics appears in the summary following each investigation.

Each investigation is followed by a Practice and Applications section in which students can revisit ideas presented within the lesson. These exercises may be assigned as homework, given as group work during class, or omitted altogether if students are ready to move ahead.

Student assessments occur after two units in the student book. These can be assigned as long-range take-home tasks, as group assessment activities, or as in-class work. The assessment pages provide a summary of the lessons up to that point and can serve as a vehicle for students to demonstrate what they know and what they can do with the mathematics. Commenting on the strategies students use to solve a problem can encourage students to apply different strategies. Students also learn to recognize those strategies that enable them to find solutions efficiently. Also included are two student projects.
Where to Use the Module in the Curriculum

This module is about solving systems of equations and graphing and working with linear inequalities. It can be used in a first-year algebra or second-year algebra course in a variety of ways. The module can be used as a complete unit, starting with the solving of a system of equations and finishing with the solving of a system of inequalities. The three units can also be used separately or at different times during the school year.

Following are suggestions for using the module:

- Use with the standard chapter on solving systems of equations to provide real-world applications.
- Replace the standard chapter on graphing linear inequalities in any traditional mathematics text.
- Use after students have completed a section on graphing linear inequalities to illustrate how to apply those concepts in real-world contexts.
- Use in a second-year algebra course as a review for students solving a system of equations and inequalities.
- Use as the third unit in a course, following Exploring Symbols: An Introduction to Expressions and Functions and Exploring Linear Relations, from the Data-Driven Mathematics series.

Prerequisites

Students should have had experience in using variables to describe relationships, plotting points, simplifying expressions, solving equations, drawing a line that best fits a set of ordered pairs, finding the slope of a line, finding the intercepts of a line, and writing the equation of a line.
The table below provides a possible sequence and pacing for the lessons.

<table>
<thead>
<tr>
<th>LESSON</th>
<th>OBJECTIVE</th>
<th>PACING</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit I: Solving Systems of Equations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introductory Activity: Men’s and Women’s Olympic Times</td>
<td>Observe trends in data sets.</td>
<td>½ class period</td>
</tr>
<tr>
<td>Lesson 1: Systems of Equations</td>
<td>Find the point of intersection of two lines on a graph; solve a system of equations.</td>
<td>2–3 class periods</td>
</tr>
<tr>
<td>Lesson 2: Lines with the Same Slope</td>
<td>Recognize that parallel lines have the same slope or rate of change; recognize systems that do not have a solution.</td>
<td>1 class period</td>
</tr>
<tr>
<td>Assessment for Unit I</td>
<td>Apply knowledge of systems of equations.</td>
<td>1 class period or homework</td>
</tr>
<tr>
<td><strong>Unit II: Graphing Inequalities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introductory Activity: Estimating the Number of Raisins</td>
<td>Compare an estimate to an actual count.</td>
<td>½ class period</td>
</tr>
<tr>
<td>Lesson 3: Shading a Region</td>
<td>Find the solution to an inequality using the line ( y = x ).</td>
<td>1 class period</td>
</tr>
<tr>
<td>Lesson 4: Graphing Inequalities</td>
<td>Graph and interpret a linear inequality.</td>
<td>1 class period</td>
</tr>
<tr>
<td>Assessment for Unit II</td>
<td>Solve problems by graphing inequalities.</td>
<td>1 class period or homework</td>
</tr>
<tr>
<td><strong>Unit III: Solving Systems of Inequalities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Introductory Activity: Fast Foods</td>
<td>Investigate ordered pairs that satisfy a constraint.</td>
<td>½ class period</td>
</tr>
<tr>
<td>Lesson 5: Graphing Conditions</td>
<td>Graph and interpret systems of inequalities in the form ( x &lt; a ) and ( y &gt; b ).</td>
<td>1 class period</td>
</tr>
<tr>
<td>Lesson 6: Systems of Inequalities</td>
<td>Graph and interpret systems of inequalities in the form ( y &lt; mx ).</td>
<td>2 class periods</td>
</tr>
<tr>
<td>Lesson 7: Applying Systems of Inequalities</td>
<td>Graph and interpret systems of inequalities in the form ( ax + by &lt; c ).</td>
<td>2 class periods</td>
</tr>
<tr>
<td>Assessment for Unit III</td>
<td>Apply knowledge of systems of inequalities.</td>
<td>1 class period or homework</td>
</tr>
</tbody>
</table>

Approximately 3 weeks total time
**Use of Teacher Resources**

At the back of this Teacher's Edition are the following:

- Quizzes for selected lessons
- End-of-Module Test
- Solution Key for quizzes and test
- Activity Sheets

These items are referenced in the *Materials* section at the beginning of the lesson commentary.

<table>
<thead>
<tr>
<th>LESSONS</th>
<th>RESOURCE MATERIALS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit I: Solving Systems of Equations</strong></td>
<td></td>
</tr>
<tr>
<td>Introductory Activity: Men's and Women's Olympic Times</td>
<td>Activity Sheet 1 (Problems 3–6)</td>
</tr>
</tbody>
</table>
| Lesson 1: Systems of Equations | Activity Sheet 2 (Problem 3)  
 | Activity Sheet 3 (Problem 9)  
 | Lesson 1 Quiz |
| Lesson 2: Lines with the Same Slope | Activity Sheet 4 (Problems 2 and 7) |
| Assessment for Unit I | Activity Sheet 5 (Problem 2) |

**Unit II: Graphing Inequalities**

Introductory Activity: Estimating the Number of Raisins

| Lesson 3: Shading a Region | Activity Sheet 6 (Problem 6) |
| Lesson 4: Graphing Inequalities | Activity Sheet 7 (Problems 1 and 2)  
 | Lesson 4 Quiz |
| Assessment for Unit II | |

**Unit III: Solving Systems of Inequalities**

Introductory Activity: Fast Foods

| Lesson 5: Graphing Conditions | Activity Sheet 8 (Problems 1–7, 11, and 12)  
 | Activity Sheet 9 (Problem 15) |
| Lesson 6: Systems of Inequalities | Activity Sheet 10 (Problems 2–6 and 8)  
 | Activity Sheet 11 (Problems 11–13 and 18)  
 | Lesson 6 Quiz |
| Lesson 7: Applying Systems of Inequalities | Activity Sheet 12 (Problems 2, 3, and 7)  
 | Activity Sheet 13 (Problems 9, 14, and 18)  
 | Activity Sheet 14 (Problems 27 and 28)  
 | Lesson 7 Quiz |
| Assessment for Unit III | Activity Sheet 15 (Problems 1 and 2)  
 | End-of-Module Test |
Solving Systems of Equations
INTRODUCTORY ACTIVITY

Men's and Women's Olympic Times

Materials: rulers, Activity Sheet 1
Technology: graphing calculators (optional)
Pacing: \( \frac{1}{2} \) class period

Overview
This opening activity uses the men's and women's 800-meter Olympic times to focus on the concept of slope. Students are asked to draw a line that best fits the data and compare the slopes. The slopes that they find are then compared to the information regarding the speeds of men and women racers.

Teaching Notes
This activity can be used as homework or completed during class. This activity can also be used as a review of fitting a line to data and writing the equation of a line. These skills are used throughout the module, and this activity serves as an introduction to demonstrate how these skills will be used to solve a system of equations. You may wish to review finding the equation of a line and interpreting the slope and intercepts. The article used at the beginning of the activity will be referred to in Lesson 1. The data used in this activity can be found at the end of Lesson 1.

It is important to note that there are a number of prerequisite skills needed for this activity as well as the entire module. Students should be able to find the slope of a line, the x- and y-intercepts, and the equation of the line in both point-slope and slope-intercept form. Students are only expected to fit a line to the data using an “eyeball” fit. If students know how to draw a median-fit or least-squares regression line, they could use either one to draw the line. Students are expected to find the point of intersection of the two lines by estimating from the graph. The algebra will be developed in Lesson 1.

Technology
Graphing calculators are optional. Students should be encouraged to draw a line by hand, since this is a skill that is used throughout the module.
INTRODUCTORY ACTIVITY:

MEN'S AND WOMEN'S OLYMPIC TIMES

INTRODUCTORY ACTIVITY

Men's and Women's Olympic Times

Do you think that women's times will ever catch up to men's times in Olympic races?

"Women may outrun men, researchers suggest."

Analyzing trends in records for sporting events is of great interest to many people. Often these trends can be determined by graphing. In this unit, you will learn how to find relationships between two sets of data by studying their graphs.

EXPLORE

Within a few generations, female runners may start beating male runners in world-class competitions. An analysis of world records for a variety of distances found that women's rates have been improving about twice as quickly as men's rates.
INTRODUCTORY ACTIVITY: MEN'S AND WOMEN'S OLYMPIC TIMES

Solution Key

1. Times for men and women in the 200-, 400-, 800-, and 1500-meter events

2. 0.16; twice as fast as men

3. and 4. Possible answers:

If this trend continues, the top female and male runners may start performing equally well between the years 2015 and 2055 in the 200-, 400-, 800-, and 1500-meter events. These findings were reported in a letter in the journal Nature.

"None of the current women's world-record holders at these events could even meet the men's qualifying standard to compete in the 1992 Olympic Games," researchers Brian Wipp and Susan Ward wrote. "However, it is the rates of improvement that are so strikingly different—the gap is progressively closing."

But other researchers said they doubted the projections because they believed women's rate of improvement would decrease.

Source: Staff and wire services, Milwaukee Journal, January, 1992

1. What data could you collect that would help you answer the question on page 3?

2. If you found that the men's times in an event were decreasing 0.08 second per year, by what amount must the women's times decrease to prove the researchers' claim?

The graph below shows men's and women's times for the 800-meter race.

5. Based on Problems 3 and 4 graph, point of intersection: approximately (2125, 84)

6. The point represents the location where the two lines intersect. The ordered pair gives the year that men and women will run the 800-meter race in the same time. This assumes that trends will continue at the same rate of change.

On the graph on Activity Sheet 1, draw a line that you think represents the trend for the men's times.

On the same graph, draw a line that you think represents the trend for the women's times.

Extend the two lines. Estimate their point of intersection.

What does this point represent in terms of the data?
LESSON 1

Systems of Equations

Materials: rulers, Activity Sheets 2 and 3, Lesson 1 Quiz
Technology: graphing calculators with list capabilities
Pacing: 2–3 class periods

Overview

This lesson introduces students to three methods of solving a system of equations. First, the students are asked to estimate the point of intersection of two lines on a graph. Next, the students use a graphing calculator to make a table of values for each equation and look for a common ordered pair in the table. The remaining part of the lesson is devoted to solving the system algebraically. The equations are given in the form $y = mx + b$. The algebraic method asks the students to use substitution and set both equations equal to each other.

Teaching Notes

This lesson assumes that students know how to write the equation of a line and are able to interpret the slope of the line. For review of these topics, you may wish to see Exploring Linear Relations for examples and development of these concepts. You may wish to present the graphical approach and the table approach on one day and the algebraic approach on another day.

When drawing a line that best summarizes the data, students do not need to draw the median-fit or least-squares regression line. The intent is for the students to draw an “eyeball” fit line. This does mean that students' equations may be different but their slopes should be approximately the same.

Technology

It is suggested that students use a graphing calculator to make a table of values for Problem 4. They could also use a spreadsheet instead of a graphing calculator. The focus should be to have students start the table by counting by 1s and then change the increment value ($\Delta$Tbl) to smaller values until the $y$-values for both equations are equal for a given $x$-value.

Steps for the TI-83:

1. Enter the equation $y = 0.97x - 1893$ into $Y_1$.
2. $2^{nd}$ TABLE
   Tbl Start = 1975
   $\Delta$Tbl = 1
3. $2^{nd}$ TABLE
   Use $\uparrow$
4. $2^{nd}$ TBLSET
   Tbl Start = 1982
   $\Delta$Tbl = .1

Follow-Up

Students could investigate other Olympic events, such as swimming events, in which both men and women compete. Students can also do research on the history of women’s competing in the Olympic Games.
Systems of Equations

Does your family subscribe to a daily paper?

If your family subscribes, who reads the paper?

What sections do people read the most often?

We often want to observe trends in related events. In this lesson, you will investigate the relationships between morning and evening newspaper circulation, as well as several other sets of data.

INVESTIGATE
Trends in circulation of daily newspapers have been changing in recent years. Some cities in the United States have both an evening and a morning paper while others have one daily newspaper. A recent trend for evening newspapers has been to either stop publishing or to convert to a morning paper. The graph on page 6 shows the total circulation in millions of morning and evening newspapers across the United States.
Solution Key

Discussion and Practice

1. a. Morning newspaper circulation is increasing.
   Evening newspaper circulation is decreasing.
   b. Evening papers may go out of business and morning papers will continue to increase in circulation.
   c. Owners of newspapers and advertisers; owners may want to know whether to keep the paper in production or not.


3. a. Slope of morning newspaper line is about 1; slope of evening newspaper line is about −1.
   b. Possible answers: Morning newspaper, \( y = 1x - 1950 \); evening newspaper, \( y = -1x + 2208 \)
   c. Possible answers: Morning newspaper circulation: about 50 million; evening newspaper circulation: about 8 million; extend the lines and estimate from the graph or use the equations in part b.
   d. Use the graph to see if the lines intersect near 32 million.
   e. Year: 1982; circulation: 30 million; estimate the point where the two lines intersect on the graph.

Discussion and Practice

1. Refer to the graph above.
   a. What trends do you observe in the plots?
   b. Describe what you think will happen in the next few years if the trends continue. This is often called extrapolation, or predicting beyond the given information.
   c. Who might be interested in these trends? Why?

2. Use the graph to find the total combined circulation of morning and evening newspapers for the years 1975, 1980, 1985, 1990, and 1995. Has the total circulation remained about the same since 1975?

3. On the graph on Activity Sheet 2, draw a line that you think best represents each set of data.
   a. Find the slope of each line. Describe the trends you observe in terms of the slopes.
   b. Write an equation of each line.
   c. Predict the circulation for evening and morning newspapers in 2000. How did you make your prediction?
   d. Tanya claims that in 1982 both morning and evening newspapers had a circulation of about 32 million. How can you evaluate her claim?
   e. When do you estimate that the circulation was the same for morning and evening newspapers? Approximately how large a circulation did each have when they were equal? Explain how you found your answer.
4. a. Answer depends on what equations are used.
   b. Compare values under Y1 and Y2 and find the years in which the Y1 values become greater than the Y2 values. Answers may vary depending on equations used, but the two years will probably include 1982.
   c. The year when evening and morning newspapers have the same circulation.

A set of two or more equations in the same variables is a system of equations. Finding the point where two or more lines intersect, or cross, is called solving a system of equations. A solution of a system of equations in two variables is a set of values that makes all the equations in the system true.

To solve some systems of equations, you can graph each equation and find the point where the graphs intersect.

Sometimes you can find the point where the lines intersect by constructing a table of values. Tanya drew lines on her graph and found the following equations: Y1 = 0.97X - 1893 and Y2 = -1.1X + 2210. She used her calculator to make the following table of values:

<table>
<thead>
<tr>
<th>X</th>
<th>Y1 = 0.97X - 1893</th>
<th>Y2 = -1.1X + 2210</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>32.45</td>
<td>26.5</td>
</tr>
<tr>
<td>1986</td>
<td>33.42</td>
<td>25.4</td>
</tr>
<tr>
<td>1987</td>
<td>34.39</td>
<td>24.3</td>
</tr>
<tr>
<td>1988</td>
<td>35.36</td>
<td>23.2</td>
</tr>
<tr>
<td>1989</td>
<td>36.33</td>
<td>22.1</td>
</tr>
<tr>
<td>1990</td>
<td>37.3</td>
<td>21</td>
</tr>
<tr>
<td>1991</td>
<td>38.27</td>
<td>19.9</td>
</tr>
</tbody>
</table>

X = 1991

4. a. Construct a table of values for the years from 1975 to 1985 for each of your equations.
   b. Look at a table of values for each equation. Is any ordered pair the same for the two equations?
   c. How can you determine from the table between what two years the ordered pairs will be the same? List the years.
   d. Change the increment for your table to 0.1. Are any of the ordered pairs the same? If not, continue to change the increment until you find the value for X for which the values in Y1 and Y2 are the same. What does the ordered pair tell you?
s. It is the same.

You can also find the solution to the system algebraically. Let
C represent the circulation and T the year. Rewrite the equa-
tions for this system as shown, letting \( C_m \) represent morning
newspaper circulation and \( C_e \) represent evening newspaper
circulation.

\[
C_m = 0.97T - 1893 \quad C_e = -1.1T + 2210
\]

You would like to know when \( C_m = C_e \).

Since \( C_m = 0.97T - 1893 \), you can substitute
\( 0.97T - 1893 \) for \( C_e \) in the second equation and solve for \( T \).

\[
\begin{align*}
0.97T - 1893 &= -1.1T + 2210 \\
0.97T + 1.1T &= 2210 + 1893 \\
2.07T &= 4103 \\
T &= \frac{4103}{2.07}
\end{align*}
\]

To find \( C_m \), substitute 1982.1 for \( T \) in the equation.

\[
C_m = 0.97(1982.1) - 1893
\]

The solution is (1982.1, 29.64).

The solution (1982.1, 29.64) means that at the beginning of
1982 the circulation of both morning and evening newspapers
was about 29.6 million.

s. Use 1982.1 for \( T \) in the equation for \( C_e \) and solve for
\( C_e \). How does your answer compare to the solution
(1982.1, 29.64)?

Summary
Some problems have relationships that can be described by a
set of two equations. To find values that make both equations
true, you have to solve the two equations at the same time.
You can do this
- by studying the graph to see where the lines intersect,
- by looking at a table of values for each equation to see where
  they match, or
- by using algebra and substituting from one equation into the
  other.
LESSON 1: SYSTEMS OF EQUATIONS

Practice and Applications

6.  
   a.  \( x = 9 \)
   b.  \( 4x + 51 = 175 \)
   c.  \( x = 31 \)
   d.  All three lines intersect at the point (31, 237).

