

A Sweet Task

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Overview of Lesson

Working in pairs and with class discussion, students will gain an understanding of how to create two-way frequency tables and interpret relative frequencies in the context of types and colors of candy.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report. The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. This is a GAISE Level B Activity.

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

Common Core State Standards Grade Level Content (High School)

S-ID. 5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

S-CP. 3. Understand the conditional probability of A given B as $P(A \text{ and } B) / P(B)$.

[Extension only.]

S-CP. 4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to approximate conditional probabilities.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability Standards for Grades 9-12

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them:

- compute basic statistics and understand the distinction between a statistic and a parameter.

Select and use appropriate statistical methods to analyze data:

- display and discuss bivariate data where at least one variable is categorical.

Develop and evaluate inferences and predictions that are based on data:

- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

Understand and apply basic concepts of probability:

- understand the concepts of conditional probability and independent events;
- understand how to compute the probability of a compound event.

Prerequisites

Students will need to be familiar with creating and using single-variable frequency tables.

Students also need to have a basic understanding of probability.

Learning Targets

Students will learn how to summarize categorical data in a two-way frequency table and interpret the data within a context of types and colors of candies. After an exploration activity and a class discussion, students will learn how to compute joint, marginal, and conditional probabilities from a two-way frequency table.

Time Period

This lesson may take place in one 50-minute class period; however, teachers may need 1.5 periods or a 90-minute block.

Materials Required

- “A Sweet Task” Activity Worksheet (located at the end of this lesson)
- A large bag of plain M&Ms and a large bag of regular Skittles – Each team of two students should receive 30 of each candy type
- Bag of trail mix or snack mix in a transparent bag
- Preformatted spreadsheets which will sum the rows/columns (rows represent the type of candy and columns represent the colors)
- Written assignments of problems

Instructional Lesson Plan

The GAISE Statistical Problem-Solving Procedure

I. Formulate Question(s)

Hold up the bag of trail or snack mix. Ask students to think about their favorite ingredient in the trail or snack mix and discuss sharing the mix with their families or friends. Talk about how some people just eat the pieces they like and leave the rest. Discuss what the actual proportions of each type of piece might be and what you might need to include in order to create a “perfect” mix to share with their families.

Introduce the task – “You and your partner will determine the probabilities of drawing different types of candy from our own two-ingredient candy mix of M&Ms and Skittles.” The candy mix will include all data from each team of two. Have students guess what the probability of picking a green M&M would be out of this “mix”. Also, have them guess what would be the probability that, if they pick a Skittle, it would be yellow. Finally, ask the students “If you know a candy is red, is it more likely to be an M&M or a Skittle?” Discuss ways that they could find out the answers to these questions.

Skittle colors are reportedly distributed evenly with 20% of each color. A distribution of plain M&M colors is provided below according to a 2008 correspondence with Mars, Inc.

From: <http://www.exeter.edu/documents/mandm.pdf>

The M&Ms website no longer posts the proportions of colors found in bags of M&Ms.

The following is their response to an inquiry from the fall of 2008.

Our color blends were selected by conducting consumer preference tests, which indicate the assortment of colors that pleased the greatest number of people and created the most attractive overall effect.

M&M'S MILK CHOCOLATE: 24% cyan blue, 20% orange, 16% green, 14% bright yellow, 13% red, 13% brown.

M&M'S PEANUT: 23% cyan blue, 23% orange, 15% green, 15% bright yellow, 12% red, 12% brown.

M&M'S DARK: 17% cyan blue, 16% orange, 16% green, 17% bright yellow, 17% red, 17% brown.

M&M'S PEANUT BUTTER and ALMOND: 20% cyan blue, 20% orange, 20% green, 20% bright yellow, 10% red, 10% brown.

Each large production batch is blended to those ratios and mixed thoroughly. However, since the individual packages are filled by weight on high-speed equipment, and not by count, it is possible to have an unusual color distribution.

II. Design and Implement a Plan to Collect the Data

Students should be put into groups of two. Distribute the samples ($n = 30$ each) of plain M&Ms and Skittles to each pair. Additionally, provide each student with an Activity Sheet (see the end of this lesson).

Students will create frequency tables in Question 1. They will then share their counts with the teacher who will quickly enter them into a spreadsheet. As an alternative, students could enter the data into a spreadsheet on the teacher's computer. Students will copy the compiled class data into their own tables in Question 2 on their Activity Sheets, and then calculate the totals for each row and column.

