

Odd or Even? The Addition and Complement Principles of Probability

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Student Handout and Appendix with Technology Directions

Odd or Even? The Addition and Complement Principles of Probability Student Handouts

Launch:

Work with a partner to analyze the game “Odd or Even”. A trial occurs when two cubes are rolled once and points are assigned. The game consists of many trials, the number of which are decided upon by Player A and B before the first trial.

Odd or Even Rules:

Roll two standard number cubes.

If the sum is 6 or an odd number, then Player A scores a point.

If the sum is 6 or an even number, then Player B scores a point.

Repeat the above steps an agreed number of times.

Whomever has the most points at the end of the game wins.

1. Suppose you play a game with a certain number of trials. Predict which partner is likely to score more points in “Odd or Even”. Justify your reasoning.
2. Next, ask your teacher for two standard numbered cubes. Then, play the game with 30 trials, keeping track of scores by tallying points:

Player A’s points:

Probability that Player A wins:

Player B’s points:

Probability that Player B wins:

3. Looking at your results, which player was more likely to win?

How do your experimental results compare with your prediction in question 1? Are your results surprising? In what way(s)?

4. Using the technology specified by your teacher, simulate tossing a pair of numbered cubes 1000 times. How many points did each player win?

Player A’s points:

Probability that Player A wins:

Player B’s points:

Probability that Player B wins:

How do your results using technology to simulate playing the game compare with the prediction you made in question 1 and your experimental result in question 3?

5. Do you think the game is fair? Why or why not?
6. We want to be sure of our answer to the previous question. What ideas do you have to find the exact/theoretical probability of each player winning?

Explore:
On Your Own...

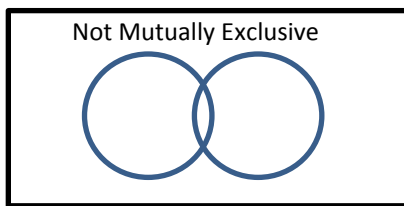


Diagram X

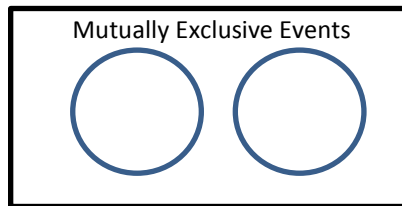


Diagram Y



Diagram Z

7. Consider the Venn Diagrams above where one circle in each diagram represents $S =$ “the sum is 6” and the other represents $O =$ “the sum is odd”.
 - a. The Venn diagram that represents Player A’s events is diagram _____. Explain why.
 - b. How is the probability of Player A winning related to the diagram you chose in the previous part?

With your Partner...

8. The table to the right represents the sample space for the sum of the values when two cubes are tossed. Complete the table.
9. Using the table to the right, shade the cells that represent Player A winning the game.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

10. What is the probability that Player A wins the game? That is, what is the probability of rolling a sum of 6 or a sum that is odd?

$$P(\text{Player A wins}) = P(S \text{ or } O) =$$

11. For each part, shade the corresponding event and use it to calculate the probability.
 a. Shade event “S”, and find $P(S)$.
 b. Shade event “O”, and find $P(O)$.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

c. Compare the diagrams for “S” and “O” in the previous question with the diagram for “S or O” in question 9. What, if anything do you notice?

d. How do you think $P(S)$, $P(O)$, and $P(S \text{ or } O)$ are related? Explain your answer.

e. Test your conjecture to part d. by computing probabilities. Were you correct or do you want to change your conjecture?

f. Why do you think the relationship you found in part e. is true? Think about the event and sample spaces and the Venn diagram.

g. Summarize

If two events, M and N , are _____,
 then $P(M \text{ or } N) = P(M \cup N) =$ _____.

If two events, M and N , are _____, then
 $P(M \text{ or } N) =$ _____



On Your Own...

12. Now suppose we re-label the circles in the Venn Diagrams above so that one circle in each diagram represents $S =$ “the sum is 6” and the other represents $E =$ “the sum is even”.

a. The Venn diagram before question 7 that represents Player B’s events is diagram _____. Explain why.

b. Redraw the Venn diagram from the previous part. Shade and describe in your own words the region where S and E overlap.



c. How is the probability of Player B winning related to the diagram you chose in the previous part?

With your Partner...

13. Using the table to the right, shade the cells that represent Player B winning the game.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

14. What is the probability that Player B wins? That is, calculate the probability of rolling a sum of 6 or a sum that is even.

$P(S \text{ or } E) =$

15. Create two sample spaces and represent event “ S ” and event “ E ”.

16. What are $P(S)$ and $P(E)$?

$P(S) =$

$P(E) =$

17. Compare the diagrams of your sample spaces for the events “S” and “E” with that of “S or E”.

a. What do you notice?

b. Explain your discovery using the Venn diagram from question 12.

c. How do you think $P(S)$, $P(E)$, and $P(S \text{ or } E)$ are related? Explain your answer.

d. How is your answer in part c related to your answer from question 11d?


e. What is the probability that Player B wins?

$$P(\text{Player B wins}) = P(S \text{ or } E) =$$

f. **Summarize**

If two events, H and I, are _____,
then $P(H \text{ or } I) = P(H \cup I) =$ _____.