   ![Graph](https://via.placeholder.com/150)

   \[ y = 2x + 175 \]
   \[ y = 237 \]
   \[ y = 6x + 51 \]

7.  a. \((25, 75)\)

   ![Graph](https://via.placeholder.com/150)

   \[ y_1 = 150 + 3x \]
   \[ y_2 = 75 - 2(x - 100) \]

   b. Yes; \( x = 25 \), \( y_1 = 225 \), \( y_2 = 225 \)

8.  a. Possible answer:

   ![Graph](https://via.placeholder.com/150)

   **Sales of Electronic and Conventional Pianos**

   **a**. Conventional pianos
   **b**. Electronic pianos


   Answers to parts b and c are based on part a graph.

   b. Electronic pianos: \( y = 7000x - 13,815,000 \); conventional pianos: \( y = -9000x + 18,060,000 \); point of intersection: \((1992.1875, 130,312.5)\)

   c. x-intercept = 2006.7; this is the first year when people will buy no conventional pianos.
9. a. City at sea level; Lander, WY
   b. Anchorage, AK; Tampa, FL
   c. Possible answer:

   Elevation and Number of Cloudy Days out of 365

<table>
<thead>
<tr>
<th>City</th>
<th>Elevation (feet)</th>
<th>Number of Clear Days per Year</th>
<th>Number of Cloudy Days per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albany, NY</td>
<td>275</td>
<td>67</td>
<td>190</td>
</tr>
<tr>
<td>Albuquerque, NM</td>
<td>5311</td>
<td>144</td>
<td>120</td>
</tr>
<tr>
<td>Anchorage, AK</td>
<td>114</td>
<td>64</td>
<td>238</td>
</tr>
<tr>
<td>Boise, ID</td>
<td>8210</td>
<td>111</td>
<td>153</td>
</tr>
<tr>
<td>Boston, MA</td>
<td>15</td>
<td>90</td>
<td>179</td>
</tr>
<tr>
<td>Billings, MT</td>
<td>3025</td>
<td>94</td>
<td>153</td>
</tr>
<tr>
<td>Lander, WY</td>
<td>5557</td>
<td>117</td>
<td>133</td>
</tr>
<tr>
<td>Milwaukee, WI</td>
<td>972</td>
<td>77</td>
<td>201</td>
</tr>
<tr>
<td>Minneapolis, MN</td>
<td>131</td>
<td>67</td>
<td>184</td>
</tr>
<tr>
<td>Mobile, AL</td>
<td>590</td>
<td>86</td>
<td>161</td>
</tr>
<tr>
<td>Nashville, TN</td>
<td>7</td>
<td>54</td>
<td>188</td>
</tr>
<tr>
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<tr>
<td>Tampa, FL</td>
<td>19</td>
<td>53</td>
<td>111</td>
</tr>
</tbody>
</table>

   Sources: Statistical Abstract of the United States, 1995

   d. Based on part c graph, 
   \[ y = -0.01x + 183.7 \]

   Does elevation, or the height above sea level, have any effect on the weather? Are the numbers of clear days and cloudy days affected by the elevation of a city? The table below contains the elevation and the numbers of clear and cloudy days per year for selected cities in the United States.

   a. What would it mean if a city had an elevation of zero? Which city has the highest elevation?
   b. Which city has the greatest number of cloudy days? The least?
   c. On the first graph on Activity Sheet 3, draw a line that best fits the data for (elevation, cloudy days).
   d. Write an equation for the line you drew in part c. This equation expresses the relationship between elevation and number of cloudy days per year.
e. Possible answer:

### Elevation and Number of Clear Days out of 365

<table>
<thead>
<tr>
<th>- Elevation (feet)</th>
<th>Number of Clear Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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</tr>
<tr>
<td>100</td>
<td>100</td>
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<tr>
<td>1100</td>
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<tr>
<td>1200</td>
<td>1200</td>
</tr>
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</table>

Answers to parts f and g are based on parts c and e graphs.

f. \( y = 0.0089x + 78.4 \)
g. Elevation of approximately 5570 feet; sample explanation: I solved the equation \(-0.01x + 183.7 = 0.0089x + 78.4\).

10. Check students' graphs.

a. (5, 16)
b. (-2.5, -2.5)
c. (8, 8)
d. (3, 12)

11. The values of the variables that make both equations true

12. a. The angular distance north or south of the equator, measured in degrees

- On the second graph on Activity Sheet 3, draw a line that best fits the data for (elevation, clear days).
- Write an equation for the line you drew in part e. This equation expresses the relationship between elevation and number of clear days per year.
- For what elevation will the expected number of cloudy days and the number of clear days be the same? Explain how you got your answer.
- Find the intersection point for each system of equations both by graphing and by finding the solution algebraically.
  a. \( y = 12 + 2(x - 3) \)
  b. \( y = x \)
  c. \( y = 2x - 8 \)
  d. \( y = 12 + 2(x - 3) \)
- When you solve a system of equations algebraically, what are you finding?

<table>
<thead>
<tr>
<th>City</th>
<th>Latitude ('N)</th>
<th>January ('F)</th>
<th>July ('F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atlanta, CA</td>
<td>33</td>
<td>51</td>
<td>69</td>
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<tr>
<td>Baltimore, MD</td>
<td>39</td>
<td>41</td>
<td>67</td>
</tr>
<tr>
<td>Jackson, MI</td>
<td>39</td>
<td>57</td>
<td>68</td>
</tr>
<tr>
<td>Bismarck, ND</td>
<td>46</td>
<td>28</td>
<td>56</td>
</tr>
<tr>
<td>Boise, ID</td>
<td>43</td>
<td>37</td>
<td>59</td>
</tr>
<tr>
<td>Cleveland, OH</td>
<td>40</td>
<td>23</td>
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</tr>
<tr>
<td>Chicago, IL</td>
<td>41</td>
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<tr>
<td>Detroit, MI</td>
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<td>31</td>
<td>61</td>
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<tr>
<td>Houston, TX</td>
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<td>62</td>
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</tr>
<tr>
<td>Key West, FL</td>
<td>27</td>
<td>72</td>
<td>80</td>
</tr>
<tr>
<td>New York, NY</td>
<td>49</td>
<td>37</td>
<td>69</td>
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<tr>
<td>Oklahoma City, OK</td>
<td>35</td>
<td>47</td>
<td>77</td>
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<tr>
<td>Portland, ME</td>
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<td>31</td>
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<tr>
<td>Portland, OR</td>
<td>44</td>
<td>44</td>
<td>56</td>
</tr>
<tr>
<td>Las Vegas, NV</td>
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<td>16</td>
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<tr>
<td>Las Vegas, CA</td>
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<td>76</td>
</tr>
<tr>
<td>Sarasota, FL</td>
<td>42</td>
<td>44</td>
<td>54</td>
</tr>
</tbody>
</table>


a. What is latitude? If you don't know, refer to a dictionary for the definition.

**Systems of Equations**
False; the greater the degree of latitude, the colder it is in January and July.

Possible answer:

Average January High and July Low Temperatures

Estimate for intersection point: (8, 102)

Answers to parts d, e, and f are based on part c graph.

d. January: \( y = -1.92x + 117.02 \);
July: \( y = -1.19x + 111.6 \)

e. Average July low temperature drops 11.9 degrees.

f. Point of intersection: (7.42, 102.76)

g. The point of intersection tells the latitude where the average high January temperature and the average low July temperature are equal.

13. Possible answer: 1978.7; I used the data to write the equations
\( y = 4x - 7,825 \) and \( y = -7.867x + 15,656.7 \). Then I solved the equations.

14. a. Time rate of change = -0.1; Newsweek rate of change = 0.017; U.S. News & World Report rate of change = 0.033

b. Yes; equation for Time: \( y = -0.1(x - 1985) + 4.7 \); equation for Newsweek: \( y = 0.017(x - 1985) + 3.1 \); point of intersection: (1998.7, 3.33)
c. Equation for U.S. News & World Report: \( y = 0.033(x - 1985) + 2.0 \)
Point of intersection (2053.7, 4.27)
d. No; rates will probably not remain constant as people find other means to read about the news.

15. Possible lines are given and equations are based on those lines.

**Men's 200-Meter Run**

<table>
<thead>
<tr>
<th>Year</th>
<th>Male Runner</th>
<th>Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900</td>
<td>Walter Tewksbury, US</td>
<td>22.2s</td>
</tr>
<tr>
<td>1904</td>
<td>Archie Hahn, US</td>
<td>21.6s</td>
</tr>
<tr>
<td>1908</td>
<td>Robert Kene, Canada</td>
<td>22.5s</td>
</tr>
<tr>
<td>1912</td>
<td>Ralph Craig, US</td>
<td>21.7s</td>
</tr>
<tr>
<td>1916</td>
<td>Allan Waddington, US</td>
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</tr>
<tr>
<td>1920</td>
<td>Jackson Scholz, US</td>
<td>21.8s</td>
</tr>
<tr>
<td>1924</td>
<td>Percy Williams, Canada</td>
<td>21.6s</td>
</tr>
<tr>
<td>1928</td>
<td>Eddie Tolan, US</td>
<td>21.2s</td>
</tr>
<tr>
<td>1932</td>
<td>Jesse Owens, US</td>
<td>20.7s</td>
</tr>
<tr>
<td>1936</td>
<td>Joe monstrous, US</td>
<td>20.7s</td>
</tr>
<tr>
<td>1940</td>
<td>Milt Fountain, US</td>
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<tr>
<td>1944</td>
<td>Archie Hahn, US</td>
<td>21.8s</td>
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<tr>
<td>1948</td>
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<tr>
<td>1952</td>
<td>Andrew Spenley, US</td>
<td>20.7s</td>
</tr>
<tr>
<td>1956</td>
<td>Bobby Morrow, US</td>
<td>20.6s</td>
</tr>
<tr>
<td>1960</td>
<td>Betty Cuthbert, Australia</td>
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<td>1964</td>
<td>Harry Car, US</td>
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<tr>
<td>1968</td>
<td>Mannie Smith, US</td>
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<td>1972</td>
<td>Valery Borzov, USSR</td>
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</tr>
<tr>
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<td>Donald Quarrie, Jamaica</td>
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<tr>
<td>1980</td>
<td>Pietro Mennea, Italy</td>
<td>20.17s</td>
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<td>1988</td>
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<tr>
<td>1992</td>
<td>Mike Marsh, US</td>
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</tr>
<tr>
<td>1996</td>
<td>Michael Johnson, US</td>
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**Women's 200-Meter Run**

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<th>Year</th>
<th>Female Runner</th>
<th>Time (seconds)</th>
</tr>
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<tbody>
<tr>
<td>1900</td>
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</tr>
<tr>
<td>1904</td>
<td>Marjorie Jackson, Australia</td>
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<td>1920</td>
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<td>Valerie Stecher, Germany</td>
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</tr>
<tr>
<td>1952</td>
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<tr>
<td>1956</td>
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<tr>
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<td>1964</td>
<td>Edith McKinley, US</td>
<td>23.0s</td>
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<tr>
<td>1968</td>
<td>Uta Rettig, Germany</td>
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<tr>
<td>1972</td>
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<td>1976</td>
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<tr>
<td>1980</td>
<td>Wilma Rudolph, US</td>
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<tr>
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<td>Valerie Stecher, Germany</td>
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<td>1988</td>
<td>Cornelia Hagemann, Germany</td>
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</tr>
<tr>
<td>1992</td>
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<tr>
<td>1996</td>
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**Men's 400-Meter Run**

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<td>1912</td>
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<td>1996</td>
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**Women's 800-Meter Run**

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<th>Female Runner</th>
<th>Time (seconds)</th>
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<tr>
<td>1996</td>
<td>Remedios Garcia, Mexico</td>
<td>22.1s</td>
</tr>
</tbody>
</table>
LESSON 1: SYSTEMS OF EQUATIONS

STUDENT PAGE 14

Women's 800-Meter Run

Men's 1500-Meter Run

Women's 1500-Meter Run

b. Possible answers:
Equation of men's 200-meter run:
\[ y = -0.028x + 75.7 \]
Equation of women's 200-meter run:
\[ y = -0.055x + 131 \]
Equation of men's 400-meter run:
\[ y = -0.075x + 193 \]
Equation of women's 400-meter run:
\[ y = -0.117x + 282.2 \]
Equation of men's 800-meter run:
\[ y = -0.142x + 387 \]
Equation of women's 800-meter run:
\[ y = -0.321x + 757.7 \]
Equation of men's 1500-meter run:
\[ y = -0.333x + 880 \]

Equation of women's 1500-meter run:
\[ y = -0.221x + 677 \]

In the 200-, 400-, and 800-meter runs, the women's rate of change was about twice that of men. In the 1500-meter run, the men's rate of change is about twice the women's rate of change.

d. Answers will vary.
16. a. Olympic times in particular events; specifically, the slopes of the best-fit lines for the data

b. In order to predict the future, we must assume that the rates of change will remain constant. Other researchers are probably saying that it is impossible to maintain these decreases in time.

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>1896</td>
<td>3000m</td>
<td>Edwin Flack, Australia</td>
<td>4m33.2s</td>
</tr>
<tr>
<td>1900</td>
<td>3000m</td>
<td>Jarno Tiihonen, Finland</td>
<td>4m56.4s</td>
</tr>
<tr>
<td>1904</td>
<td>3000m</td>
<td>Albert Hill, GB</td>
<td>4m18s</td>
</tr>
<tr>
<td>1908</td>
<td>3000m</td>
<td>Harry Lame, Finland</td>
<td>4m23.2s</td>
</tr>
<tr>
<td>1912</td>
<td>3000m</td>
<td>Luigi Brocasi, Italy</td>
<td>4m15.2s</td>
</tr>
<tr>
<td>1920</td>
<td>3000m</td>
<td>Jack Lovelock, New Zealand</td>
<td>3m47.8s</td>
</tr>
<tr>
<td>1924</td>
<td>3000m</td>
<td>Hans Helmer, Sweden</td>
<td>3m49.8s</td>
</tr>
<tr>
<td>1928</td>
<td>3000m</td>
<td>Joseph Barthelet, Luxembourg</td>
<td>3m45.2s</td>
</tr>
<tr>
<td>1932</td>
<td>3000m</td>
<td>Ron Delaney, Ireland</td>
<td>3m45.2s</td>
</tr>
<tr>
<td>1936</td>
<td>3000m</td>
<td>Erich Eckstein, Austria</td>
<td>3m35.6s</td>
</tr>
<tr>
<td>1940</td>
<td>3000m</td>
<td>Peter Snell, New Zealand</td>
<td>3m38.1s</td>
</tr>
<tr>
<td>1944</td>
<td>3000m</td>
<td>Ediho Kirabo, Kenya</td>
<td>3m34.9s</td>
</tr>
<tr>
<td>1948</td>
<td>3000m</td>
<td>Peri Zdansky, Finland</td>
<td>3m39.4s</td>
</tr>
<tr>
<td>1952</td>
<td>3000m</td>
<td>Tatsuya Kazama, USSR</td>
<td>3m56.6s</td>
</tr>
<tr>
<td>1956</td>
<td>3000m</td>
<td>Sebrtmane Cao, China</td>
<td>3m53.9s</td>
</tr>
<tr>
<td>1960</td>
<td>3000m</td>
<td>Gabriele Dorsa, Italy</td>
<td>3m51.7s</td>
</tr>
<tr>
<td>1964</td>
<td>3000m</td>
<td>Peter Rono, Kenya</td>
<td>3m35.9s</td>
</tr>
<tr>
<td>1968</td>
<td>3000m</td>
<td>Hasiba Aroub, Algeria</td>
<td>3m35.7s</td>
</tr>
<tr>
<td>1972</td>
<td>3000m</td>
<td>Noureddine Morcelle, Algeria</td>
<td>3m35.78s</td>
</tr>
</tbody>
</table>

The researchers Brian Whipp and Susan Ward wrote, "None of the current women's world-record holders at these events could even meet the men's qualifying standard to compete in the 1992 Olympic games. However, it is the rates of improvement that are so strikingly different—the gap is progressively closing."

a. What numerical information were Whipp and Ward using to make their claim?

b. Other researchers doubted the projections. List some of the reasons they might have given for their skepticism.
Lesson 2

Lines with the Same Slope

Materials: rulers, Activity Sheet 4
Technology: graphing calculators (optional)
Pacing: 1 class period

Overview
This lesson presents data sets that have approximately the same rate of change, or slope. The lesson opens with a data set comparing the percent of homes wired for cable to the percent of homes subscribing to cable. Students are asked to take values from the graph and place them in a table. Students then find the difference between the percent wired for cable and the percent that subscribe to cable. These percents will not be exactly equal but students should begin to make the connection that for parallel lines the difference is constant. The goal of this lesson is for students to conclude that lines with the same slope are parallel.

Teaching Notes
The focus of the lesson is on the slope of the lines. Having completed Lesson 1, students should be able to interpret the slope and conclude that lines with the same slope are parallel. In some of the examples, the lines that students draw will not have slopes that are exactly equal. Some students will want to say that the lines are not parallel. Suggest that students find the point of intersection and interpret this point.

Technology
The list feature of the graphing calculator can be used for Problem 1. Students can enter the years into L1 and the percent wired into L2 and percent subscribed into L3. L4 can be defined as the difference between L2 and L3. You might have students make a scatter plot of the ordered pairs (years, difference) or (L1, L4). The graph should be an approximately horizontal line showing, graphically, that the difference is approximately constant. Students can also use the graphing calculator to find the median-fit or least-squares regression line for each set of data and then compare the slopes of these lines.

Follow-Up
Have students use an almanac to update the cable data and determine if the rate of change for each line has remained about the same.
LESSON 2

Lines with the Same Slope

Do two lines always have to intersect? How can you tell?

If two lines do not intersect, what characteristics will their graphs have? What is true about their equations?

OBJECTIVES
Recognize that parallel lines have the same slope or rate of change.
Recognize systems that do not have a solution.

INVESTIGATE
The scatter plot below shows the percent of homes that are wired for cable and the percent of homes subscribing to a cable TV service for the years 1980 to 1990.

LESSON 2: LINES WITH THE SAME SLOPE

Solution Key

Discussion and Practice

1. a. Percent wired for cable and percent subscribing to cable are increasing at about the same rate.
   b. See table below.
   c. From 1984, the difference remained about the same.

2. a. Possible answer:

   ![Graph showing cable TV usage over the years]

   b. It appears that they will never be the same.
   c. They have approximately the same rate of change; or the lines are nearly parallel, diverging very slightly.

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent Wired</th>
<th>Percent Subscribing</th>
<th>Difference Between Percent Wired and Percent Subscribing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>47</td>
<td>26</td>
<td>21</td>
</tr>
<tr>
<td>1981</td>
<td>50</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>1982</td>
<td>59</td>
<td>35</td>
<td>24</td>
</tr>
<tr>
<td>1983</td>
<td>65</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>1984</td>
<td>71</td>
<td>42</td>
<td>29</td>
</tr>
<tr>
<td>1985</td>
<td>78</td>
<td>49</td>
<td>29</td>
</tr>
<tr>
<td>1986</td>
<td>80</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>1987</td>
<td>81</td>
<td>52</td>
<td>29</td>
</tr>
<tr>
<td>1988</td>
<td>88</td>
<td>56</td>
<td>32</td>
</tr>
<tr>
<td>1989</td>
<td>89</td>
<td>58</td>
<td>31</td>
</tr>
<tr>
<td>1990</td>
<td>90</td>
<td>60</td>
<td>30</td>
</tr>
</tbody>
</table>

d. Based on part a graph, slope for % wired line is ≈4.5; slope for % subscribing line is ≈3.5; the two lines are not parallel because the two slopes are not equal.
3. a. 10 pounds  
   b. 5 pounds  
   c. They are equal.