Checkpoint #1 – Be sure all students have copied the data onto their Activity Sheets. Remind students how to find the probability of an event. The probability of an event = Number of ways the event can occur divided by the Number of possible outcomes. An example is:

The probability of choosing a blue candy =

Number of blue candies in the class sample / The total number of candies

III. Analyze the Data

If the students have had little to no experience with joint and marginal probability a discussion prior to having them start Question 3 is recommended. A joint probability embodies the likelihood that two events will occur at the same time. However, marginal probability is concerned with only one event occurring. When viewing the two-way frequency table, be sure to highlight that the marginal frequencies can be found along the edges or margins of the table (totals), which represent one event, such as a candy being orange. The joint frequencies can be found only in the middle of the table where combinations of characteristics occur, such as a candy being yellow **and** a Skittle.

Have students complete Question 3 with partners. Question 3 introduces the students to marginal and joint probabilities and how to use the frequency table to compute them.

Checkpoint #2 – Discuss students' answers for Question 3 problems a through h. Have a class discussion about marginal probability and joint probability. Recall: marginal probability is concerned with the probability that one characteristic occurs and is found using the numbers in the bottom row or right column in the numerator, and the table total in the denominator. Joint probability addresses two characteristics occurring at the same time, and is found using the cells in the middle of the table in the numerator and the table total in the denominator. Ask students to suggest additional problems for the class to solve and discuss the type of probability (marginal or joint).

Question 4 is intended to walk students through the process of discovering how to find conditional probabilities using counts in a two-way frequency table. Let students work with their partners on this section up to Checkpoint #3. Be sure to walk around the class to ensure students are on the right track before working problems in Question 5.

IV. Interpret the Results

Checkpoint #3 – Discuss answers to Question 5 in terms of both their numerical value and the meaning given the particular problem context. Ask students “What the answer was to problem 1?” and then “What does this tell us?” A response should convey that this is the likelihood that a candy is green if we know that the candy is also a Skittle. Follow this process for each problem. At the conclusion, ask students if they now know the likelihood that a candy is an M&M given that you know it is red (answer from Question 4, part b). Compare this to the probability determined in Question 5, part d, the probability that a candy is a Skittle if it is red. What do the students notice about these two probabilities? Hopefully, they will see that they add to 1, as the two events are compliments.

With the class, determine the answers to the other questions posed in the launch: “What is the probability of picking a green M&M given our data?” and “If we choose a Skittle, what is the probability that it would be yellow?”.

Assessment

The table below shows the number of M&Ms and Skittles in another class' candy mixture. Use the data in the table to complete the problems that follow.

	Red	Orange	Yellow	Green	Blue	Purple	Brown	Total
M&M's	12	6	8	14	10	0	8	58
Skittle's	9	11	7	10	7	5	0	49
Total	21	17	15	24	17	5	8	107

1. Marginal probability.

a) Write an example event in words.

b) Find the value of the probability.

2. Joint probability.

a) Write an example event in words.

b) Find the value of the probability.

3. Conditional probability.

a) Write an example event in words.

b) Find the value of the probability.

Answers

1.
 - a. An example of a student response might be to randomly select an M&M from the classroom “mix” of candy. This is marginal probability because only one characteristic needs to occur.
 - b. Then, the probability of this would be 58/107.

2.
 - a. An example of a student response might be to select the event of a purple Skittle selected at random. This is a joint probability because the outcome requires that two characteristics occur at the same time.
 - b. Then, the probability would be 5/107.

3.
 - a. An example of a student response might be to pose the question, “If a green candy is selected, what is the probability that it is a Skittle?”. This is a conditional probability because one condition is given, restricting the sample space that needs to be considered.
 - b. Then, the probability would be 10/24, as only the “Green” candy column is considered.

