Which Hand Rules?

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Overview of Lesson
In this lesson students drop and catch a yardstick to determine if there is significant evidence (via a matched pairs randomization test) that people are faster with their dominant than their non-dominant hand. Students will engage in data collection and the randomization process of generating possible values from the sampling distribution of mean differences. Finally, they will interpret the estimated *p*-value in the context of the problem.

GAISE Components
This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report*. The four components are: formulate a question, design and implement a plan to collect data, analyze the data, and interpret results in the context of the original question.
This is a **GAISE Level C** activity.

Common Core State Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Learning Objectives Alignment with Common Core and NCTM PSSM

Learning Objectives	Common Core State Standards	NCTM Principles and Standards for School Mathematics
Students will systematically collect and pair data and interpret a dot plot of differences to informally reason whether people catch a yardstick faster with their dominant hand than with their non-dominant hand.	**S-ID.3.** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**Grades 9-12** Select and use appropriate statistical methods to analyze data: for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics.
Students will perform multiple tactile simulations, pool the simulated values of the statistic (mean of the differences of reaction	**S-IC.5.** Use data from a randomized experiment to compare two treatments; use	**Grades 9-12** Develop and evaluate inferences and predictions that are based on data: use simulations to

distances), and estimate the proportion of people with faster dominant hands if there was no difference in reaction times (p-value) from the pooled data.	simulations to decide if differences between parameters are significant.	explore the variability of sample statistics from a known population and to construct sampling distributions.

Prerequisites
Students will have been introduced to data collection, creating and interpreting dot plots, computing basic statistics, and constructing an argument based on data. They may also have exposure to significance tests and p-values (for differentiation).

Time Required
Two 50-minute class periods: one for Part I and one for Part II

Materials and Preparation Required

- pencil and activity sheets
- yardstick with centimeters for each pair of students
- calculator
- coin for each pair of students
- For extension, access to computer with internet to use the following applet (http://www.rossmanchance.com/applets/MatchedPairs/MatchedPairs.htm)

Which Hand Rules?
Teacher's Lesson Plan

This lesson has two parts. The first part focuses on class-level data; the second part takes the results from Part 1 and extends it to investigate whether this could have happened by chance using a matched pairs randomization test. The idea for this lesson plan came from a conversation with my colleague, Tom Linton.

Part I. A Class-Level Experiment

Describe the Context and Formulate a Question

Talk about the process of reacting to a stimulus such as an egg rolling toward the edge of the counter: the eye sees, the brain receives the visual message, the brain tells the hand to react, and the body physically reacts. Are there other examples where one's reaction time to a visual stimulus is important? Examples include reacting to a red light while driving, a goal keeper reacting to a soccer ball during a penalty kick, ice cream getting ready to slide off a cone, or a toddler getting too close to an open stairwell.

After discussing possible scenarios, lead the class into the experiment to be conducted by asking whether everyone's reaction time is the same, and lead into whether reaction times between our two feet or hands are the same. Ask the class if they think people are faster with their right or left hands. This will naturally lead into talking about dominant versus non-dominant reaction times. Which hand rules? Then explain that we are going to see if there is a difference between the reaction times of our two hands by dropping and catching a yardstick.

Collect Data

Before collecting data, lead a discussion about how to collect data to ensure that results can be generalized. The key components to elicit are

- have each student do the experiment with both hands; the dominant hand is the one with which you write on a regular basis
- randomly determine which hand people use first to minimize any learning that may occur during the experiment
- collect the data in the same way each time the experiment is done (everyone measures the distance the yardstick dropped the same way)
- have a partner drop the yardstick to ensure reaction to a stimulus is spontaneous
- we are measuring reaction to a stimulus using the distance the yardstick drops before it is caught; thus faster reaction times will correspond to shorter yardstick measurements which we will call "reaction distance"
- calculate the difference in reaction distances the same way; because it is more likely that people will have greater reaction distances with their non-dominant hand than with their dominant hand and it is more natural to work with positive changes, we will subtract the dominant reaction distance, **D**, from the non-dominant reaction distance, **ND**: **ND – D**

Before passing out the student handouts, yardsticks, and coins, verbally describe the process outlined in questions #1-4. Students will complete the activity in pairs. (Question numbers noted in this plan refer to the student handout.) Each student completes the table following question 4 with his/her individual data. Note that if a student misses grabbing the yardstick, they should repeat that trial. Once each pair has completed their tables, they should proceed to question 5. Question 5 sets up an important component of a matched pairs randomization: if there is no difference in reaction distances between dominant and non-dominant hands, then it shouldn't matter which hand we call "dominant". Hence it shouldn't matter in which order we subtract, and changing which hand we call dominant will change the sign of the difference in reaction distances.

Once students add their data to the class data (question 6), pause and discuss briefly why we are pooling our data: we want to be able to generalize our results to a population, so we need multiple data points.