4. a. No; the algebra was correct. 
   b. Both lines have the same slope. The lines are parallel and will never intersect, so they will not have a point of intersection. 
   c. No; the lines do not intersect so there is no solution.

3. The scatter plot below shows the average weight of American men by height and age for men 20–24 years old and 60–69 years old.

   ![Scatter Plot]

   a. For each height, find the average difference in weight between a 20–24-year-old and a 60–69-year-old.
   b. For each 1-inch increase in height, what is the increase in weight for a 20–24-year-old and a 60–69-year-old?
   c. What do the values you found in part b tell you about the rate of change in weight for the two age groups?

4. Sam wanted to find where the two lines \( y_1 = 2x - 10 \) and \( y_2 = 15 + 2(x - 4) \) intersect. His solution is shown here.

   \[
   \begin{align*}
   y_1 &= 2x - 10 \\
   y_2 &= 15 + 2(x - 4) \\
   2x - 10 &= 15 + 2x - 8 \\
   2x - 10 &= 2x + 7 \\
   -10 &= 7 \\
   \end{align*}
   \]

   a. Did he make a mistake? If so, where?
   b. Describe the graphs of the two lines. How does the graph help you understand the problem?
   c. Can Sam conclude that \( x = -10 \) and \( y = 7 \)? Explain.
LESSON 2: LINES WITH THE SAME SLOPE

Practice and Applications

5. **a.** The lines are parallel, because they have the same slope. 5.
   **b.** The lines will intersect, because they have different slopes, $-2$ and 2.

6. Possible answer:

   Prices for Honda Civics, Toyota Celicas, Ford Mustangs

   ![](chart.png)

   - Civic
   - Celica
   - Mustang

   **Summary**

   If two lines have the same slope, they are parallel and will not have a point of intersection. If two equations have the same slope, you must determine whether they represent the same line or parallel lines. You can do this by investigating the points that work in the equations or the graphs of the lines they represent. Or you can find the solution algebraically.

   - If two distinct lines have the same slope, or rate of change, then they are parallel and have no points in common.
   - If two distinct lines intersect, their slopes are different, and they have one point in common.

   **Practice and Applications**

   5. Make a conjecture about the relation between the graphs of the lines represented by the pairs of equations in each problem. Verify your conjecture in some way.
   **a.** $C = 250 + 5(x-1980)$
   **b.** $r = 18 + 2(x-4)$

   6. Below are the list prices in dollars for new cars for the years 1971-1997. On the same set of axes, make a plot of years and prices for the Honda Civic, the Toyota Celica, and the Ford Mustang. Then fit a line to each set of data.

<table>
<thead>
<tr>
<th>Year</th>
<th>Honda Civic</th>
<th>Chevrolet Camaro</th>
<th>Toyota Celica</th>
<th>Ford Mustang</th>
<th>Mercury Cougar</th>
<th>BMW</th>
<th>Corvette</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>1395</td>
<td>3790</td>
<td>2847</td>
<td>3783</td>
<td>4046</td>
<td>5945</td>
<td>6327</td>
</tr>
<tr>
<td>1972</td>
<td>2150</td>
<td>3829</td>
<td>3150</td>
<td>3723</td>
<td>4045</td>
<td>8230</td>
<td>7367</td>
</tr>
<tr>
<td>1973</td>
<td>2799</td>
<td>4735</td>
<td>3604</td>
<td>4906</td>
<td>6371</td>
<td>10,605</td>
<td>9,426</td>
</tr>
<tr>
<td>1974</td>
<td>2849</td>
<td>5423</td>
<td>3252</td>
<td>4814</td>
<td>6225</td>
<td>14,640</td>
<td>11,506</td>
</tr>
<tr>
<td>1975</td>
<td>3649</td>
<td>5897</td>
<td>4994</td>
<td>5339</td>
<td>6423</td>
<td>20,185</td>
<td>12,556</td>
</tr>
<tr>
<td>1976</td>
<td>5189</td>
<td>5142</td>
<td>7554</td>
<td>7191</td>
<td>8762</td>
<td>24,005</td>
<td>15,144</td>
</tr>
<tr>
<td>1977</td>
<td>4090</td>
<td>5852</td>
<td>8824</td>
<td>8406</td>
<td>10,735</td>
<td>24,760</td>
<td>19,340</td>
</tr>
<tr>
<td>1978</td>
<td>4879</td>
<td>10,273</td>
<td>5949</td>
<td>8441</td>
<td>11,615</td>
<td>20,970</td>
<td>26,901</td>
</tr>
<tr>
<td>1979</td>
<td>7566</td>
<td>11,874</td>
<td>12,808</td>
<td>9946</td>
<td>14,642</td>
<td>24,070</td>
<td>26,834</td>
</tr>
<tr>
<td>1980</td>
<td>6473</td>
<td>15,273</td>
<td>5949</td>
<td>8441</td>
<td>11,615</td>
<td>20,970</td>
<td>26,901</td>
</tr>
<tr>
<td>1981</td>
<td>9140</td>
<td>15,198</td>
<td>15,528</td>
<td>11,145</td>
<td>15,995</td>
<td>25,520</td>
<td>32,445</td>
</tr>
<tr>
<td>1982</td>
<td>3655</td>
<td>12,454</td>
<td>12,688</td>
<td>11,823</td>
<td>16,089</td>
<td>26,700</td>
<td>33,410</td>
</tr>
<tr>
<td>1983</td>
<td>4730</td>
<td>15,379</td>
<td>15,863</td>
<td>12,847</td>
<td>17,823</td>
<td>26,200</td>
<td>26,230</td>
</tr>
<tr>
<td>1984</td>
<td>10,130</td>
<td>17,336</td>
<td>19,410</td>
<td>17,550</td>
<td>18,960</td>
<td>34,230</td>
<td>22,972</td>
</tr>
<tr>
<td>1985</td>
<td>12,449</td>
<td>18,930</td>
<td>20,825</td>
<td>18,525</td>
<td>19,150</td>
<td>35,060</td>
<td>38,160</td>
</tr>
</tbody>
</table>

Source: Kelley Blue Book
Lesson 2: Lines with the Same Slope

a. Prices are increasing at different rates.
b. Based on the graph, slope of Civic line = $3903/yr; slope of Celica line = $7003/yr; slope of Mustang line = $5103/yr
c. The Celica is the most expensive, then the Mustang, and finally the Civic.

7. a. Possible answer:

```
<table>
<thead>
<tr>
<th>Attendance at Disney World and Universal Studios</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>1988</td>
</tr>
<tr>
<td>1990</td>
</tr>
<tr>
<td>1992</td>
</tr>
<tr>
<td>1994</td>
</tr>
<tr>
<td>1996</td>
</tr>
<tr>
<td>1998</td>
</tr>
</tbody>
</table>
```

Attendance at Disney World and Universal Studios will never be the same, as the lines appear to be nearly parallel, diverging only slightly. Answers to parts b and c are based on part a graph.
b. Slope of the Disney line = 1.0; slope of the Universal line = 0.67
c. Disney prediction: 36 million; Universal prediction: 10 million
ASSESSMENT

Assessment for Unit I

Materials: rulers, graph paper, Activity Sheet 5
Technology: graphing calculators (optional)
Pacing: 1 class period or homework

Overview
Problems 1 and 2 assess students' understanding of Lesson 1. Problem 3 assesses the objectives of Lesson 2, and Problem 4 asks students to solve systems of equations algebraically.

Teaching Notes
This assessment can be used in a number of ways. It can be used strictly as an assessment that is completed by students in one class period. It could also be used as a take-home test or as additional practice on the objectives of Lessons 1 and 2. Students will be asked throughout the rest of the module to solve systems of equations, so it is important that students are able to find the intersection point of two equations.

Technology
Graphing calculators are optional. You may wish to allow students to use graphing calculators with Problem 3.

Follow-Up
Students can use ads from the newspaper to collect data on the cost of lumber in their location. These prices could be used as a comparison of the prices presented in Problem 2. Students could use an almanac to update the data on life expectancies. They could also find the data on life expectancies for women.
Solution Key

1. a. Slope of incandescent line = 5.2; slope of fluorescent line = 1; for every additional 1000 hours of lighting, the slope expresses the additional amount of money it costs to burn this type of bulb.

b. Equation of incandescent line: y = 5.2x + 1; equation of fluorescent line: y = x + 15; the y-intercept is the initial cost of the bulb.

c. (3.3, 18.3); both bulbs cost the same amount if they are used for about 3 hours and 20 minutes.

Assessment for Unit I

1. a. Find the slope of each line. Describe the slope in terms of the data.

b. Write an equation for each line. Describe the y-intercept of each line in terms of the data.

c. Solve the system of equations. Then interpret your answer.
2. a. Possible answer:

![Graph showing the costs of lumber at a local lumber store. The prices are for 2-inch-by-4-inch and 2-inch-by-8-inch boards of various lengths.]

Equation of 2-by-4 line: \( y = 0.31x - 0.25 \); equation of 2-by-8 line: \( y = 0.64x - 0.28 \)

Answers to parts b, c, and d are based on part a answer.

b. 2-by-4 line: For each additional foot in length, the cost of the lumber increases by 31 cents; 2-by-8 line: For each additional foot in length, the cost of the lumber increases by 64 cents.

c. \( $1.92 \)

d. \((0.9, -0.22)\); this point is very close to \((0, 0)\), which means that for either size, to buy nothing costs nothing.

3. The life expectancy of males born in certain years is given in the table below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Life Expectancy (years)</th>
<th>Year</th>
<th>Life Expectancy (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>White Male</td>
<td>Black Male</td>
<td></td>
</tr>
<tr>
<td>1920</td>
<td>54.4</td>
<td>45.5</td>
<td>1980</td>
</tr>
<tr>
<td>1930</td>
<td>59.7</td>
<td>47.3</td>
<td>1985</td>
</tr>
<tr>
<td>1940</td>
<td>62.1</td>
<td>53.5</td>
<td>1986</td>
</tr>
<tr>
<td>1950</td>
<td>66.5</td>
<td>59.1</td>
<td>1987</td>
</tr>
<tr>
<td>1960</td>
<td>67.6</td>
<td>61.7</td>
<td>1988</td>
</tr>
<tr>
<td>1965</td>
<td>67.8</td>
<td>61.1</td>
<td>1989</td>
</tr>
<tr>
<td>1970</td>
<td>68.0</td>
<td>61.3</td>
<td>1990</td>
</tr>
<tr>
<td>1975</td>
<td>69.5</td>
<td>63.7</td>
<td>1992</td>
</tr>
</tbody>
</table>

3. a. Possible answer:

![Graph showing Life Expectancy for White Males and Black Males]

Equation for white-male line: 
\[ y = 0.23x - 384 \]  
Equation for black-male line: 
\[ y = 0.295x - 520 \]

b. Based on part a answer, slope of white-male line = 0.23; slope of black-male line = 0.295

Possible comparison paragraph:
Life expectancy of white males is higher than that of black males at this time. The rate of change for life expectancy of black males is 0.295, which is greater than the 0.23 for white males. This means that if the trend continues, in about the year 2114 black males and white males will have the same life expectancy of 102 years. This seems unlikely.

4. a. (1.83, 1.33)
   b. (9.6, 4.8)
   c. (1.841269, 6.64)

STUDENT PAGE 23
Graphing Inequalities
INTRODUCTORY ACTIVITY

Estimating the Number of Raisins

Materials: \( \frac{1}{2} \)-ounce boxes of raisins (one for each student), graph paper, rulers
Technology: graphing calculators (optional)
Pacing: \( \frac{1}{2} \) class period

Overview
The purpose of this activity is to have students begin to think about what it means for a point to be above or below a line. Students are asked to estimate the number of raisins in a \( \frac{1}{2} \)-ounce box of raisins and to compare their estimates to the actual counts.

Teaching Notes
When students are working through this activity, stress that a point below the line \( y = x \) represents a point where the \( x \)-coordinate is greater than the \( y \)-coordinate. If students struggle with this concept, have them list ordered pairs and compare the \( x \)- and \( y \)-coordinates. The first problem in Lesson 3 presents another activity that is very similar to this one. You do not have to introduce the < and > symbols, since they are first used in Lesson 3.

The module Exploring Linear Relations in the Data-Driven Mathematics series contains an activity in which students estimate ages of famous people. This activity could be used in place of the activity here or as another example.

Technology
Students can use graphing calculators to make the scatter plot. The estimates can be entered into L1 and the actual counts into L2. The line \( Y = X \) can then be graphed, and students can use their graph for the problems in this activity.

Follow-Up
You may wish to have students construct a line graph of the actual number of raisins in each box. They could find the average number and write a description of the graph and how their number compared to the class average.
Estimating the Number of Raisins

How many raisins do you suppose there are in a small box?

How did you make your estimate?

Do you think your estimate is greater than or less than the actual number of raisins in the box?

In this unit, you will investigate data that are less than or greater than a given standard, such as the weight or number of raisins in a box of a given size.

EXPLORE
Each of you should have your own \(\frac{1}{2}\)-ounce box of raisins. Look carefully at your box of raisins without opening it. Then write your estimate of the number of raisins contained in the box. Now open your box and count the raisins. How close were you to the actual number? Was your estimate high or low?
INTRODUCTORY ACTIVITY: ESTIMATING THE NUMBER OF RAISINS

Solution Key

1. Answers will vary.
2. Answers will vary.
3. On the line $y = x$
4. The estimate was less than the actual count.
5. The estimate was greater than the actual count.

STUDENT PAGE 28

1. Collect from each student in class his or her estimate for the number of raisins and the actual number. Record the data in a table similar to the one below:

<table>
<thead>
<tr>
<th>Student</th>
<th>Estimate</th>
<th>Actual Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Make a scatter plot of the ordered pairs (estimate, actual number) for the class data.
3. Where will the point lie if your estimate is correct? Draw the line that represents 100% accurate estimates. Write an equation for the line.
4. What does it mean if a point is above this line?
5. Describe, in terms of estimating raisins, a point found below the line.
LESSON 3

Shading a Region

Materials: graph paper, rulers, Activity Sheet 6
Technology: calculators (optional)
Pacing: 1 class period

Overview

This lesson begins with an activity similar to the estimating-raisins introductory activity. It is used to introduce the use of the symbols < and > to describe a region that is below or above the line \( y = x \).

Teaching Notes

The main focus of the lesson is to have students understand how to describe a region on a graph with the symbols <, \( \leq \), >, and \( \geq \). Be sure students know the difference between the symbols < and \( \leq \) and the symbols > and \( \geq \), and that they can graphically represent inequalities involving these symbols. It is not appropriate to have students draw a median-fit or least-squares regression line. Initially, Problem 8 may be difficult for some students. You may wish to review graphing the lines of the type \( x = 5 \) and \( y = 3 \) before students work through Problem 8.

Technology

Graphing calculators are not needed. Students should draw the line \( y = x \) and shade the region by hand.

Follow-Up

Students could generate a list of situations that involve the comparison of two variables in which it would be appropriate to draw the line \( y = x \).
LESSON 3: SHADING A REGION

STUDENT PAGE 29

LESSON 3

Shading a Region

How good were you at estimating the number of raisins in a \( \frac{1}{2} \)-ounce box?

How good do you think you are at estimating the number of calories in certain fast-food items?

When comparing prices between two stores, how can you tell whether one store is generally higher priced or lower priced than the other?

In this lesson, you will use inequalities to investigate these questions.

INVESTIGATE

Many fast-food chains base their advertising on the quality and prices of their food or on claims that their food is nutritious and tasty.

Discussion and Practice

A single hamburger at McDonald's has 270 calories. On page 30 is a list of some other items from McDonald's.

OBJECTIVE

Find the solutions to an inequality using the line \( y = x \).
Solution Key

Discussion and Practice

1. Estimates will vary.

<table>
<thead>
<tr>
<th>Item</th>
<th>Estimated Calories</th>
<th>Actual Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Bag of French Fries</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>Filet o' Fish</td>
<td>360</td>
<td></td>
</tr>
<tr>
<td>6 Chicken McNuggets</td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>Chicken Sandwich</td>
<td>510</td>
<td></td>
</tr>
<tr>
<td>Big Mac</td>
<td>530</td>
<td></td>
</tr>
<tr>
<td>Small Chocolate Shake</td>
<td>340</td>
<td></td>
</tr>
<tr>
<td>Small Diet Coke</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Apple Pie</td>
<td>260</td>
<td></td>
</tr>
<tr>
<td>Cheeseburger</td>
<td>320</td>
<td></td>
</tr>
<tr>
<td>Quarter Pounder</td>
<td>430</td>
<td></td>
</tr>
</tbody>
</table>

2. a. Answers will vary.
   b. On the line $y = x$
   c. The estimate was less than the actual value or the actual value was greater than the estimate. The actual value is greater than the estimate.
   d. Answers will vary.
   e. The estimate is greater than the actual value or the actual value is less than estimate; $y < x$

1. Copy the table above and estimate the number of calories to the nearest 10 calories in each item. Then your teacher will give you the correct number of calories to the nearest 10 calories.

2. To help you determine how well you estimated the number of calories, complete the following.
   a. Make a scatter plot of the ordered pairs (estimated calories, actual calories).
   b. Where would the point lie if your estimate were correct? Draw the line that represents 100% accurate estimates. Write an equation for that line.
   c. What does it mean if a point is above this line? Shade the region above the line. Make a list of five ordered pairs in the shaded region. What is true about the relationship between the actual calories (y-value) and the estimated calories (x-value) in each ordered pair you wrote?
   d. A point in the region shaded above the line can be described with the inequality $y > x$. Verify that the ordered pairs you listed above satisfy this inequality.
   e. Describe a point found in the region below the line in words. Write a symbolic description with an inequality.
3. **Possible answer:**

   **Grocery Prices**

   3. **a.** Listed below are randomly selected items and their prices from two grocery stores.