Possible Extension: A short extension to the lesson that builds upon the work completed thus far is to connect the two-way frequency approach to the ratio of probabilities formula given below:

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

This formula is needed when counts are not available. In Question 5, students computed conditional probabilities by finding a joint frequency and then dividing by a marginal frequency. Demonstrate to students that this formula follows the same process. For example, for problem b in Question 5, show that $P(\text{M\&M} \cap \text{Blue})$ is similar to the joint frequency value where the “M&M” row and “Blue” column intersect divided by the total number of candies in the two-way frequency table. Then, the $P(\text{Blue})$ is similar to the total for the “Blue” column divided by the total number of candies. Therefore, the frequency representation similar to the ratio of probabilities is:

$$\frac{\text{BlueM \& Ms}}{\text{TotalCandies}} \div \frac{\text{BlueCandies}}{\text{TotalCandies}}, \text{ which reduces to } \frac{\text{BlueM \& Ms}}{\text{BlueCandies}}.$$

Note: Wendy Weber, Central College, provided guidance on this lesson plan during the Mathematics and Science Partnership project “*Statistics and Probability for Teaching Secondary Students*” (2013).

A Sweet Task – A Lesson in Exploring Conditional Probability Activity Sheet

Name _____ Date _____

One member of each group should obtain a bag of M&Ms and a bag of Skittles from the teacher.

1. Collect Data: Count the number of M&Ms and Skittles of each color in your bag and record this information in the frequency tables below. Then, either enter the data on your teacher’s computer or give the data to your teacher.

	Red	Orange	Yellow	Green	Blue	Brown	Total
M&M’s							

	Red	Orange	Yellow	Green	Blue	Purple	Total
Skittle’s							

2. Create a Two-Way Frequency Table: We can combine individual frequency tables into a two-way frequency table. The rows represent the **types of candy** and the columns represent the **color of the candy**. Use the combined class data (provided by your teacher) to fill in the two-way frequency table below. **Be sure to total each column and row.**

	Red	Orange	Yellow	Green	Blue	Purple	Brown	Total
M&M’s								
Skittle’s								
Total								

We read a two-way frequency table in a similar way as a regular frequency table. For example, the number of orange Skittles is listed where the “Orange” column and the “Skittles” row meet. This is called a **joint frequency**.

We can also find the total number of blue candies in the bag. We just look at the total of the “Blue” column. This is a **marginal frequency**.

Checkpoint #1

3. Analyzing the Data – Finding Marginal and Joint Probabilities: We can compute the probability of an event occurring from the frequency counts within the candy “mix” two-way frequency table. Find the probability of randomly choosing a candy with the listed attributes. Also, identify each event as either a joint or marginal probability. A joint probability requires two or more characteristics to hold true, whereas a marginal probability requires only one.

Probability

Joint or Marginal Probability

a. Any Color M&M

b. A Purple Skittle

c. A Blue M&M

d. An Orange Skittle

e. Any Green candy

f. A Blue Skittle

g. In your own words describe how you compute a joint probability given counts in a two-way frequency table.

h. In your own words describe how you compute a marginal probability given counts in a two-way frequency table.

Checkpoint #2

4. Finding Conditional Probability with Counts: Imagine that your friend chooses a candy piece from the class “mix”. She looks at it, tells you that it is red, but doesn’t tell you if it is an M&M or a Skittle.

Knowing that your friend has a red candy in her hand, we can find the probability that it is a red M&M. This is called the **conditional probability** of an event because **we already know something (a condition) about the event in question.**

Answer the following questions to help you find the conditional probability.

a. What is the “total number of possible outcomes” for your friend’s candy? (*Remember we know the candy is red.*)

b. What is the probability of your friend having an M&M, if we know the candy is red? (*Keep in mind we only are worried about M&Ms that are red.*)

c. In your own words, explain how to compute conditional probabilities given a two-way frequency table.

What you just found can be written as $P(\text{M\&M} \mid \text{red})$, which we read as “the probability of a candy being an M&M *given* that it is red”.

5. Computing Conditional Probabilities: Using the class data about the candy “mix”, find the following conditional probabilities.

a. $P(\text{green} \mid \text{Skittle})$

b. $P(\text{M\&M} \mid \text{blue})$

c. $P(\text{brown} \mid \text{M\&M})$

d. $P(\text{Skittle} \mid \text{red})$

e. $P(\text{Skittle} \mid \text{purple})$

f. $P(\text{M\&M} \mid \text{purple})$

g. $P(\text{yellow} \mid \text{M\&M})$

h. $P(\text{M\&M} \mid \text{yellow})$

Checkpoint #3

If you draw a red candy, is it more likely to be an M&M or Skittle? Why?