Note: if you plan to extend this activity by using the applet found at http://www.rossmanchance.com/applets/MatchedPairs/MatchedPairs.htm you will need a modified data collection method where each student records the **ID** number, his/her non-dominant reaction distance, **ND**, his/her dominant reaction distance, **D**, in this order. An example of such a table is shown in the solutions.

Analyze Data

Once students have recorded their data in the class data table, they may proceed to analyze the data by answering questions 7a (create a dot plot) and 7b (calculate and locate the mean on the dot plot). Following #7b, pause and bring the class together and check that everyone is obtaining the same mean difference.

Interpret the Data

In #7c, students interpret the data. If the data is skewed right (left) with a positive (negative) mean, then it is reasonable, at this point, to conclude there is evidence that people are faster with their dominant (non-dominant) hand. If the data is unimodal with mean near zero, then one may reasonably conclude that there is no evidence there is a difference in reaction distances and hence it is plausible that people have similar reaction distances with both hands. It is unlikely that the data will be mostly negative as this would mean the non-dominant hand is faster for most people. At this point, begin to question whether the dot plot shows that the dominant hand (if the mean is positive) is really faster than the non-dominant hand or if these results are just due to chance alone and we'd get the same results no matter which hand we call dominant?

Part II: Could this have happened by Chance?

Formulating a New Question

Questions 8-9 are aimed at setting up the idea of a matched pairs randomization test. As noted in the student pages, #8-9 are the basis of determining whether the differences we obtained provide strong evidence that people are faster with their dominant hand. If there really is no difference between reaction distances (question #8), then it really does *not* matter which hand we call "dominant". In this case we should obtain differences that are close to zero.

For #9, if there really is a significant difference between reaction distances, then it *does* matter which hand we call "dominant", and we should obtain differences, **ND – D**, that are positive. In fact, the further the differences are from zero, the stronger the evidence that one's dominant hand is faster.

Following the Checkpoint after the hypotheses are defined, the key ideas behind a matched pair randomization test should be carefully discussed with students before proceeding to question #10. We are looking for evidence that tells us that the results we obtained in the experiment didn't happen by chance alone. That is, it matters which hand we call "dominant", and dominant reaction distances are less than non-dominant reaction distances.

The idea behind matched pairs randomization is assuming a positive difference is just as likely as a negative difference for each set of paired data points and asking how likely it is that we would obtain a mean difference like ours.

Collect a New Set of Data
To answer our new question, we are going to randomly assign which hand we call dominant by flipping a coin. This is equivalent to randomly assigning the difference in reaction distances as positive or negative by flipping a coin. Flip a coin, and illustrate the process of obtaining a tails (keeping the original assignment of dominance) is the same as keeping the original difference **ND – D**, and that obtaining a heads is the same as switching which hand we call dominant or switching the sign of the difference (switching the difference and using **D – ND = –** (**ND – D**)). We flip a coin, and find all possible rearrangements of which hand we call dominant *within each pair* of measurements. If there are 20 paired data points, we will have 2^{20}, or over one million, such rearrangements. We can't possibly find all such rearrangements within our paired data, so we will instead simulate several rearrangements, pool our data, and use it to estimate how likely it is that out of all those rearrangements, we would get one that would yield a mean difference as extreme (farther from zero) as what the class found via their data collection.

Following this discussion, students should proceed to answer questions 10- 14. For #12, students should have realized that no matter what their reaction distance was, when you write **D – ND**, you get the same magnitude answer, just opposite in sign. For #11, have students proceed to flip coins and record data. Then, have them compute their mean difference, $\bar{x}_{\text{rearrangement\_1}}$. Mean differences will vary from group to group since they are dependent on the data collected which will vary. Question 13 asks students to repeat this simulation 2 more times, calculating the mean of their differences for each set of data. Question 14 asks students to add their three mean differences to the board. The instructor should draw on the board a well-labeled large dot plot for the simulated mean differences with range from about -2 to 2. Students should copy the class dot plot to answer question #14.

Analyze New Data

Once everyone has added their data to the class data plot (#14) of simulated mean differences, students may continue by analyzing the dot plot (#15-16). Note that students are asked to estimate the mean and standard deviation of the dot plot, not necessarily calculate it, though the instructor may wish to have students record their differences in a table in addition to the dot plot so that students can numerically calculate the mean difference and standard deviation. Answers for these questions are dependent upon the collected class data, but the dot plot of mean differences should be roughly symmetric with mean near zero.