<table>
<thead>
<tr>
<th>Item</th>
<th>Store A</th>
<th>Store B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soda (12-pack)</td>
<td>$1.48</td>
<td>$3.68</td>
</tr>
<tr>
<td>Head of lettuce</td>
<td>$0.98</td>
<td>$1.79</td>
</tr>
<tr>
<td>Salad dressing (24 oz)</td>
<td>$2.38</td>
<td>$2.88</td>
</tr>
<tr>
<td>Punch (1/2 gal)</td>
<td>$0.88</td>
<td>$0.98</td>
</tr>
<tr>
<td>Catalo (50 gal)</td>
<td>$1.58</td>
<td>$2.75</td>
</tr>
<tr>
<td>Peanut butter (28 oz)</td>
<td>$5.38</td>
<td>$5.68</td>
</tr>
<tr>
<td>1 pouch of Raisins</td>
<td>$0.19</td>
<td>$0.22</td>
</tr>
<tr>
<td>1 box of Hamburger</td>
<td>$0.82</td>
<td>$0.48</td>
</tr>
<tr>
<td>Can of corn (14 oz)</td>
<td>$0.40</td>
<td>$0.69</td>
</tr>
<tr>
<td>Box of rice</td>
<td>$1.06</td>
<td>$1.35</td>
</tr>
<tr>
<td>Pizza sauce (12 oz)</td>
<td>$1.80</td>
<td>$1.98</td>
</tr>
<tr>
<td>Baked beans (27 oz)</td>
<td>$0.78</td>
<td>$0.99</td>
</tr>
<tr>
<td>Mushroom soup (10.5 oz)</td>
<td>$0.49</td>
<td>$0.55</td>
</tr>
<tr>
<td>Breakfast cereal (21 oz)</td>
<td>$2.18</td>
<td>$2.38</td>
</tr>
<tr>
<td>Canned vegetables (16 oz)</td>
<td>$0.98</td>
<td>$6.55</td>
</tr>
</tbody>
</table>

   **b.** On the line $y = x$
   
   **c.** Below the line $y = x$; $y > x$
   
   **d.** Store A has cheaper prices because many of the points are above the line $y = x$, which means that the x-coordinate (Store A) is less than the y-coordinate (Store B).

4. **a.** $y = x$
   
   **b.** $y < x$
**Practice and Applications**

5. **a.** Any point above the line $y = x$
   
   **b.** Any point below the line $y = x$

   **c.** Store B is slightly cheaper than Store C, because there are more points above the line $y = x$, which means that prices in store B are less than prices in store C.

---

**Summary**

To compare two quantities with the same units, you can draw the line $y = x$. This line divides the plane into two regions. One region contains points where $y$ is greater than $x$, and the other region contains points where $y$ is less than $x$. When graphing the inequality $y > x$ or the inequality $y < x$, the line $y = x$ is usually shown as a dashed line to indicate that the points on the line do not satisfy the inequality.

**Practice and Applications**

5. In the graph that follows, the prices from Store C are plotted against the prices from store B.

   a. Identify a point where Store B has lower prices than Store C does. Describe the costs for that item.
   
   b. Identify a point where Store B is more expensive than Store C. Describe the costs for that item.
   
   c. Make a sketch of the plot and shade in the area where Store B has lower prices than Store C. In general, which store has lower prices? How can you tell?
6. The following data are the percents of teens and children who watched America's favorite prime time network television programs in 1992-93, according to the Nielsen Media Research.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Program</th>
<th>% Teens</th>
<th>% Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60 Minutes</td>
<td>28.3</td>
<td>29.9</td>
</tr>
<tr>
<td>2</td>
<td>Roseanne</td>
<td>15.3</td>
<td>17.2</td>
</tr>
<tr>
<td>3</td>
<td>Home Improvement</td>
<td>13.4</td>
<td>11.8</td>
</tr>
<tr>
<td>4</td>
<td>Murphy Brown</td>
<td>6.6</td>
<td>4.9</td>
</tr>
<tr>
<td>5</td>
<td>Murder, She Wrote</td>
<td>2.6</td>
<td>2.4</td>
</tr>
<tr>
<td>6</td>
<td>Coach</td>
<td>10.4</td>
<td>8.4</td>
</tr>
<tr>
<td>7</td>
<td>NFL Monday Night Football</td>
<td>7.5</td>
<td>4.1</td>
</tr>
<tr>
<td>8</td>
<td>CBS Sunday Night Movie</td>
<td>3.9</td>
<td>2.6</td>
</tr>
<tr>
<td>9</td>
<td>Cheers</td>
<td>7.0</td>
<td>5.1</td>
</tr>
<tr>
<td>10</td>
<td>Full House</td>
<td>13.0</td>
<td>10.9</td>
</tr>
<tr>
<td>11</td>
<td>Northern Exposure</td>
<td>3.9</td>
<td>2.0</td>
</tr>
<tr>
<td>12</td>
<td>Basquiat</td>
<td>6.4</td>
<td>2.3</td>
</tr>
<tr>
<td>13</td>
<td>20/20</td>
<td>4.6</td>
<td>4.8</td>
</tr>
<tr>
<td>14</td>
<td>CBS Tuesday Night Movie</td>
<td>5.0</td>
<td>4.2</td>
</tr>
<tr>
<td>15</td>
<td>Love and War</td>
<td>4.5</td>
<td>3.1</td>
</tr>
<tr>
<td>16</td>
<td>Fresh Prince of Bel Air</td>
<td>17.1</td>
<td>12.4</td>
</tr>
<tr>
<td>17</td>
<td>Hangin' With Mr. Cooper</td>
<td>13.3</td>
<td>14.0</td>
</tr>
<tr>
<td>18</td>
<td>Jackie Thomas Show</td>
<td>16.3</td>
<td>7.0</td>
</tr>
<tr>
<td>19</td>
<td>Evening Shade</td>
<td>3.6</td>
<td>3.7</td>
</tr>
<tr>
<td>20</td>
<td>Hauty Affe</td>
<td>3.8</td>
<td>3.6</td>
</tr>
<tr>
<td>21</td>
<td>Unsolved Mysteries</td>
<td>4.7</td>
<td>4.5</td>
</tr>
<tr>
<td>22</td>
<td>Prime TIME LIVE</td>
<td>2.5</td>
<td>1.6</td>
</tr>
<tr>
<td>23</td>
<td>NBC Monday Night Movie</td>
<td>7.6</td>
<td>4.6</td>
</tr>
<tr>
<td>24</td>
<td>Dr. Quinn, Medical Woman</td>
<td>5.3</td>
<td>3.0</td>
</tr>
<tr>
<td>25</td>
<td>Jeep</td>
<td>5.0</td>
<td>3.2</td>
</tr>
<tr>
<td>26</td>
<td>Dinosaurs</td>
<td>17.1</td>
<td>11.5</td>
</tr>
<tr>
<td>27</td>
<td>48 Hours</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>28</td>
<td>ABC Sunday Night Movie</td>
<td>6.9</td>
<td>4.9</td>
</tr>
<tr>
<td>29</td>
<td>Matlock</td>
<td>1.5</td>
<td>2.2</td>
</tr>
<tr>
<td>30</td>
<td>The Simpsons</td>
<td>15.5</td>
<td>10.2</td>
</tr>
</tbody>
</table>

LESSON 3: SHADING A REGION

6. a. Possible answer:

Percent of Teens and Children Watching Top 30 TV Shows in 1993

<table>
<thead>
<tr>
<th>Teens (%)</th>
<th>Children (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

b. \( t > c \), where \( t = \% \) of teens and \( c = \% \) of children watching

c. Blossom, the greatest distance from the line \( y = x \)

7. a. The \( y \)-coordinate is less than the \( x \)-coordinate.

A scatter plot of the percent of children and the percent of teens who watched the top thirty programs is given below and on Activity Sheet 6.

b. \( t > c \), where \( t = \% \) of teens and \( c = \% \) of children watching

c. Blossom, the greatest distance from the line \( y = x \)

7. Consider the equations \( y = x \) and \( y = -x \).

a. Describe the graph of \( y = x \) in words and with a picture.

b. Graph the line \( y = -x \). Shade the region that describes the set of all points that satisfy the inequality \( y < -x \).

c. Graph the inequalities \( y > x \) and \( y \geq x \). How can you show that the two inequalities are different?

8. Graph each inequality.

a. \( x < 5 \)  
b. \( y \geq -3 \)  
c. \( x \geq 2 \)  
d. \( y < 0 \)
LESSON 3: SHADING A REGION

8. a.

b.

c.

d.
LESSON 4

Graphing Inequalities

Materials: graph paper, rulers, Activity Sheet 7, Lesson 4 Quiz
Technology: graphing calculators (optional)
Pacing: 1 class period

Overview
This lesson extends the students' understanding of the use of the symbols < and > to describe a region. The lesson begins by presenting a graph and a line of best fit. Students are asked to graph the line; and as the lesson progresses, they are asked to shade the region that is below the line and describe this region with an inequality. At the end of the lesson, students should be able to graph a given inequality and to describe a shaded region with an inequality.

Teaching Notes
All of the inequalities given in this lesson are in the form $y < mx + b$ or $y > mx + b$. The main emphasis is placed on developing an understanding of the relationship between the inequality and the shaded region.

In presenting this lesson, you may wish to use the data on curl-ups, since they are not used in the lesson. Students should draw a line that fits the data and then discuss the meaning of the points that are in the region below the line. This region can then be described with an inequality in the form $y < mx + b$.

Technology
It is important that students graph and shade the proper region by hand. If students are using graphing calculators, they should be encouraged to make a sketch of their graph or print the results.

Follow-Up
Ask students to think of situations where “benchmarks” or levels are set to achieve an award that changes for different ages.
LESSON 4: GRAPHING INEQUALITIES

STUDENT PAGE 35

LESSON 4

Graphing Inequalities

How old do you have to be to drive a car?

How many points do you have to earn in track to qualify for a letter?

Many situations involve inequalities. You will get a grade of B if you have more than a certain number of points; renting a car can be more economical if you drive more than a certain number of miles; selecting the best telephone company and package depends on the range of calls you make per month. In this lesson, you will learn to find a region that contains a set of points that satisfy a statement of inequality.

INVESTIGATE

Each year, students are encouraged to participate in the National Physical-Fitness Tests to demonstrate how physically fit they are and to qualify for a national award. The qualifying standards for boys for the Presidential Award are given below.

<table>
<thead>
<tr>
<th>Ages</th>
<th>Curl-Ups (1 minute)</th>
<th>Shuttle Run (seconds)</th>
<th>Sit and Reach (Inches)</th>
<th>1-Mile Run (minutes)</th>
<th>Pull-Ups</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>33</td>
<td>12.1</td>
<td>3.5</td>
<td>10:15</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>11.5</td>
<td>3.5</td>
<td>9:22</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>11.1</td>
<td>3.0</td>
<td>8:30</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>41</td>
<td>10.9</td>
<td>3.0</td>
<td>8:11</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>45</td>
<td>10.3</td>
<td>4.0</td>
<td>7:27</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>47</td>
<td>10.0</td>
<td>4.0</td>
<td>7:11</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>9.8</td>
<td>4.0</td>
<td>7:08</td>
<td>7</td>
</tr>
<tr>
<td>12</td>
<td>53</td>
<td>9.5</td>
<td>3.5</td>
<td>6:50</td>
<td>7</td>
</tr>
<tr>
<td>13</td>
<td>56</td>
<td>9.1</td>
<td>4.5</td>
<td>6:29</td>
<td>10</td>
</tr>
<tr>
<td>14</td>
<td>57</td>
<td>9.0</td>
<td>5.0</td>
<td>6:20</td>
<td>11</td>
</tr>
<tr>
<td>15</td>
<td>58</td>
<td>8.7</td>
<td>6.0</td>
<td>6:08</td>
<td>11</td>
</tr>
<tr>
<td>16</td>
<td>55</td>
<td>8.7</td>
<td>7.0</td>
<td>8:08</td>
<td>13</td>
</tr>
<tr>
<td>17</td>
<td>52</td>
<td>8.7</td>
<td>7.0</td>
<td>8:09</td>
<td>13</td>
</tr>
</tbody>
</table>

Source: The Orlando Sentinel, November 17, 1991

OBJECTIVE

Graph and interpret a linear inequality.
Solution Key

Discussion and Practice

1. Be sure students realize that the seconds in the data have been changed to fractions of minutes.
   a. As the boy's age increases, the time needed to earn a Presidential Award in the 1-mile run decreases.
   b. No; the units on the x- and y-axes are different, and the graph has a negative slope.
   c. Presidential Award in 1-Mile Run

   \[
   y = -0.4x + 11.9
   \]

   a. What trend do you see in the plot?
   b. Would the line \( y = x \) make sense in this plot? Explain.
   c. Suppose that the equation of a line that seems to fit the data is \( y = -0.4x + 11.9 \). Graph this line on the graph on Activity Sheet 7. Describe how you graphed the line.
   d. How can you determine the qualifying standard for an 8-year-old boy using the equation? How does this value compare with the time listed in the table?

2. a. Substitute the value of 8 for \( x \). The equation gives a value of 8.7 or 8 minutes and 42 seconds. This is faster than the 8 minutes 48 seconds from the table.

   Presidential Award in 1-Mile Run

   a. On the graph for Problem 1c, plot the point representing the boy's time. Shade the region of the graph that contains this point. What does this region represent?
   b. List three ordered pairs that lie in the shaded region.
   c. Write an inequality for the shaded region. Show how each ordered pair you listed in part b satisfies the inequality.
   d. If a 15-year-old boy ran the mile in 7 minutes and 10 seconds, would he qualify for a Presidential Award? How can you use your graph to determine your answer?

All the points that are below the Presidential Award marks.

   a. Answers will vary.
   b. \( y < -0.4x + 11.9 \); demonstrations of how each ordered pair satisfies the inequality will vary.
   c. No; the point (15, 7 minutes 10 seconds) is above the line.
3. a. Possible answer: \( y = -0.03x + 13.6 \)
   
   b. Based on part a equation, \( y > -0.3x + 13.6 \)

   Showed below is the graph of boy’s age in years and time in seconds necessary to earn a Presidential Award in the shuttle run.

   a. Write an equation for the line drawn on the plot.
   b. Sketch the plot and the line, and then shade the region that represents the times for boys who would not earn an award for the shuttle run.
   c. Write an inequality that represents the shaded region drawn in part b.

   **Summary**
   - If two variables have the same unit, you can plot them and use the graph of \( y = x \) to determine whether the \( x \)-variable is greater than or less than the \( y \)-variable. The region above the graph for \( y = x \) can be represented by the inequality \( y > x \). The regions below the graph for \( y = x \) can be represented by the inequality \( y < x \).
   - If two variables do not have the same unit but seem to have a linear relationship, you can draw a line summarizing the relationship. This line can be used to determine an inequality for the relationship. The area on one side of the line is greater than the relationship, the area on the other side is less. To find which side is greater than the relationship, you must investigate what the ordered pairs on each side represent and whether or not they satisfy the inequality. The inequality \( y > ax + b \) represents the region above the graph of \( y = ax + b \), and the inequality \( y < ax + b \) represents the region below the graph of \( y = ax + b \).
Practice and Applications

4. a. Shaded above a dashed line
   b. Shaded below a dashed line
   c. Shaded below a solid line

5. a. As the girl's age increases, the time needed to qualify for a Presidential Award in the shuttle run decreases. Times start to level off around the age of 15.

   b. 
   
   
   c. Possible answer:

   
   

   
   

   
   

\[ y = -0.2x + 13.4 \]

   d. Based on part c graph,
   \[ y < -0.2x + 13.4 \]

6. Times needed to qualify for a Presidential Award in the shuttle run for boys are less than for girls. At the early ages, there is not much difference; but as ages increase, the gap between the times increases.

Practice and Applications

4. If the equation of a line is \( y = sx + d \), describe each region.
   a. \( y > sx + d \)
   b. \( y < sx + d \)
   c. \( y \leq sx + d \)

5. The data for the physical fitness qualifying standards for the Presidential Award for girls are in the table below.

   **Physical-Fitness Qualifying Standards for Girls, The Presidential Award**

<table>
<thead>
<tr>
<th>Ages</th>
<th>Curl-Ups (1 minute)</th>
<th>Shuttle Run (seconds)</th>
<th>Sit and Reach (inches)</th>
<th>1-Mile Run (minutes)</th>
<th>Pull-Ups</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>30</td>
<td>12</td>
<td>5.5</td>
<td>11.20</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>34</td>
<td>11</td>
<td>5.0</td>
<td>10.36</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>38</td>
<td>10</td>
<td>4.5</td>
<td>10.02</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>39</td>
<td>11</td>
<td>5.5</td>
<td>9.30</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>10</td>
<td>6.0</td>
<td>9.19</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>42</td>
<td>10</td>
<td>6.5</td>
<td>9.07</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>45</td>
<td>9</td>
<td>7.0</td>
<td>8.23</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>46</td>
<td>9</td>
<td>8.0</td>
<td>8.13</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>47</td>
<td>10</td>
<td>8.0</td>
<td>7.59</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>48</td>
<td>10</td>
<td>8.0</td>
<td>8.08</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>45</td>
<td>10</td>
<td>9.0</td>
<td>8.23</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>44</td>
<td>10</td>
<td>8.0</td>
<td>8.15</td>
<td>1</td>
</tr>
</tbody>
</table>

   Source: The Orlando Sentinel, November 17, 1991

   a. Look carefully at the data for age and time for the shuttle run. Describe the relationship between age and time.
   b. Make a scatter plot of the girls' Presidential Award qualifying times for the shuttle run.
   c. Draw a line to fit the data. Find an equation for the line.
   d. Write an inequality that represents the region that describes those girls who would qualify for an award.
   e. Compare the times for boys and the times for girls to receive a Presidential Award for the shuttle run. What are your conclusions?
7. a. Possible answer:

As the week number increases, the time walking increases.

Answers to parts b and c are based on part a graph.

b. \( y = 7.2x \); this describes the relationship between the week number and the number of minutes a person should walk.

c. The region below the line represents the values where a person walks less than the amount suggested in the chart; \( y < 7.2x \)

8. a. 

b. 

8. Graph each inequality.

a. \( y \geq 2x - 3 \)

b. \( y < \frac{1}{2}x + 4 \)

c. \( y \leq 5 - 0.5(x - 2) \)

d. \( y > 30 + 4(x - 2) \)

9. Write an inequality to describe each graph.

a. \( y \geq -x + 50 \)

b. \( y > x + 100 \)
ASSessment

Assessment for Unit II

Materials: graph paper, rulers
Technology: graphing calculators (optional)
Pacing: 1 class period or homework

Overview

Problem 1 assesses students' understanding of the inequality \( y \leq x \). Problem 2 asks students to draw a line that best fits the data, write its equation, and then change the equation to an inequality that represents a shaded region.

Teaching Notes

This assessment can be used in a number of ways. It can be used strictly as an assessment that is completed by students in one class period. It could also be used as a take-home test or as additional practice on the objectives of Lessons 3 and 4. The rest of the module uses inequalities extensively; as a result, students will need a good understanding of graphing a given inequality or describing a shaded region with an inequality.