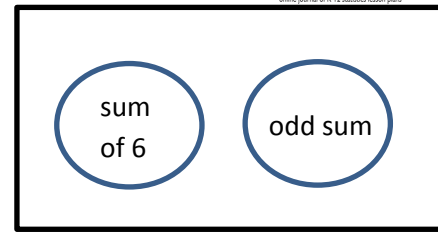
If two events, H and I, are _____
then $P(H \text{ or } I) =$ _____



18. We found the theoretical probability that Player A wins one point and the probability that Player B wins one point. Return to the original question: if you play a game with many trials, who is likely to win? Explain your answer.

On Your Own...

19. a. Shade the area of the Venn diagram to the right that represents the probability of not getting either a sum of 6 or an odd sum.



- b. Using the sample space table to the right, calculate the probability of getting a sum that is neither 6 nor odd using simple probability.

$$P([S \cup O]^c) =$$

Mutually Exclusive

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

With your Partner...

- c. Compare your sample space diagram for $S \cup O$ (question 9) with the sample space diagram for $[S \cup O]^c$ (question 19a). Compare also the Venn diagram for $S \cup O$ (question 7) with the Venn diagram for $[S \cup O]^c$ (question 19b). What do you notice?

- d. How do you think the probabilities $P(S \cup O)$ and $P([S \cup O]^c)$ are related? Explain your answer.

- f. Use $P(S \cup O)$ to compute $P([S \cup O]^c)$.

g. Summarize

Two events A and B are called *complementary* if A and B are mutually exclusive events whose union is the universal space. Symbolically, we write $A = B^c$ and $B = A^c$.

If S and T are complementary events, then $P(T) = P(S^c) =$ _____.

If Q and R are _____,
then $P(R) = P(Q^c) =$ _____



20. Are the events of Player A winning a point and Player B winning a point mutually exclusive? Complementary? Support your answers using theoretical probabilities computed in previous questions.

Close:

21. Now it's your turn. Design a game using two six-sided number cubes so that each of Player A and Player B can win in one of two ways, but the two players do not have the same probability of winning. Calculate the probability that each player wins.

Odd or Even? The Addition and Complement Principles of Probability TI-Nspire and TI-Nspire CAS Simulation Directions

Directions can also be found at

http://math.kendallhunt.com/documents/daa2/cntns/daa2cntns_014_10.pdf

To simulate tossing two standard numbered cubes, we need to assume that each number has an equally likely chance of occurring. In other words, we need a uniform distribution of the numbers 1 through 6 so that we can use the random number generator in the List & Spreadsheet application.

1. Open the List & Spreadsheet app.
2. We have two numbered cubes and want to find their sum, so label columns A, B, and C “cube1”, “cube2”, “sumcube”, respectively.
3. To simulate rolling a six-sided cube 1000 times, type “randint(1, 6, 1000)” into the formula cell of columns “cube1” and “cube2”.
4. To compute the sum of the two cubes, in the formula cell for “sumcube” type “cube1 + cube2”. Then, press “Enter” to populate the column.
5. To see the distribution of the sums, you can create a histogram: from the Home screen, go to **Data & Statistics**.
6. Add “sumcube” to the x-axis. Select “Menu” and “PlotType | Histogram”
7. From the Histogram you can tally the number of outcomes that make up Player A’s and B’s outcome.

Core Math Tools Simulation Instructions:

Another way of making a simulation if you didn’t have access to the TI-Nspire calculator is using the program Core Math Tools. You can download it from the website <http://www.nctm.org/coremathtools/> by clicking on “downloadable suite”.

Once Core Math Tools is downloaded, open it and start a Simulation under Statistics & Probability.

1. From the column on the left, click the six-sided dice twice so we have two dice to work with.
2. Next highlight both die (on a PC this is done holding the “Control” key, while on a Mac you hold the “Command” key.)
3. Then, from the Build tab, click “Add” to simulate rolling two six-sided dice and adding the two together to show the sum.
4. We then want to see the number of times we roll the sum of an even number or a sum of 6. To simulate this, highlight the simulation, and click “Count # of” from the “Build” tab. A box will pop up with numbers 2-12. To count the number of even sums, you need to click on all the even numbers. First double click on the 2. Then, if you have a Mac, use “Command” while choosing “4”. Continue doing that until you have highlighted all of the even numbers. On a PC, do the same thing but use “Control” instead of “Command”.

5. Then, to find the number of times we roll the sum of an odd number or a sum of 6, we repeat the process above (begin with step #1) selecting 3, 5, 6, 7, 9, 11.
6. Finally, conduct 1000 Runs via the menu at the top. The simulation will show a “1” if the sum was even or 6 (odd or 6) and a “0” if not.
7. To find the total count of sums for each simulation, select “Graph” and “Labels Bars” in the “View” drop-down menu.