Interpret New Results

Students may be surprised to find that their value of p (also called the p-value) in #19 is larger than 0.05. Instructors may wish to pause the class after question 19 and discuss what the value of p represents in general and then more specifically for this experiment. **The p-value of the randomization test is the proportion of observations from the process that are at least as extreme (farther from zero) as our original mean difference.** Be prepared for students to misinterpret the p-value as the probability the dominant hand is faster than the non-dominant hand. Rather, emphasize the definition and the fact that it comes from answering the question of whether our results (or ones more extreme) could have happened by chance alone. Large p-values indicate that it is plausible we obtained results like we did just by chance and that there is no difference in reaction distances for our two hands. Small p-values indicate that we have strong evidence that our results were due to something other than chance: there is strong evidence that the dominant hand is faster than the non-dominant hand.

The instructor may choose to lead a class discussion that illuminates why our dominant hand is not faster. Several points that may be brought out include

- Often, the dominant hand is only slightly faster than the non-dominant hand, and in classroom (as opposed to laboratory) experiments, error is likely to occur and the difference will not be statistically significant.
- Does it matter if a person was male or female? Left- or right-hand dominant? Should we have taken these factors into account?
- Could the data collection process have been faulty? What could have been done to ensure more accurate results?
- Does it matter what time of day we conduct the experiment? Are people more alert at different times of the day?
- Would results change if we include an auditory stimulus, such as a buzzer or bell, along with the visual stimulus of dropping the yardstick?
- How would results change if we were to physically exercise before we do the test? (It is possible that exercise improves reaction times.)
- What might we expect if the data collection process was altered and everyone performed three drops with each hand and we used the middle value? How would that affect the distribution of values? (There will likely be less variability and fewer extreme measurements.)

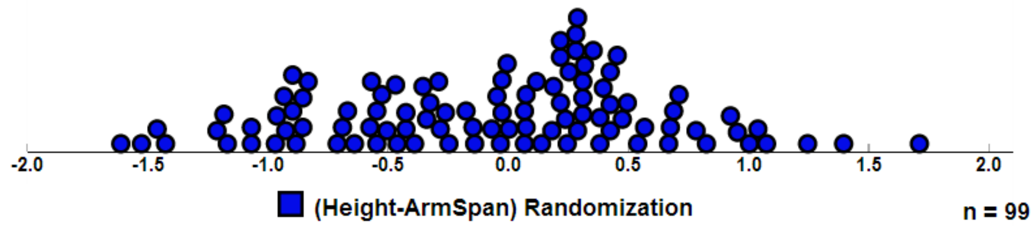
Suggested Assessment

Leonardo Da Vinci's Vitruvian Man showed several proportions found within the male human body. One such proportion stated that the length of the outspread arms (arm span) of a man is equal to the height of that man. You will investigate this claim by looking at a random sample of 28 *female* students and investigate whether there is evidence that height is actually greater than arm span.

The following data from 2014 was collected from a random sample of U.S. female students in grade 11 from self-reported data at Census at School (http://ww2.amstat.org/censusatschool/).

ID	Height (cm)	Arm Span (cm)	Height – Arm Span (cm)	ID	Height (cm)	Arm Span (cm)	Height – Arm Span (cm)
1	162	163	-1	15	167.6	168	-0.4
2	170	168	2	16	168	166	2
3	164	165	-1	17	154.9	148	6.9
4	152.4	153.4	-1	18	156	150	6
5	171	165	6	19	172	171	1
6	166	160	6	20	161	165	-4
7	170	170	0	21	173	168	5
8	166	164	2	22	160	157	3
9	170	165	5	23	161	165	-4
10	163	163	0	24	158.9	160.5	-1.6
11	165	169	-4	25	153	155	-2
12	163	159	4	26	169	174	-5
13	169	162	7	27	165	164	1
14	170	168	2	28	169	168	1

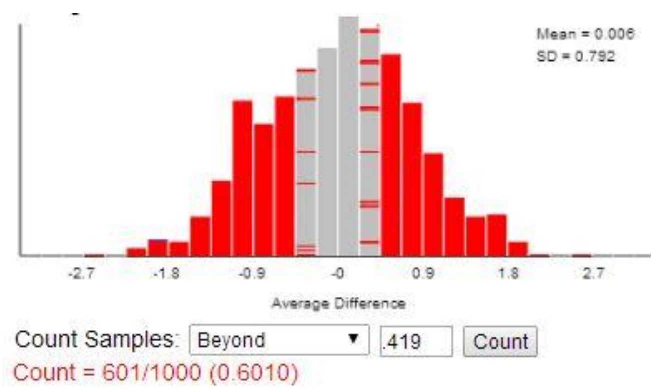
1. Create a well-labeled dot plot of the difference Height – Arm Span (cm).
2. Calculate the mean difference and add a vertical line representing $\bar{x}_{\text{difference}}$, to your dot plot.
3. Does the data suggest Da Vinci's claim for 11th grade females in the U.S. is correct? Explain your answer using statistical reasoning and your previous answers.
4. Do we have evidence that height is greater than arm span? Explain how you would perform a matched pairs randomization test to answer this question.
5. Carry out *one* matched pairs randomization. Record the results of your randomization, and calculate the value $\bar{x}_{\text{rearrangement}}$.
6. Add your mean to the dot plot below (which contains 99 data points).