Technology

The use of graphing calculators is optional. You may wish to allow students to use a calculator to make the scatter plot asked for in Problems 1 and 2.

Follow-Up

Students could use a current issue of U.S. News & World Report to update the data presented and then draw and find the equation of a line that relates the number of faculty to the enrollment. Students could compare this equation to the equation they found in Problem 2c.
Solution Key


b. Draw the line showing all the points where the rank in 1970 is the same as in 1995. Write an equation for the line.

c. Below the line $y = x$, 4 in this region

d. Above the line $y = x$, 7 in this region

$y = x$

e. Age group 35–39; changed from a rank of 11 to 1
2. **a.** Yes; higher enrollment would mean more faculty are needed to teach.

**b.**

_As student enrollment increases, the number of faculty increases._

**c.** Possible answer:

\[ y = 0.11x + 51 \]

Answers to parts d and e are based on part c graph.

**d.**

\[ y < 0.11x + 51 \]

2. Listed below are the 15 highest-ranked national universities among 204 schools that are research-oriented. The ranks were determined by the _U.S. News and World Report_ magazine. The chart gives the enrollment and the number of faculty for each school for the 1992–93 school year.

<table>
<thead>
<tr>
<th>Rank</th>
<th>College</th>
<th>Enrollment</th>
<th>Faculty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Harvard University</td>
<td>18,275</td>
<td>2,278</td>
</tr>
<tr>
<td>2</td>
<td>Princeton University</td>
<td>7,438</td>
<td>955</td>
</tr>
<tr>
<td>3</td>
<td>Yale University</td>
<td>11,120</td>
<td>672</td>
</tr>
<tr>
<td>4</td>
<td>Massachusetts Institute of Technology</td>
<td>9,798</td>
<td>675</td>
</tr>
<tr>
<td>5</td>
<td>California Institute of Technology</td>
<td>2,099</td>
<td>770</td>
</tr>
<tr>
<td>6</td>
<td>Stanford University</td>
<td>13,893</td>
<td>1,406</td>
</tr>
<tr>
<td>7</td>
<td>Duke University</td>
<td>11,476</td>
<td>2,704</td>
</tr>
<tr>
<td>8</td>
<td>Dartmouth College</td>
<td>5,475</td>
<td>468</td>
</tr>
<tr>
<td>9</td>
<td>University of Chicago</td>
<td>10,231</td>
<td>1,643</td>
</tr>
<tr>
<td>10</td>
<td>Cornell University</td>
<td>18,450</td>
<td>1,593</td>
</tr>
<tr>
<td>11</td>
<td>Columbia University School of Engineering and Applied Sciences</td>
<td>1,832</td>
<td>101</td>
</tr>
<tr>
<td>12</td>
<td>Brown University</td>
<td>7,593</td>
<td>663</td>
</tr>
<tr>
<td>13</td>
<td>Northwestern University</td>
<td>12,032</td>
<td>1,079</td>
</tr>
<tr>
<td>14</td>
<td>Rice University</td>
<td>4,035</td>
<td>547</td>
</tr>
<tr>
<td>15</td>
<td>Johns Hopkins University</td>
<td>4,613</td>
<td>473</td>
</tr>
</tbody>
</table>

(Source: _U.S. News and World Report, 1994 College Guide_)

**a.** Would you expect to see some relation between the number of faculty members and the enrollment? Why or why not?

**b.** Make a scatter plot of (enrollment, number of faculty). Describe any pattern you see.

**c.** Draw a line that seems to fit the data. Write an equation of this line that relates the number of faculty members to the enrollment.

**d.** Shade the region where the number of faculty members is less than expected. Write an inequality to represent this region.

**e.** The University of Texas at Austin has an enrollment of 49,253 and 2,358 faculty members. Is the relationship better or worse than you expected? Tell how you made your decision.

**e.** Worse than expected. Substitute 49,253 into the equation. The predicted number of faculty is about 5470.
3. a. $y \leq x$

b. $y \geq x$

c. $y \leq 2x + 6$

d. $y \geq 2x - 7$

4. a. $y \geq -x$

b. $y \leq x + 2$

c. $y > -x + 2$
Solving Systems of Inequalities
INTRODUCTORY ACTIVITY

Fast Foods

Materials: none needed
Technology: calculators
Pacing: \( \frac{1}{2} \) class period

Overview

This is a very brief activity to give students an opportunity to preview the next few lessons. Students are given the amount of cholesterol and sodium in a chicken sandwich and in 1 cup of 1% milk. They are asked to determine how many chicken sandwiches and how much milk they could consume and still remain under the recommended dietary limits.

Teaching Notes

This activity could be assigned as homework after students have finished the assessment for Unit II. Lesson 5 will build on the ideas presented in this activity, so there is no need to proceed too deeply with your students.

Technology

Calculators will help students with the necessary computation.
Solution Key

1. a. Yes; $2(50) + 2(10) = 120$, which is less than 300.
b. Yes; $2(820) + 2(115) = 1870$, which is less than 2400.

INTRODUCTORY ACTIVITY: FAST FOODS

Fast Foods

Do you ever think about how many calories are in the food you eat?

Are you concerned about the amount of fat and salt in your diet?

Many people need to be aware of the amount of cholesterol and sodium in the food that they eat, because high cholesterol levels can lead to heart attacks and high sodium levels may be a cause of high blood pressure. If your daily dietary intake is about 2,500 calories a day, then your daily diet should include less than 300 mg of cholesterol and less than 2,400 mg of sodium.

The amounts of cholesterol and sodium in a chicken sandwich and in 1 cup (8 fluid ounces) of 1% lowfat milk at a fast-food restaurant are listed below.

<table>
<thead>
<tr>
<th>Chicken Sandwich</th>
<th>1 Cup of 1% Milk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cholesterol (mg)</td>
<td>50</td>
</tr>
<tr>
<td>Sodium (mg)</td>
<td>820</td>
</tr>
</tbody>
</table>

2. Suppose you ate 2 chicken sandwiches and drank 2 cups of 1% milk.
   a. Would you stay under the dietary levels of 300 mg of cholesterol? Show how you determined your answer.
   b. Would you stay under the dietary levels of 2,400 mg of sodium? Show how you determined your answer.
INTRODUCTORY ACTIVITY: FAST FOODS

2. a. Less than 6
   b. 2.9, just less than 3

3. a. Less than 30
   b. 20.9, just less than 21

4. (0, 20), representing no chicken sandwiches and 20 cups of milk; if at least one chicken sandwich must be eaten, then the ordered pair is (1, 13), representing 1 chicken sandwich and 13 cups of milk.

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2. If you ordered just chicken sandwiches and no milk,
   a. how many sandwiches could you eat and stay under the dietary levels of 300 mg of cholesterol?
   b. how many sandwiches could you eat and stay under the dietary levels of 2,400 mg of sodium?

3. If you just ordered just 1% milk and no chicken sandwiches,
   a. how many cups of milk could you drink and stay under the dietary levels of 300 mg of cholesterol?
   b. how many cups of milk could you drink and stay under the dietary levels of 2,400 mg of sodium?

4. Find an ordered pair that represents the greatest number of chicken sandwiches and cups of 1% milk that you could consume and stay under dietary levels for cholesterol and sodium. Do you think one person actually might eat the combination that you found?
LESSON 5

Graphing Conditions

**Materials:** graph paper, rulers, Activity Sheets 8 and 9
**Technology:** graphing calculators (optional)
**Pacing:** 1 class period

**Overview**

The main objective of this lesson is to have students graph two inequalities on the same coordinate system. All of the inequalities in this lesson are in the form $x \leq a$ or $y \leq b$. The lesson begins with data from the 1996 NBA Championship Series. Students are asked to graph the inequalities total points $\geq 80$ and rebounds $\geq 40$. These inequalities form four regions on the graph, and students are asked to interpret each region and write an inequality that represents each region. The remainder of the lesson uses different data sets to emphasize the meaning of the region where the two given inequalities overlap.

**Teaching Notes**

As you present this lesson, you will want to remind students of the difference and similarities between the inequalities $x \leq 5$ and $x < 5$. In addition, the emphasis should be placed on the interpretation of the shaded region where the two inequalities overlap. Most students will not have too much difficulty with this lesson, but it is important that they be able to graph the given inequalities and understand that the solution set is the shaded region where the shading overlaps.

**Technology**

Graphing calculators are optional. Students can make their scatter plots on a calculator and use the DRAW function to draw the horizontal and vertical lines.

**Follow-Up**

Have students find the data for the most recent NBA Championship Series. The data can usually be found at the NBA Web site. Have students use the same criteria, total points $\geq 80$ and total rebounds $\geq 40$, to determine the MVP.
LESSON 5: GRAPHING CONDITIONS

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LESSON 5

Graphing Conditions

Do you watch professional basketball or other sports on television?

Who is your favorite player?

Led by Michael Jordan, the Chicago Bulls won the 1996 NBA Championship. They defeated the Seattle SuperSonics, led by Shawn Kemp, in six games. Jordan was voted the most valuable player by the media covering the game. If you had been allowed to vote for MVP, would you have voted for Jordan?

INVESTIGATE

The following tables contain the statistics for the 1996 Championship series. Only those players who played a total of at least 40 minutes in the series are listed.

<table>
<thead>
<tr>
<th>Player</th>
<th>Min</th>
<th>FG-A</th>
<th>FT-A</th>
<th>RB</th>
<th>AST</th>
<th>PF</th>
<th>PT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kemp</td>
<td>243</td>
<td>49-89</td>
<td>42-49</td>
<td>86</td>
<td>11</td>
<td>28</td>
<td>140</td>
</tr>
<tr>
<td>Schrempf</td>
<td>238</td>
<td>35-60</td>
<td>21-24</td>
<td>39</td>
<td>10</td>
<td>18</td>
<td>95</td>
</tr>
<tr>
<td>Payton</td>
<td>274</td>
<td>40-80</td>
<td>19-26</td>
<td>41</td>
<td>44</td>
<td>22</td>
<td>156</td>
</tr>
<tr>
<td>Hawkins</td>
<td>253</td>
<td>25-55</td>
<td>24-26</td>
<td>28</td>
<td>6</td>
<td>21</td>
<td>82</td>
</tr>
<tr>
<td>Perkins</td>
<td>190</td>
<td>22-51</td>
<td>17-21</td>
<td>27</td>
<td>12</td>
<td>13</td>
<td>67</td>
</tr>
<tr>
<td>Wilkens</td>
<td>43</td>
<td>5-10</td>
<td>4-4</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>McMillan</td>
<td>31</td>
<td>3-7</td>
<td>1-3</td>
<td>11</td>
<td>8</td>
<td>6</td>
<td>31</td>
</tr>
<tr>
<td>Bricken</td>
<td>58</td>
<td>2-9</td>
<td>0-0</td>
<td>14</td>
<td>3</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

Min (minutes played), FG-A (field goals made-field goals attempted), FT-A (free throws made-free throws attempted), RB (rebounds), AST (assists), PF (personal fouls), PT (total points)

Source: Chicago Tribune

OBJECTIVE

Graph and interpret systems of inequalities in the form $x < a$ and $y < b$. 

GRAPHING CONDITIONS 59
Solution Key

Discussion and Practice

1. Most of the data points lie roughly along a line.

2. Yes, generally, as the number of points increase the number of rebounds increases. Exceptions are Michael Jordan and Dennis Rodman.

3. Suppose you felt that the most valuable player should have scored at least 80 points in the six games. Draw a line on your scatter plot showing all the ordered pairs whose x-coordinate, or total points, is 80.

4. Shade the region that shows the location of all the ordered pairs with at least 40 rebounds. List all the players who had more than 40 rebounds.

5. See the graph in Problem 4; Kemp, Schrempf, Payton, Hawkins, Pippen, Jordan

6. See the graph in Problem 6; Kemp, Payton, Pippen, Rodman, Kukoc
LESSON 5: GRAPHING CONDITIONS

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8. a. x-coordinate less than 80 and y-coordinate less than 40
b. x-coordinate greater than 80 and y-coordinate less than 40
c. x-coordinate greater than 80 and y-coordinate greater than 40

9. a. \( t < 80 \) and \( r > 40 \)
b. \( t > 80 \) and \( r < 40 \)
c. \( t > 80 \) and \( r > 40 \)

10. a. Region IV
b. Kemp, Payton, Pippen
c. No; Jordan did not satisfy both conditions.

Your scatter plot should be separated into four nonoverlapping regions.

The points in region I are those whose x-coordinate is less than 80 and whose y-coordinate is greater than 40.

b. Describe the ordered pairs that are in each other region.
   a. Region II  b. Region III  c. Region IV
   If you let \( t \) represent the total points and let \( r \) represent the total rebounds, then region II could be represented by the inequalities \( t < 80 \) and \( r < 40 \).

9. Write a pair of inequalities to represent each other region.
   a. Region I  b. Region III  c. Region IV

10. Consider the region that satisfies the conditions that the most valuable player should have at least 80 points and at least 40 rebounds.
    a. Which region satisfies these conditions?
    b. Identify the players in the region.
    c. Based on these two conditions, do you agree that Michael Jordan should have been voted the MVP of the Championship series? Why or why not?
LESSON 5: GRAPHING CONDITIONS

11. a. Price and Mileage for Selected 1995 Cars

b. Let \( c \) = price of a car; \( c \leq 25,000 \)
c. Price and Mileage for Selected 1995 Cars

d. Let \( m \) = miles per gallon; \( m \geq 25 \)
e. 5 cars

12. a. \( c \geq 25,000 \) and \( m \geq 25 \)
b. \( c \leq 25,000 \) and \( m \leq 25 \)

Buying a Car

The scatter plot below shows the prices and the miles per gallon for a sample of 1995 cars. A car buyer decides that the car she will purchase must cost no more than $25,000 and get at least 25 miles per gallon.

11. Use the second grid on Activity Sheet 8 for this problem and the next.
   a. Shade the region of the graph that represents all the cars that cost no more than $25,000.
   b. Write an inequality to represent this region.
   c. Shade the region of the graph that represents all the cars that get at least 25 miles per gallon.
   d. Write an inequality to represent this region.
   e. How many cars satisfy both conditions?
12. Write a pair of inequalities and sketch a region that represents each set of conditions.
a. Costs at least $25,000 and gets at least 25 miles per gallon
b. Costs no more than $25,000 and gets no more than 25 miles per gallon
LEsson 5: Graphing Conditions

Practice and Applications

13. a.

Summary
The graph of the conditions $x > a$ and $y > b$ is the doubly-shaded region shown at the right.

Practice and Applications

13. Graph each pair of inequalities.
   a. $x < 5$   b. $x > 1$   c. $x \geq 0$
      $y > 2$   $y < 3$   $y \geq 0$

14. Write a pair of inequalities to represent the doubly-shaded region of each graph.

14. a. $x < -2$ and $y \leq -3$
       b. $x \leq 2$ and $y < 4$
       c. $x \leq 4$ and $y \leq 0$
       d. $x \leq 0$ and $y \leq 0$
15. a. The table below contains the watts per channel and suggested retail price in 1996 for selected stereo minisystems. A consumer wants a stereo minisystem with at least 50 watts per channel and wishes to pay no more than $500.

<table>
<thead>
<tr>
<th>Model</th>
<th>Power (watts)</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yamaha DX-5</td>
<td>35</td>
<td>550</td>
</tr>
<tr>
<td>JVC MX-C3000</td>
<td>40</td>
<td>530</td>
</tr>
<tr>
<td>AIWA NSX-916</td>
<td>40</td>
<td>480</td>
</tr>
<tr>
<td>AIWA NSX-999</td>
<td>120</td>
<td>600</td>
</tr>
<tr>
<td>Onkyo PCS-207</td>
<td>25</td>
<td>580</td>
</tr>
<tr>
<td>JVC MX-C300</td>
<td>30</td>
<td>440</td>
</tr>
<tr>
<td>Onkyo DX-400</td>
<td>50</td>
<td>890</td>
</tr>
<tr>
<td>Onkyo MX-C6005</td>
<td>60</td>
<td>590</td>
</tr>
<tr>
<td>AIWA NSX-K400</td>
<td>50</td>
<td>580</td>
</tr>
<tr>
<td>Pioneer CS-204</td>
<td>100</td>
<td>650</td>
</tr>
<tr>
<td>Sony MHC-C40055</td>
<td>160</td>
<td>490</td>
</tr>
<tr>
<td>Fisher DCS-M37</td>
<td>50</td>
<td>450</td>
</tr>
<tr>
<td>Onkyo CS-204</td>
<td>33</td>
<td>440</td>
</tr>
<tr>
<td>Kenwood UD-303</td>
<td>50</td>
<td>730</td>
</tr>
<tr>
<td>Kenwood UD-403</td>
<td>55</td>
<td>730</td>
</tr>
</tbody>
</table>

Source: Consumer Reports, February, 1996

a. On the grid on Activity Sheet 9, make a scatter plot of the ordered pairs (power, price).

b. Shade the region of the graph that represents all minisystems that have 50 or more watts per channel. Write an inequality to represent this region.

c. On the scatter plot, shade the region of the graph that represents all minisystems that cost $500 or less. Write an inequality to represent this region.

d. Which minisystems satisfy both conditions?

e. Write a pair of inequalities to represent the region where cost is more than $500 and watts per channel is greater than 50.

d. Sony MHC-40555, Fisher DCS-M37, Kenwood UD-303

e. Let $W =$ watts and $C =$ cost; $C \leq 500$ and $W \geq 50$
16. a. New Zealand, Denmark, Sweden, Finland, Norway, Switzerland, Netherlands
   b. \( J = \) judicial system rating and \( A = \) anticorruption rating; \( J \geq 8 \) and \( A \geq 8 \)
   c. \( J \leq 4 \) and \( A \geq 6 \); Singapore

17.

1996 NBA Championship Series
Bulls Versus Supersonics

Conditions will vary, but minutes should be at least 150 minutes and field goals made should be at least 25.

16. The scatter plot below shows the relationship between a country's judicial system rating and anticorruption rating.

   a. Identify the countries that have a rating of 8 or higher on both scales.
   b. Write a pair of inequalities to represent the region of the graph where countries have a rating of 8 or higher on each scale.
   c. Write a pair of inequalities to represent the region of the graph where countries have a judicial system rating of 4 or lower and an anticorruption rating of 6 or higher. Identify the countries in this region.

Extension

17. Use the categories of total minutes played and field goals made from the 1996 NBA Championship series data presented in this section to decide which players are the most valuable. Your work should include a scatter plot and the conditions, written as inequalities, used to make your decisions.
Systems of Inequalities

Materials: graph paper, rulers, Activity Sheets 10 and 11, Lesson 6 Quiz
Technology: graphing calculators (optional)
Pacing: 2 class periods

Overview
This lesson begins by presenting data on the number of calories and calories from fat for selected items from McDonald's. Students are asked to graph a line through the origin that best represents the data. The students are then asked to compare actual data values to values on the line. From this comparison, the inequalities in the form $y \leq mx$ are developed.

Teaching Notes
The beginning part of the lesson is very similar to Lesson 4. Students draw a line and then compare ordered pairs above and below the line. This lesson extends Lesson 4 by having students investigate the relationship between two inequalities both in the form $y \leq mx$. The main focus is on the understanding of the shaded regions and the solution set to two given inequalities. This is why all the given equations pass through the origin.

Technology
If your students are using graphing calculators, they should not use the MED-MED line or LINReg options on the calculator. When using these options, the calculator will find a line that has a y-intercept other than zero. Except for Problem 18, the intent of the lesson is to use lines whose y-intercepts are zero.

Follow-Up
Have students list all the items that they eat in one day. Have them use a nutrition guide to determine the calories and calories from fat for all the items. Have them make a scatter plot of their data and draw a line through the origin that fits their data. Have them interpret the slope of their line. They can then add the line $y = 0.30x$ and compare their line to this line.
How often do you eat breakfast, lunch, or dinner at a fast-food restaurant?

How healthy do you think the food is at these restaurants?

Many people think that the food at fast-food restaurants is high in calories and saturated fats. In this lesson, you will investigate how to use inequalities to evaluate how the number of calories from fast food compares to suggested dietary recommendations.

INVESTIGATE

Calories from Fat

With the increase in spending at fast-food restaurants comes a greater awareness of dietary concerns. In 1989, the National Research Council published Recommended Daily Allowances stating that no more than 30% of a person's daily caloric intake should come from fats. The table on page 55 shows the calories and calories from fat for selected items from McDonald's in a recent year.
Lesson 6: Systems of Inequalities

Solution Key

Discussion and Practice

1. As calories increase, calories from fat also increase.

2. a. 0 calories and 0 calories from fat

b. Possible answer:

<table>
<thead>
<tr>
<th>Item</th>
<th>Calories</th>
<th>Calories from Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamburger</td>
<td>270</td>
<td>90</td>
</tr>
<tr>
<td>Cheeseburger</td>
<td>320</td>
<td>130</td>
</tr>
<tr>
<td>Quarter Pounder</td>
<td>430</td>
<td>150</td>
</tr>
<tr>
<td>Quarter Pounder with Cheese</td>
<td>530</td>
<td>270</td>
</tr>
<tr>
<td>Arch Deluxe</td>
<td>570</td>
<td>280</td>
</tr>
<tr>
<td>Anti Deluxe with Bacon</td>
<td>610</td>
<td>310</td>
</tr>
<tr>
<td>Big Mac</td>
<td>530</td>
<td>250</td>
</tr>
<tr>
<td>McGrilled Chicken Classic</td>
<td>260</td>
<td>35</td>
</tr>
<tr>
<td>Garden Salad</td>
<td>80</td>
<td>35</td>
</tr>
<tr>
<td>Tofu Salad</td>
<td>160</td>
<td>60</td>
</tr>
<tr>
<td>Lite Vinaigrette Dressing</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>Apple Muffin</td>
<td>170</td>
<td>0</td>
</tr>
<tr>
<td>Apple Danish</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>1% Lowfat Milk</td>
<td>100</td>
<td>20</td>
</tr>
</tbody>
</table>

Below is a scatter plot of the ordered pairs (calories, calories from fat).

Discussion and Practice

1. What patterns do you observe in the scatter plot above?

2. Refer to the scatter plot above.

a. What does the point (0, 0) mean in terms of the data?

b. On the first graph on Activity Sheet 10, draw a line that passes through the point (0, 0) that you feel summarizes the data.
LESSON 6: SYSTEMS OF INEQUALITIES

3. a. Possible answer: \( y = 0.43x \)
   b. Based on part a, for every 100 calories, 43 calories come from fat.

4. a. Based on Problem 2 graph, 95 calories from fat
   b. Prediction is less than actual amount.

5. Based on Problem 3, predicted calories from fat: 155, 220, 125

6. a. Filet o' Fish below the line, McChicken Sandwich above the line, and Egg McMuffin below the line.
   c. The actual number of calories from fat is greater than the predicted number of calories from fat.

   Calculations and Calories from Fat in Fast Foods

<table>
<thead>
<tr>
<th>Calories from Fat</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>200</td>
<td>80</td>
</tr>
<tr>
<td>300</td>
<td>120</td>
</tr>
<tr>
<td>400</td>
<td>160</td>
</tr>
<tr>
<td>500</td>
<td>200</td>
</tr>
<tr>
<td>600</td>
<td>240</td>
</tr>
<tr>
<td>700</td>
<td>280</td>
</tr>
</tbody>
</table>

   b. Filet o' Fish below the line, McChicken Sandwich above the line, and Egg McMuffin below the line.
   c. The actual number of calories from fat is greater than the predicted number of calories from fat.

STUDENT PAGE 56

3. Refer to Problem 2b.
   a. Write an equation for the line you have drawn.
   b. Explain the meaning of the slope of this line in terms of the data.

4. The number of calories in a small bag of French fries is 220.
   a. Use your line and predict the number of calories from fat for a small bag of French fries.
   b. If the actual number of calories from fat is 110, how accurate was your prediction?

5. The following items were not shown on the scatter plot above. Use the equation for the line you drew and predict the number of calories from fat for each of the given number of calories.

<table>
<thead>
<tr>
<th>Item</th>
<th>Calories</th>
<th>Predicted Calories from Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filet o' Fish</td>
<td>360</td>
<td>150</td>
</tr>
<tr>
<td>McChicken Sandwich</td>
<td>510</td>
<td>270</td>
</tr>
<tr>
<td>Egg McMuffin</td>
<td>290</td>
<td>110</td>
</tr>
</tbody>
</table>

6. The actual numbers of calories and calories from fat are listed in the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Calories</th>
<th>Actual Calories from Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filet o' Fish</td>
<td>360</td>
<td>150</td>
</tr>
<tr>
<td>McChicken Sandwich</td>
<td>510</td>
<td>270</td>
</tr>
<tr>
<td>Egg McMuffin</td>
<td>290</td>
<td>110</td>
</tr>
</tbody>
</table>

   a. On the graph for Problem 2, add the ordered pairs (calories, calories from fat) for the three items listed in the table above.
   b. Are the ordered pairs above, below, or on the line that you have drawn?
   c. Explain what it means if a point is above the line.
LESSON 6: SYSTEMS OF INEQUALITIES

7. a. Mc Grilled Chicken Classic, Apple Bran Muffin, 1% Lowfat Milk
   b. Calories from fat are less than 30% of calories.
   c. Yes

8. Possible answer:

   Calories and Calories from
   Fat in Fast Foods

   The National Research Council suggests that we should get no more than 30% of our calories from fat. To express this recommendation as an inequality, let \( c \) = number of calories and let \( f \) = number of calories from fat. Then, \( f \leq 0.30c \).

   The graph of the inequality is shown below.

   Refer to the graph above.
   a. Identify the items that are in the shaded region.
   b. What do the items in this region represent?
   c. Could you make a meal out of these items?

   Draw the line you drew in Problem 2 on the plot of the inequality given on the second graph on Activity Sheet 10.

9. a. Hamburger, Cheeseburger, Fajita Chicken Salad, Apple Danish
   b. They are above the recommended daily amount but below the general McDonald's average.
   c. Let \( c \) = calories from fat and \( x \) = calories; \( c < 0.43x \) and \( c > 0.3x \)

   Refer to the line in Problem 8.
   a. Identify the items that lie in the unshaded region but below the original line that you drew.
   b. What do the items in this region represent?
   c. Write a pair of inequalities to describe this region.
10. As the number of calories increases, the number of grams of saturated fat increases.

11. Possible answer:

**Calories and Saturated Fats in Fast Foods**

<table>
<thead>
<tr>
<th>Item</th>
<th>Calories</th>
<th>Saturated Fat (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamburger</td>
<td>270</td>
<td>3.5</td>
</tr>
<tr>
<td>Cheeseburger</td>
<td>330</td>
<td>6</td>
</tr>
<tr>
<td>Quarter Pounder</td>
<td>430</td>
<td>8</td>
</tr>
<tr>
<td>Quarter Pounder with Cheese</td>
<td>530</td>
<td>13</td>
</tr>
<tr>
<td>Arch Deluxe</td>
<td>570</td>
<td>11</td>
</tr>
<tr>
<td>Arch Deluxe with Bacon</td>
<td>610</td>
<td>12</td>
</tr>
<tr>
<td>Big Mac</td>
<td>530</td>
<td>10</td>
</tr>
<tr>
<td>McGrilled Chicken Classic</td>
<td>240</td>
<td>1</td>
</tr>
<tr>
<td>Garden Salad</td>
<td>80</td>
<td>1</td>
</tr>
<tr>
<td>Fajita Chicken Salad</td>
<td>160</td>
<td>1.5</td>
</tr>
<tr>
<td>Lite Vinaigrette Dressing</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>Apple Bran Muffin</td>
<td>170</td>
<td>0</td>
</tr>
<tr>
<td>Apple Danish</td>
<td>360</td>
<td>5</td>
</tr>
<tr>
<td>1% Lowfat Milk</td>
<td>130</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Source: McDonald's Nutrition Facts

Below is a scatter plot of the ordered pairs (calories, saturated fat) of the items in this table.

10. Describe any trends that you observe.

11. On the first graph on Activity Sheet 11, draw a line that passes through the origin and summarizes the data.
12. **a.** Possible answer: \( y = 0.017x \)
   **b.** Based on part a, grams of saturated fat are less than 1.7% of calories.
   **c.** Let \( c = \) calories and \( s = \) grams of saturated fat; \( s < 0.017c \).

13. **a.** \( s \leq 0.10c \)
   **b.**
   
   **Calories and Saturated Fats in Fast Foods**

   - Region I contains ordered pairs such that the grams of saturated fat comprise more than 10% of calories. Region II contains ordered pairs such that the grams of saturated fat comprise less than 1.7% of calories and more than 1.7% of calories. Region III contains ordered pairs such that the grams of saturated fat comprise less than 1.7% of calories.
   **b.** Region I: \( s > 0.1c \); region II: \( s < 0.1c \) and \( s > 0.017c \); region III: \( s < 0.017c \)
Given a system of two inequalities in the form \( y \geq ax \) and \( y \geq bx \), the coordinate system can be divided into four regions.

In the graph below, region I represents the ordered pairs that satisfy both inequalities. Region II satisfies the inequality \( y \geq ax \) but not the inequality \( y \geq bx \). Region III satisfies the inequality \( y \geq bx \) but not the inequality \( y \geq ax \). Region IV represents the ordered pairs that satisfy neither inequality.

**Practice and Applications**

15. a. Graph each inequality.
   a. \( y \geq 2x \)  
   b. \( y \geq \frac{1}{3}x \)  
   c. \( y \geq -3x \)

16. Write an inequality to represent the shaded region in each graph.
   a. 
   b. 

17. Graph each system of inequalities.
   a. \( y \geq \frac{1}{3}x \)  
   b. \( y \geq 1x \)  
   \( y \leq 2x \)  
   \( y \leq 1.5x \)
18. a. Possible answer:

Gross Monthly Income

and Housing Costs

b. Based on part a, graph,
y = 0.29x - 84; for every increase
of $10 in monthly gross income,
monthly housing costs will increase
by $2.90.

c. The person is paying more of his
or her income for housing than is
the average person in the sample.

d. \( y \leq 0.20x \)

e. Ordered pairs where the month-
ly housing costs are 20% or less of
monthly gross income

g. Based on part b, \( y < 0.29x - 84 \)
and \( y > 0.20x \)
LESSON 7

Applying Systems of Inequalities

Materials: graph paper, rulers, Activity Sheets 12–14, Lesson 7 Quiz
Technology: graphing calculators (optional)
Pacing: 2 class periods

Overview

In this lesson, students develop inequalities in the form $ax + by \geq c$ and investigate the idea of linear programming. The first investigation involves a student who wishes to burn calories by jogging and weight lifting. Students are given the number of calories burned for each activity and are then asked to determine the total number of calories burned for a given number of minutes for each activity. Students then generalize these relationships and form two inequalities. The two inequalities are graphed on the same coordinate system. The region formed by the overlap of the two inequalities is defined as the feasible region. Students are then asked to find the corner points and use these points to find the total number of calories burned.

The next investigation involves how much money a conservation club can make from a cookie sale. The constraints given include the number of cookies to be made and the amount of money the club can spend. Inequalities developed are in the form $ax + by \leq c$. The main focus is on the meaning of the feasible region and finding the maximum profit.

The last investigation involves how many hamburgers and apple pies a person can eat and still remain under the daily dietary constraints of at most 2400 mg of sodium and at most 65 grams of fat. The students are again asked to write inequalities, graph the inequalities, describe the feasible region, and find and interpret the corner points.

Teaching Notes

This lesson can be very difficult for students. Students need to be able to graph an inequality in the form $ax + by \geq c$. You may wish to have students find the $x$- and $y$-intercepts of the inequality and use the intercepts to graph the inequality. These intercepts become two corner points of the feasible region. Students also need to be able to solve a system of inequalities to find the remaining corner point. Even though corner points are introduced in this lesson, the idea that one of the corner points will be the answer to the question or which point will maximize or minimize the constraint is not formally covered. The intent of the lesson is to give students a beginning understanding of feasible regions and corner points.

Technology

Students can use graphing calculators to help them graph the inequalities and find the corner points. All of the inequalities are in the form $ax + by \geq c$, so students will need to solve for $y$ before entering them into the calculator. Once equations are entered, students can graph the equations and shade the correct regions. Graphing calculators can then be used to find the intersection of the two equations.

Follow-Up

You may wish to have students choose two menu items other than hamburger and apple pie and repeat Problems 22–28 using their choices.
Solution Key

Discussion and Practice

1. Let $j = \text{number of minutes jogging}$ and $w = \text{number of minutes weight lifting}$. Write an inequality that shows the relationship between minutes jogging and minutes weight lifting and the condition of at least 180 minutes of exercise a week.
2. **Exercise: Jogging and Weight Lifting**

```
<table>
<thead>
<tr>
<th>Minutes Jogging (j)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes Lifting (w)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

3. Possible answer: (100, 100); the x-coordinate is the number of minutes jogging and the y-coordinate is the number of minutes weight lifting.

4. a. 1215 calories  
   b. 2295 calories  
   c. 1665 calories

5. a. Yes; 1395 calories  
   b. Yes; 1305 calories  
   c. No; 1125 calories

6. Possible answer: (100, 80)

7. **Exercise: Jogging and Weight Lifting**

```
<table>
<thead>
<tr>
<th>Minutes Jogging (j)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes Lifting (w)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

8. The x-coordinate is the number of minutes jogging and the y-coordinate is the number of minutes weight lifting; these two values satisfy the inequality $9j + 4.5w \geq 1200$. 

9. On Activity Sheet 12, graph your inequality from Problem 1 on the first grid.

3. Find an ordered pair in the shaded region. Describe what the ordered pair represents.

Josh knows that for his weight and size jogging burns about 9 calories per minute and weight lifting burns approximately 4.5 calories per minute.

4. How many calories will Josh burn if he jogs
   a. for 90 minutes and lifts weights for 90 minutes each week?
   b. for 240 minutes and lifts weights for 30 minutes each week?
   c. for 60 minutes and lifts weights for 250 minutes each week?

5. If Josh wants to burn at least 1200 calories per week, could he reach this goal by jogging
   a. for 350 minutes and lifts weights for 100 minutes?
   b. for 100 minutes and lifts weights for 90 minutes?
   c. for 50 minutes and lifts weights for 150 minutes?

6. What do you think is the minimum number of minutes that Josh can jog and lift weights and still reach his goal of burning 1200 calories in a week? Remember he must exercise a total of at least 180 minutes a week.

You can use the following equation to find the total number of calories burned:

\[ 9j + 4.5w = c, \]

where \( j = \) number of minutes jogging, \( w = \) number of minutes weight lifting, and \( c = \) number of calories burned.

Since Josh wants to burn at least 1200 calories per week, you can use the following inequality:

\[ 9j + 4.5w \geq 1200 \]

7. On Activity Sheet 12, graph this inequality on the second grid.

8. Describe all the ordered pairs in the shaded region.
Josh wants to exercise at least 180 minutes and burn at least 1200 calories in a week. In order for him to determine the minimum number of minutes that he must jog and lift weights to reach this goal, he needs to consider the two inequalities together.

9. On Activity Sheet 13, graph the two inequalities \( j + w \geq 180 \) and \( 9j + 4.5w \geq 1200 \) on the first grid.

The region that is represented by the overlap of the two inequalities is called the feasible region. This region represents all the possible combinations of minutes jogging and weight lifting that meet the constraints of at least 180 minutes exercising and at least 1200 calories burned. Consider this graph.

Points A, B, and C are known as corner points of the feasible region. Point A is the x-intercept of the equation \( j + w = 180 \). Point B is the y-intercept of the equation \( 9j + 4.5w = 1200 \). Point C is the point whose ordered pair satisfies both equations.
10. a. The points in the feasible region satisfy both inequalities.
b. A (180, 0); B (0, 266.7); C (86.7, 93.3)
c. See table below.
d. Point A

11. Answers will vary.

12. Point C

13. \( m + c \leq 144 \)

---

### Lesson 7: Applying Systems of Inequalities

**Student Page 65**

10. Refer to the graph on page 64.
   a. The shaded region is the feasible region. Explain why this is the feasible region.
   b. Find the ordered pairs for points A, B, and C. List them in a table like the one below. Show your work.
   
<table>
<thead>
<tr>
<th>Point</th>
<th>Ordered Pair</th>
<th>Total Minutes Exercising</th>
<th>Total Calories Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(180, 0)</td>
<td>180</td>
<td>1620</td>
</tr>
<tr>
<td>B</td>
<td>(0, 266.7)</td>
<td>266.7</td>
<td>1200</td>
</tr>
<tr>
<td>C</td>
<td>(86.6, 93.3)</td>
<td>180</td>
<td>1200</td>
</tr>
</tbody>
</table>
   
   c. Find the total number of minutes exercising and the total calories burned for each corner point. List the results in your table.
   d. Of the three corner points, which one gives the greatest calorie burn for the fewest minutes exercising?

11. In your table, list three more ordered pairs that are in the feasible region. Find the total number of minutes exercised and the total calories burned for each ordered pair.

12. Suppose Josh wants to exercise for 180 minutes to stay in shape but does not want to lose weight. Of all the points listed in the table, which point gives the least number of calories burned for 180 minutes exercised? Compare answers within your group.

### Cookie Sale

The Conservation Club at a high school decides to sell cookies to earn some money to buy trees to plant around school. They decide to make no more than 12 dozen, or 144, cookies. They also decide to make only two types of cookies, one with M&Ms® and the other with chocolate chips.

13. Let \( m \) = number of M&M cookies and let \( c \) = number of chocolate-chip cookies. Write an inequality that shows the relationship between the number of each type of cookie under the condition that club members are planning to make no more than 144 cookies.
LESSON 7: APPLYING SYSTEMS OF INEQUALITIES

14. On Activity Sheet 13, graph your inequality from Problem 13 on the second grid.

15. List an ordered pair in the shaded region of your graph in Problem 14. Describe what this ordered pair represents.

The Conservation Club members know that each M&M cookie will cost about $0.15 to make and each chocolate-chip cookie will cost about $0.10 to make.

16. About how much will it cost to make
a. 72 cookies of each kind?
   b. 44 M&M cookies and 100 chocolate-chip cookies?
   c. 100 M&M cookies and 44 chocolate-chip cookies?

17. Use $m$ to represent the number of M&M cookies made, $c$ the number of chocolate-chip cookies made, and $t$ the total cost to write an equation that can be used to find the total cost of making all the M&M and chocolate-chip cookies.

18. The advisor of the club said that the club members can spend no more than $\$20.00$ on supplies for the cookies.
   a. Express the total cost as an inequality that shows the condition that the total cost cannot exceed $\$20.00$.
   b. Graph the inequality on the same grid you used for Problem 14.
   c. In a table like the one below, list three ordered pairs in the feasible region. Describe what each point represents.

<table>
<thead>
<tr>
<th>Ordered Pair</th>
<th>Number of M&amp;M Cookies</th>
<th>Number of Chocolate-Chip Cookies</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 144)</td>
<td>0</td>
<td>144</td>
<td>$14.40</td>
</tr>
<tr>
<td>(133, 0)</td>
<td>133</td>
<td>0</td>
<td>$20.00</td>
</tr>
<tr>
<td>(112, 32)</td>
<td>112</td>
<td>32</td>
<td>$20.00</td>
</tr>
</tbody>
</table>

e. Answers will vary. Each ordered pair satisfies the two inequalities:
   $m + c \leq 144$ and $0.15m + 0.1c \leq 20$.

d. Answers will vary.
19. a. \( p = 0.45m + 0.40c \)
   b. Answers for first three points will vary. Profit for (0, 144) = $57.60; profit for (133.3, 0) = $60.00; profit for (112, 32) = $63.20
   c. (112, 32)
   d. (112, 32)

Practice and Applications

20. a. 

   b. See second column.
   c. 

20. The Club decides to sell M&M cookies for $0.60 each and chocolate-chip cookies for $0.50 each. This means a profit of $0.45 for each M&M cookie and $0.40 for each chocolate-chip cookie.

   a. Use \( m \) to represent the number of M&M cookies made, \( c \) the number of chocolate-chip cookies made, and \( p \) the total profit to write an equation that could be used to find the total profit.
   b. Find the profit for each ordered pair listed in your table in Problem 18.
   c. Of the points listed in the table, which point gives the maximum profit?
   d. Compare your answer with those of other members in your group. Which point in the feasible region gave the maximum profit?

Summary

Linear programming involves writing a system of inequalities based on various constraints. The graph of the system of inequalities determines the feasible region. This region represents all of the ordered pairs that satisfy the system of inequalities. The corner points are those points at which various quantities are maximized or minimized.

Practice and Applications

20. Graph the feasible region for each system of inequalities.

   a. \( y \leq -2x + 8 \) \( y \leq -0.5x + 4 \) \( y \geq 1.5x + 1 \) \( y \leq 8 \) \( y \geq 0 \) \( x \geq 0 \)
   b. \( x \leq 5 \)
21. a. \( p = 0.45m + 0.50c \)
   b. (0, 200), $100; (133.33, 0), $60; (112, 32), $66.40
   c. (0, 200)

22. a. 1260 mg
   b. 33 g
   c. 1990 mg
   d. 56 g

23. \( s = 530h + 200a \)

21. Suppose the Conservation Club decides to sell each cookie for $0.60.
   a. Write an equation for the total profit that the club could make from the sale of M&M and chocolate-chip cookies.
   b. Find the profit for each ordered pair that you listed in Problem 18.
   c. Of the points listed, which one gives the maximum profit?

The National Research Council suggests that we have at most 2400 mg of sodium and at most 65 g of fat each day. If you were going to eat at a fast-food restaurant, how many hamburgers and apple pies could you eat and stay within these dietary conditions?

22. A hamburger from McDonald's contains 530 mg of sodium and 10 g of fat. Apple pie contains 200 mg of sodium and 13 g of fat.
   a. How many milligrams of sodium are contained in 2 hamburgers and 1 apple pie?
   b. How many grams of fat are contained in 2 hamburgers and 1 apple pie?
   c. How many milligrams of sodium are contained in 3 hamburgers and 2 apple pies?
   d. How many grams of fat are contained in 3 hamburgers and 2 apple pies?

Could you eat 6 hamburgers and no apple pies and stay within the dietary conditions concerning sodium and total fat? What do you think is the maximum number of hamburgers and apple pies you could order and eat and still stay under the dietary conditions?

Let \( h = \) the number of hamburgers eaten,
let \( a = \) the number of apple pies eaten,
let \( s = \) amount of sodium in mg, and
let \( f = \) amount of fat in g.

23. Write an equation that can be used to find the total amount of sodium for a given number of hamburgers and apple pies.
LESSON 7: APPLYING SYSTEMS OF INEQUALITIES

24. \( f = 10h + 13a \)

25. \( 530h + 200a \leq 2400 \)

26. \( 10h + 13a \leq 65 \)

27. Sodium and Fat in Fast Foods

28. a. All the ordered pairs that satisfy the inequalities \( 530h + 200a \leq 2400 \) and \( 10h + 13a \leq 65 \)

   b. \( (0, 5), (3.7, 2.1), (4.5, 0) \)

   c.

<table>
<thead>
<tr>
<th>Corner Point</th>
<th>Sodium</th>
<th>Fat</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (0, 5) )</td>
<td>1000</td>
<td>65</td>
</tr>
<tr>
<td>( (3.7, 2.1) )</td>
<td>2400</td>
<td>65</td>
</tr>
<tr>
<td>( (4.5, 0) )</td>
<td>2400</td>
<td>45</td>
</tr>
</tbody>
</table>

d. Between 3 and 4 hamburgers and about 2 apple pies
ASSESSMENT

Assessment for Unit III

Materials: rulers, Activity Sheet 15, End-of-Module Test
Technology: graphing calculators (optional)
Pacing: 1 class period or homework

Overview
Problem 1 assesses students' understanding of the inequalities in the form $x \leq a$ and $y \leq b$ as presented in Lesson 5. Problem 2 assesses the objectives of Lessons 6 and 7. Students are asked to graph a pair of inequalities and find the corner points and feasible region.

Teaching Notes
This assessment can be used in a number of ways. It can be used strictly as an assessment that is completed by students in one class period. It could also be used as a take-home test or as additional practice on the objectives of Lessons 5–7.

Technology
The use of graphing calculators is optional. Students may need a calculator to help them in their calculations of milligrams of riboflavin and vitamin B6.
ASSESSMENT: ASSESSMENT FOR UNIT III

Solution Key

I. a.

<table>
<thead>
<tr>
<th>City</th>
<th>Temperature (°F)</th>
<th>Precipitation (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milwaukee, WI</td>
<td>80</td>
<td>30.9</td>
</tr>
<tr>
<td>Nashville, TN</td>
<td>80</td>
<td>48.5</td>
</tr>
<tr>
<td>Cleveland, OH</td>
<td>76</td>
<td>46.1</td>
</tr>
<tr>
<td>Omaha, NE</td>
<td>89</td>
<td>30.3</td>
</tr>
<tr>
<td>St. Louis, MO</td>
<td>89</td>
<td>33.9</td>
</tr>
<tr>
<td>Minneapolis, MN</td>
<td>83</td>
<td>24.4</td>
</tr>
<tr>
<td>Salt Lake, UT</td>
<td>75</td>
<td>33.5</td>
</tr>
<tr>
<td>Detroit, MI</td>
<td>83</td>
<td>33.0</td>
</tr>
<tr>
<td>Louisville, KY</td>
<td>80</td>
<td>49.6</td>
</tr>
<tr>
<td>Wichita, KS</td>
<td>76</td>
<td>24.6</td>
</tr>
<tr>
<td>Des Moines, IA</td>
<td>85</td>
<td>30.8</td>
</tr>
<tr>
<td>Indianapolis, IN</td>
<td>85</td>
<td>33.1</td>
</tr>
<tr>
<td>Chicago, IL</td>
<td>83</td>
<td>33.3</td>
</tr>
</tbody>
</table>

b. See graph in Problem 1a; \( t < 85 \)

c. See graph in Problem 1a; \( p > 40 \)

d. No city

e. \( t > 85 \) and \( p > 40 \)

Assessment for Unit III

Objective:

Apply knowledge of systems of inequalities.

1. The following table contains the normal July high temperature (°F) and the normal annual precipitation (inches) for selected cities in the central United States.

<table>
<thead>
<tr>
<th>City</th>
<th>Temperature (°F)</th>
<th>Precipitation (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milwaukee, WI</td>
<td>80</td>
<td>30.9</td>
</tr>
<tr>
<td>Nashville, TN</td>
<td>80</td>
<td>48.5</td>
</tr>
<tr>
<td>Cleveland, OH</td>
<td>76</td>
<td>46.1</td>
</tr>
<tr>
<td>Omaha, NE</td>
<td>89</td>
<td>30.3</td>
</tr>
<tr>
<td>St. Louis, MO</td>
<td>89</td>
<td>33.9</td>
</tr>
<tr>
<td>Minneapolis, MN</td>
<td>83</td>
<td>24.4</td>
</tr>
<tr>
<td>Salt Lake, UT</td>
<td>75</td>
<td>33.5</td>
</tr>
<tr>
<td>Detroit, MI</td>
<td>83</td>
<td>33.0</td>
</tr>
<tr>
<td>Louisville, KY</td>
<td>80</td>
<td>49.6</td>
</tr>
<tr>
<td>Wichita, KS</td>
<td>76</td>
<td>24.6</td>
</tr>
<tr>
<td>Des Moines, IA</td>
<td>85</td>
<td>30.8</td>
</tr>
<tr>
<td>Indianapolis, IN</td>
<td>85</td>
<td>33.1</td>
</tr>
<tr>
<td>Chicago, IL</td>
<td>83</td>
<td>33.3</td>
</tr>
</tbody>
</table>


a. On Activity Sheet 15, use the first grid to make a scatter plot of the ordered pairs (temperature, precipitation).

b. Shade the region of the graph that represents the cities that have a normal high temperature in July of less than 85°F. Write an inequality to represent this region.

c. Shade the region of the graph that represents the cities that have an annual precipitation of more than 40 inches. Write an inequality to represent this region.

d. Identify the cities that satisfy both conditions.

e. Write a pair of inequalities that represent the region where July temperature is higher than 85°F and annual precipitation is more than 40 inches.
a. 1.385
b. 1.43
c. \(0.154B + 0.066A = \text{number of mg of B6}\)
d. \(0.166B + 0.019A = \text{number of mg of riboflavin}\)
e. \(0.154B + 0.066A \geq 2; 0.166B + 0.019A \geq 1.7\)

### Exercises

2. Vitamins are an important part of everyone's diet. Vitamins B6 and riboflavin work together with the other B vitamins to help cells absorb and burn energy. A diet deficient in B vitamins often results in muscle weakness and in psychiatric problems. The U.S. Recommended Daily Allowance of vitamin B6 is 2 mg and of riboflavin is 1.7 mg. Listed below are the amounts of B6 and riboflavin in a half cup of broccoli and in 1 apple.

<table>
<thead>
<tr>
<th>Food</th>
<th>B6 (mg)</th>
<th>Riboflavin (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broccoli (half cup)</td>
<td>0.154</td>
<td>0.166</td>
</tr>
<tr>
<td>Apple (1)</td>
<td>0.0186</td>
<td>0.019</td>
</tr>
</tbody>
</table>

a. How many milligrams of riboflavin are contained in 4 cups of broccoli and 3 apples?
b. How many milligrams of vitamin B6 are contained in 4 cups of broccoli and 3 apples?
c. Write an expression that can be used to find the total number of milligrams of B6 for a given number of apples and number of half cups of broccoli.
d. Write an expression that can be used to find the total number of milligrams of riboflavin for a given number of apples and number of half cups of broccoli.
e. Use your expressions from parts c and d to write two inequalities that show that the recommended daily allowance of B6 is at least 2 mg and the recommended daily allowance of riboflavin is at least 1.7 mg.
f. On the second grid on Activity Sheet 15, graph your inequalities from part e and clearly show the feasible region.
g. Describe the ordered pairs in the feasible region.
h. Find the corner points of the feasible region.
i. For each corner point, find the number of milligrams of vitamin B6 and riboflavin.
j. What is the least number of apples and half cups of broccoli a person could eat to reach the recommended daily allowance for B6 and riboflavin?

<table>
<thead>
<tr>
<th>Corner Point</th>
<th>B6 (mg)</th>
<th>Riboflavin (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 89.5)</td>
<td>5.907</td>
<td>1.7</td>
</tr>
<tr>
<td>(9.24, 8.74)</td>
<td>2</td>
<td>1.7</td>
</tr>
<tr>
<td>(13.0, 0)</td>
<td>2</td>
<td>2.158</td>
</tr>
</tbody>
</table>

j. 13 half cups of broccoli and no apples.
Teacher Resources
1. The scatter plot below shows the trends in the number of pairs of jeans and dress slacks sold from 1965 to 1992.

![Sales of Jeans and Dress Slacks](image)

- Jeans sold
- Dress slacks sold

a. Draw a line that best fits the data for jeans sales and another line that best fits the data for dress-slacks sales.

b. Write an equation for each line.

c. Describe the trends in the lines in terms of the slope.

d. Use algebra to find the point of intersection of the two lines.

e. What does this point represent?

2. Find the point of intersection for the following system of equations by making a graph and by finding the solution algebraically.

\[ y = -2x + 6 \text{ and } y = \frac{1}{2}x + 1 \]
1. Shown below is a graph of boy's age and time in seconds to earn a Presidential Physical-Fitness Award in the shuttle run.

![Graph of presidential award in shuttle run](image)

a. Draw a line that best represents the data.
b. Write an equation of the line.
c. Shade the region that represents the times for boys who would not earn an award.
d. Write an inequality that represents the shaded region.

2. Graph each of the following inequalities.
   a. $y \leq -2x - 3$
   b. $y > 10 + 3(x - 2)$
3. Write an inequality for each graph.

a. 

b.
1. Some financial advisors suggest a person should save at least 10% of his or her gross income. The scatter plot below shows the relationship between gross monthly income and the amount saved by a sample of 15 people.

<table>
<thead>
<tr>
<th>Monthly Income and Amount Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
</tr>
<tr>
<td>250</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>500 1000 1500 2000 2500 3000</td>
</tr>
<tr>
<td>Gross Monthly Income ($)</td>
</tr>
</tbody>
</table>

a. Through the origin, draw a line that summarizes the data.
b. Write an equation for the line.
c. Describe the slope in terms of the data.
d. Write an inequality indicating that a person should save at least 10% of his or her gross monthly income.
e. On the graph above, graph the inequality you wrote in part d.
f. What do the ordered pairs in the shaded region represent?
g. Write a pair of inequalities that describe the ordered pairs not in the shaded region but above the original line that you drew in part a.

2. Graph the following systems of inequalities.
   a. \( y \geq \frac{1}{2}x \) and \( y \leq 2x \)
   b. \( y \geq -2x \) and \( y \leq \frac{2}{3}x \)
While planning their wedding, Laura and Jim contacted a local hotel to inquire about reserving rooms for their guests from out of town. The hotel has two types of rooms. One type of room has one queen-size bed and the other type has two double beds. They decided that they would need at most 75 rooms for their guests and at least twice as many rooms with two beds as one bed. In order for Laura and Jim to receive a reduced rate for the reception to be held at the hotel, they must guarantee that at least 12 of each type of room will be rented.

1. Let $O$ = number of rooms with one bed, and let $T$ = number of rooms with two beds.
   
   a. Write an inequality that represents the constraint that Laura and Jim will need at most 75 rooms.
   
   b. Write an inequality that represents the constraint that Laura and Jim will need at least twice as many rooms with two beds as one bed.
   
   c. Write two inequalities that represent the constraint that Laura and Jim will need at least 12 of each type of room.

2. One the grid below, determine the feasible region by graphing the four inequalities from Problems 1a, b, and c.
3. Describe the points in the feasible region.

4. Find the corner points of the feasible region. Show how you found the points.

5. If a room with one bed costs $55 per night and a room with two beds costs $65 per night, find the cost at each corner point.
1. The table at the right compares the percent of men and women earning law degrees in the United States from 1986 to 1996.

<table>
<thead>
<tr>
<th>Year</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>61%</td>
<td>39%</td>
</tr>
<tr>
<td>1987</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>1988</td>
<td>59%</td>
<td>41%</td>
</tr>
<tr>
<td>1989</td>
<td>59%</td>
<td>41%</td>
</tr>
<tr>
<td>1990</td>
<td>58%</td>
<td>42%</td>
</tr>
<tr>
<td>1991</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>1992</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>1993</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>1994</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>1995</td>
<td>57%</td>
<td>43%</td>
</tr>
<tr>
<td>1996</td>
<td>56%</td>
<td>44%</td>
</tr>
</tbody>
</table>

a. On the graph below, draw a line that you think best represents each set of data.

b. Find the slope of each line. Describe the trend you observe in terms of the slopes.

c. Write an equation for each line.

d. Predict the percent of women earning law degrees in the year 2000. Explain how you made your prediction.

e. For what year will the percent of men and the percent of women earning law degrees be the same? Show how you got your answer.
2. Find the intersection point for the following system of equations by making a graph and by finding the solution algebraically.

\[ y = -x + 8 \text{ and } y = \frac{1}{2}x - 1 \]

3. Shown below is a graph of girl's age and time in seconds to earn a Presidential Physical-Fitness Award in the shuttle run.

![Graph of girl's age and time to earn a Presidential Physical-Fitness Award in the shuttle run.]

**a.** Draw a line that best represents the data.

**b.** Write an equation for the line.

**c.** Write an inequality that represents the girls' times that are better than the standard.

4. Graph the inequality \[ y \leq -3x - 2 \].

5. Graph the following systems of inequalities.

   **a.** \[ y \geq -3 \text{ and } x \leq 4 \]

   **b.** \[ y \leq 3x \text{ and } y \geq 1.5x \]

   **c.** \[ x + y \leq 10, y \geq x, \text{ and } y \geq 0 \]
6. Cindy wants to combine swimming and walking as part of an exercise routine. She wants to exercise a total of 4 hours per week (240 minutes). She wants to spend at least 60 minutes on each activity and would like to burn at least 1000 calories a week with this exercise routine. Swimming burns 10 calories per minute and walking burns 5 calories per minute.

Let $S =$ number of minutes swimming and $W =$ number of minutes walking

a. Write an inequality for each of the following constraints.

i. Cindy wants to exercise at most 240 minutes.

ii. Cindy wants to spend at least 60 minutes on each activity.

iii. Cindy wants to burn at least 1000 calories per week.

b. On the graph below, determine the feasible region by graphing the inequalities you wrote in part a.

```
Swimming and Walking

Minutes Swimming (S)

Minutes Walking (W)
```

c. Find the corner points. Show how you found the points.
1. a. Possible answer:

Sales of Jeans and Dress Slacks

Answers to parts b, c, and d are based on part a graph.

b. Jean-sales equation: \( y = 0.28x - 549.8 \)
   Dress-slacks sales equation: \( y = -0.15x + 300 \)

c. Interpretation of jeans-sales slope: The number of jeans sold increases \( 0.28 \times 100,000 = 28,000 \) each year.
   Interpretation of dress-slacks slope: The number of dress slacks sold decreases \( 0.15 \times 100,000 = 15,000 \) each year

d. The point of intersection is approximately (1976, 3.6)

e. This point represents the year when jeans sales and dress-slacks sales were equal.

2. (2, 2)
1. a. Possible answer:

![Graph of Presidential Award in Shuttle Run](image)

Answers to parts b, c, and d are based on part a graph.

b. \( y = -0.37x + 15.2 \)

c. ![Graph of Presidential Award in Shuttle Run](image)

d. \( y > -0.37x + 15.2 \)

2. a. ![Graph of region](image)

3. a. \( y \geq \frac{6}{5} x + 6 \)

b. \( y \leq x \)
### Lesson 6 Quiz: Solution Key

#### 1. a. Possible answer:

**Monthly Income and Amount Saved**

<table>
<thead>
<tr>
<th>Gross Monthly Income ($)</th>
<th>Amount Saved ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>50</td>
</tr>
<tr>
<td>1000</td>
<td>100</td>
</tr>
<tr>
<td>1500</td>
<td>150</td>
</tr>
<tr>
<td>2000</td>
<td>200</td>
</tr>
<tr>
<td>2500</td>
<td>250</td>
</tr>
<tr>
<td>3000</td>
<td>300</td>
</tr>
</tbody>
</table>

#### 2. a. \( y \geq \frac{1}{2}x \) and \( y \leq 2x \)

- \( y \geq -2x \) and \( y \leq \frac{2}{3}x \)

#### b. There will be many different answers to this question. By having the students draw their line through the origin, the slopes could vary. You should expect equations whose slope is about 0.07, that is, \( y = 0.07x \).

#### c. For each additional $1, the amount saved is 7¢.

#### d. Amount saved \( \geq 0.10 \times \) gross monthly income

#### e. **Monthly Income and Amount Saved**

#### f. The points where the amount saved is greater than 10% of the gross monthly income.

#### g. \( y \geq 0.07x \) and \( y \leq 0.10x \)
1. a. \( O + T \leq 75 \)
   b. \( T \geq 20 \)
   c. \( T \geq 12 \) and \( O \geq 12 \)

2. The points in the feasible region represent all the ordered pairs that satisfy the four inequalities.

3. (12, 24), (12, 63), and (25, 50); students' explanations will vary.

4. Point (12, 24): cost = 12(55) + 24(65) = $2220
   Point (12, 63): cost = 12(55) + 63(65) = $4755
   Point (25, 50): cost = 25(55) + 50(65) = $4625
Answers to parts b, c, and d are based on part a graph.

b. Slope of men’s line: \(-0.436\); slope of women’s line: \(0.436\)
   Each year the percent of men earning a law degree drops about 0.44 percent; each year the percent of women earning a law degree increase about 0.44 percent.

c. Men’s equation: \(y = -0.436x + 926\); women’s equation: \(y = 0.436x - 826\)

d. Approximately 46%

e. About the year 2010. Answers may vary somewhat, depending on what slope the students use. You may wish to suggest that they use at least 3 decimal places.

2. (6, 2)
b. 

\[ \begin{align*} 
&\text{i. } S + W \leq 240 \\
&\text{ii. } S \geq 60 \text{ and } W \geq 60 \\
&\text{iii. } 10S + 5W \geq 1000 
\end{align*} \]

c. \((60, 70), (60, 180), (180, 60), (80, 60)\)
ACTIVITY SHEET 1
Introductory Activity for Unit I, Problems 3-6

NAME ______________________

Men's and Women's 800-Meter Olympic Times

- Men's times
- Women's times

Year
1900 1940 1980 2020 2060 2100 2140

Times (seconds)
70 80 90 100 110 120 130
Lesson 1, Problem 3

Circulation of Morning and Evening Newspapers

<table>
<thead>
<tr>
<th>Year</th>
<th>Morning</th>
<th>Evening</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>1974</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>1978</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>1982</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>1986</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>1990</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>1994</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>1998</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>
Lesson 1, Problem 9

Elevation and Number of Cloudy Days out of 365

Elevation and Number of Clear Days out of 365
ACTIVITY SHEET 4

Lesson 2, Problems 2 and 7

NAME

Cable TV

![Graph showing percent wired for or subscribing to Cable TV]

- □ Percent wired for cable
- ● Percent subscribing to cable

Year

Attendance at Disney World and Universal Studios

![Graph showing attendance at Disney World and Universal Studios]

- □ Disney
- ● Universal

Year
ACTIVITY SHEET 5

Assessment for Unit I, Problem 2

NAME ________________________________

![Graph showing cost of construction lumber against length.]

Cost of Construction Lumber

- 2-inch-by-4-inch
- 2-inch-by-8-inch

Cost ($) vs. Length (feet)
Lesson 3, Problem 6

Percent of Teens and Children
Watching Top 30 TV Shows in 1993

NAME

ACTIVITY SHEET 6
Activity Sheet 7

Lesson 4, Problems 1 and 2

NAME

Presidential Award in 1-Mile Run

Time (minutes)

Boy's Age (years)
ACTIVITY SHEET 8
Lesson 5, Problems 1-7, 11, and 12

NAME

1996 NBA Championship Series
Bulls Versus Supersonics

Price and Mileage for Selected 1995 Cars
Lesson 5, Problem 15

Power and Price for Selected Minisystems

<table>
<thead>
<tr>
<th>Power (watts)</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>300</td>
</tr>
<tr>
<td>40</td>
<td>400</td>
</tr>
<tr>
<td>60</td>
<td>500</td>
</tr>
<tr>
<td>80</td>
<td>600</td>
</tr>
<tr>
<td>100</td>
<td>700</td>
</tr>
<tr>
<td>120</td>
<td>800</td>
</tr>
<tr>
<td>140</td>
<td>900</td>
</tr>
</tbody>
</table>
Calories and Calories from Fat in Fast Foods

Calories from Fat

Calories

Calories and Calories from Fat in Fast Foods

Calories from Fat (f)

Calories (c)
ACTIVITY SHEET 11
Lesson 6, Problems 11-13 and 18

NAME

Calories and Saturated Fat in Fast Foods

Gross Monthly Income and Housing Costs

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Lesson 7, Problems 2, 3, and 7

Exercise: Jogging and Weight Lifting

Minutes Jogging (j)

Minutes Weight Lifting (w)
Exercise: Jogging and Weight Lifting

Cookie Sale
NAME ____________________________

**July High Temperatures and Annual Precipitation**

**Vitamins in Broccoli and Apples**
Data-Driven Mathematics
Procedures for Using the TI-83

I. Clear menus

ENTER will execute any command or selection. Before beginning a new problem, previous instructions or data should be cleared. Press ENTER after each step below.

1. To clear the function menu, Y=, place the cursor anyplace in each expression, CLEAR
2. To clear the list menu, 2nd MEM ClrAllLists
3. To clear the draw menu, 2nd DRAW ClrDraw
4. To turn off any statistics plots, 2nd STATPLOT PlotsOff
5. To remove user-created lists from the Editor, STAT SetUpEditor

II. Basic information

1. A rule is active if there is a dark rectangle over the option. See Figure 1.

On the screen above, Y1 and Plot1 are active; Y2 is not. You may toggle Y1 or Y2 from active to inactive by putting the cursor over the = and pressing ENTER. Arrow up to Plot1 and press ENTER to turn it off; arrow right to Plot2 and press ENTER to turn it on, etc.

2. The Home Screen (Figure 2) is available when the blinking cursor is on the left as in the diagram below. There may be other writing on the screen. To get to the Home Screen, press 2nd QUIT. You may also clear the screen completely by pressing CLEAR.

III. The STAT Menus

1. There are three basic menus under the STAT key: EDIT, CALC, and TESTS. Data are entered and modified in the EDIT mode; all numerical calculations are made in the CALC mode; statistical tests are run in the TEST mode.

2. Lists and Data Entry
Data is entered and stored in Lists (Figure 4). Data will remain in a list until the list is cleared. Data can be cleared using CLEAR L1 or (List name), or by placing the cursor over the List heading and selecting CLEAR ENTER. To enter data, select STAT EDIT, and with the arrow keys move the cursor to the list you want to use.
Naming Lists
Six lists are supplied to begin with. L1, L2, L3, L4, L5, and L6 can be accessed also as 2nd L1. Other lists can be named using words as follows. Put the cursor at the top of one of the lists. Press 2nd INS and the screen will look like that in Figure 6.

![Figure 6](image)

PROCEDURES FOR USING THE TI-83

Type in a numerical value and press ENTER. Note that the bottom of the screen indicates the List you are in and the list element you have highlighted. 275 is the first entry in L1. (It is sometimes easier to enter a complete list before beginning another.)

![Figure 7](image)

For data with varying frequencies, one list can be used for the data, and a second for the frequency of the data. In Figure 5 below, the L5(7) can be used to indicate that the seventh element in list 5 is 4, and that 25 is a value that occurs 4 times.

![Figure 5](image)
4. **Graphing Statistical Data**

**General Comments**
- Any graphing uses the GRAPH key.
- Any function entered in Y1 will be graphed if it is active. The graph will be visible only if a suitable viewing window is selected.
- The appropriate x- and y-scales can be selected in WINDOW. Be sure to select a scale that is suitable to the range of the variables.

**Statistical Graphs**
To make a statistical plot, select 2nd Y= for the STAT PLOT option. It is possible to make three plots concurrently if the viewing windows are identical. In Figure 9, Plots 2 and 3 are off, Plot 1 is a scatter plot of the data (Costs, Seats), Plot 2 is a scatter plot of (L3, L4), and Plot 3 is a box plot of the data in L3.

**FIGURE 9**

Activate one of the plots by selecting that plot and pressing ENTER.

Choose ON, then use the arrow keys to select the type of plot (scatter, line, histogram, box plot with outliers, box plot, or normal probability plot). (In a line plot, the points are connected by segments in the order in which they are entered. It is best used with data over time.) Choose the lists you wish to use for the plot. In the window below, a scatter plot has been selected with the x-coordinate data from COSTS, and the y-coordinate data from SEATS. (Figure 10) (When pasting in list names, press 2nd LIST, press ENTER to activate the name, and press ENTER again to locate the name in that position.)

**Statistical Calculations**
One-variable calculations such as mean, median, maximum value of the list, standard deviation, and quartiles can be found by selecting STAT CALC 1-Var Stats followed by the list in which you are interested. Use the arrow to continue reading the statistics. (Figures 11, 12, 13)
1-Var Stats
\( x = 1556.20833 \)
\( \Sigma x = 37349 \)
\( \Sigma x^2 = 135261515 \)
\( Sx = 1831.353621 \)
\( \sigma x = 1792.79449 \)
\( \downarrow n = 24 \)

**FIGURE 13**

Calculations of numerical statistics for bivariate data can be made by selecting two variable statistics. Specific lists must be selected after choosing the 2-Var Stats option. (Figure 14)

2-Var Stats L1, L2

**FIGURE 14**

Individual statistics for one- or two-data sets can be obtained by selecting VARS Statistics, but you must first have calculated either 1-Var or 2-Var Statistics. (Figure 15)

**FIGURE 15**

6. **Fitting Lines and Drawing Their Graphs**
Calculations for fitting lines can be made by selecting the appropriate model under STAT: Med-Med gives the median fit regression, LinReg the least-squares linear regression, and so on. (Note the only difference between LinReg (ax+b) and LinReg (a+bx) is the assignment of the letters a and b.) Be sure to specify the appropriate lists for \( x \) and \( y \). (Figure 16)

**FIGURE 16**

To graph a regression line on a scatter plot, follow the steps below:
- Enter your data into the Lists.
- Select an appropriate viewing window and set up the plot of the data as above.
- Select a regression line followed by the lists for \( x \) and \( y \), VARS Y-VARS Function (Figures 17, 18) and the \( Y_i \) you want to use for the equation, followed by ENTER.

**FIGURE 17**

**FIGURE 18**
The result will be the regression equation pasted into the function Y1. Press GRAPH and both the scatter plot and the regression line will appear in the viewing window. (Figures 19, 20)

\[
Y_1 = 0.52112676056338X + -37.37089201878
\]

There are two cursors that can be used in the graphing screen.

TRACE activates a cursor that moves along either the data (Figure 21) or the function entered in the Y-variable menu (Figure 22). The coordinates of the point located by the cursor are given at the bottom of the screen.

Pressing GRAPH returns the screen to the original plot. The up arrow key activates a cross cursor that can be moved freely about the screen using the arrow keys. See Figure 23.

Exact values can be obtained from the plot by selecting 2nd CALC Value. Select 2nd CALC Value ENTER. Type in the value of \( x \) you would like to use, and the exact ordered pair will appear on the screen with the cursor located at that point on the line. (Figure 24)
IV. Evaluating an expression

To evaluate \( y = 0.225x - 15.6 \) for \( x = 17, 20, \) and 24, you can:

1. Type the expression in Y1, return to the home screen, 17 STO X,T,0,n ENTER, VARS Y-VARS Function Y1 ENTER ENTER. (Figure 25)

   \[
   \begin{align*}
   17 & \rightarrow X \\
   \text{Y1} & \rightarrow -11.775
   \end{align*}
   \]
   (Figure 25)

   Repeat the process for \( x = 20 \) and 24.

2. Type \( 17^2 - 4 \) for \( x = 17 \), ENTER (Figure 26). Then use 2nd ENTRY to return to the arithmetic line. Use the arrows to return to the value 17 and type over to enter 20.

   \[
   \begin{align*}
   0.225 \cdot 17 - 15.6 & \rightarrow -11.775 \\
   0.225 \cdot 20 - 15.6 & \rightarrow -11.775
   \end{align*}
   \]
   (Figure 26)

You can also find the value of \( x \) by using the table command. Select 2nd TblSet (Figure 27). (Y1 must be turned on.) Let TBLStart = 17, and the increment \( \Delta \text{Tbl} = 1 \).

\[
\text{TABLE SETUP} \\
\begin{array}{ll}
\text{TblStart} & = 17 \\
\Delta \text{Tbl} & = 1 \\
\text{Indpnt:} & = \text{Auto} \\
\text{Depend:} & = \text{Auto}
\end{array}
\]

(FIGURE 27)

Select 2nd TABLE and the values of \( x \) and \( y \) generated by the equation in Y1 will be displayed. (Figure 28)

\[
\begin{array}{|c|c|}
\hline
X & Y1 \\
\hline
17 & -11.78 \\
18 & -11.55 \\
19 & -11.33 \\
20 & -11.1 \\
21 & -10.88 \\
22 & -10.65 \\
23 & -10.43 \\
\hline
\end{array}
\]

(FIGURE 28)

V. Operating with Lists

1. A list can be treated as a function and defined by placing the cursor on the label above the list entries. List 2 can be defined as \( L1 + 5 \). (Figure 29)

\[
\begin{array}{|c|c|c|}
\hline
L1 & L2 & L3 \\
\hline
275 & \ldots & 190 \\
5311 & 120 & \\
114 & 238 & \\
2838 & 153 & \\
15 & 179 & \\
332 & 207 & \\
3828 & 153 & \\
\hline
L2 = L1 + 5 \\
\end{array}
\]

(FIGURE 29)

Pressing ENTER will fill List 2 with the values defined by \( L1 + 5 \). (Figure 30)
2. List entries can be cleared by putting the cursor on the heading above the list, and selecting CLEAR and ENTER.

3. A list can be generated by an equation from $Y = \ldots$ over a domain specified by the values in L1 by putting the cursor on the heading above the list entries. Select VARS Y-VARS Function Y1 ENTER (L1) ENTER. (Figure 31)

4. The rule for generating a list can be attached to the list and retrieved by using quotation marks (ALPHA+) around the rule. (Figure 32) Any change in the rule (Y1 in the illustration) will result in a change in the values for L1. To delete the rule, put the cursor on the heading at the top of the list, press ENTER, and then use the delete key. (Because L1 is defined in terms of CAL, if you delete CAL without deleting the rule for L1 you will cause an error.)

VI. Using the DRAW Command

To draw line segments, start from the graph of a plot, press 2ND DRAW, and select Line. (Figure 33)

This will activate a cursor that can be used to mark the beginning and ending of a line segment. Move the cursor to the beginning point and press ENTER; use the cursor to mark the end of the segment, and press ENTER again. To draw a second segment, repeat the process. (Figure 34)
VII. Random Numbers

To generate random numbers, press MATH and PRB. This will give you access to a random number function, rand, that will generate random numbers between 0 and 1 or randInt() that will generate random numbers from a beginning integer to an ending integer for a specified set of numbers. (Figure 35) In Figure 36, five random numbers from 1 to 6 were generated.

<table>
<thead>
<tr>
<th>MATH</th>
<th>NUM</th>
<th>CPX</th>
<th>PRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: rand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2: nPr</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3: nCr</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4: !</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5: randInt()</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6: randNorm()</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7: randBin()</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 35

randInt(1, 6, 5,)
(2 4 2 5 3)

FIGURE 36

Pressing ENTER will generate a second set of random numbers.
Data-Driven Mathematics is a series of modules written by teachers and statisticians that focuses on the use of real data and statistics to motivate traditional mathematics topics. This chart suggests which modules could be used to supplement specific middle-school and high-school mathematics courses.

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