7. What proportion of values from the simulation (include your value so that you have 100 data points) is at least as extreme as the original mean?
8. What do you conclude regarding height and arm spam of the 11th grade women in the U.S.? Explain!
9. To what population can we generalize our results? Explain your answer.

Possible Differentiation

This lesson can be extended in several ways. One extension is to formally introduce the notation and meaning of a matched pairs hypothesis test. Another is to consider the two-tailed version of the problem statement: Is there a difference in reaction distance between one's dominant and non-dominant hand? In the two-tailed version, we count the number of simulated mean differences that are farther from zero in *both* directions. For our example, we estimate $p \approx 0.6010$. Thus, we do not have evidence that reaction distance between dominant and non-dominant hands is different.



Another extension is to ask students to use technology, such as the TI graphing calculators, to perform a test of significance by first verbally defining the population parameter, μ, and asking students to write the hypotheses H_0 and H_a verbally and symbolically (instead of giving students that information).

A fourth extension is to use the matched pairs randomization applet found at http://www.rossmanchance.com/applets/MatchedPairs/MatchedPairs.htm. To use this applet, one must have organized the data similar to that found in the possible solutions in #6 with three columns having the headings **ID**, **ND**, **D**. The app computes the differences. Once the paired data is entered into the dialog box, you can simulate flipping coins for each pair of data and illustrate the p-value numerically and graphically for a large number of randomizations.

A fifth extension would be to collect data and calculate the time (seconds) that corresponds to the reaction distance. Advanced students could derive the conversion formula. For example, we could convert the distance the yardstick dropped (in centimeters) into time (seconds). The general formula relating distance and time is given by $d(t) = ½\, gt^2 + v_0 t + s_0$, where g is the gravitational constant $-9.81\, {}^{m}/_{s^2} = -981\ {}^{cm}/_{s^2}$, v_0 is the initial velocity of the yardstick (hence, $v_0 = 0$), and s_0 is the initial distance between your index finger 0 cm on the yardstick (so, we also have $s_0 = 0$). Thus, the distance traveled by the yardstick is $d(t) = ½\,(-981)t^2 + 0t + 0$ which simplifies to $d(t) = -490.5t^2$. Recall that gravitation acts downward, so the distance having a negative value means the yardstick fell. Actually, all the yardsticks fell, so we know that distance will always be negative; however, what we really care about is the net distance, so we will use $d(t) = 490.5t^2$ with the understanding that distance was measured in a downward direction.

If we know the distance, we can solve for time, *t*:

$$d = 490.5t^2$$
$$\frac{d}{490.5} = t^2$$
$$t = \pm\sqrt{\frac{d}{490.5}}$$

Since time is measure after the yardstick is dropped, we only consider the positive root: $t = \sqrt{\frac{d}{490.5}}$.

References

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Further Reading About the Topic

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Acknowledgements

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Which Hand Rules?
Student Handouts

<u>Part I: Analyzing our Class Data</u>

The time it takes for the eye to see, the brain to process what the eye sees, and the body to physically react, varies from person to person. It can also vary depending on whether you are using your right or your left hand. If you are right-hand dominant, you might suspect that you have quicker reaction times with your right hand than with your left hand. But do you really?

We are going to investigate the speed with which you can catch a falling yardstick by measuring the distance it drops before you catch it. The faster you are, the closer to the 0 centimeter mark you will grab the yardstick when it falls. We will refer to where you grabbed the yardstick as your "reaction distance".

Since averages are less variability than individual observations (we don't want any really extreme values to throw off our measurements), we are going to perform the experiment three times and average the distances the yardstick drops before you catch it. We are also going to randomly determine which hand you use first by flipping a coin.

For this activity, your dominant hand is the one with which you write.

Read through steps 1-4 before you conduct the experiment for the first time.

1. Flip a coin to determine if you will begin with your dominant (Heads) or non-dominant (Tails) hand. <u>Record your result here</u>:
2. Data Collection:
    a. You will rest your arm on a table with your wrist over the edge. Your partner will hold the yardstick between your index finger and thumb of the hand determined by the coin flip; the top of your finger should be at the 0 cm mark on the yardstick.
    b. You will let your partner know when you are ready. Then, within 5 seconds, your partner will release the yardstick and you will catch the yardstick as quickly as possible.
    c. Record the distance (to the nearest tenth of a centimeter) between the bottom of the yardstick and the <u>top</u> of your index finger in the chart below.
3. Switch hands and repeat the experiment. Record all distances.
4. Now, it's your partner's turn! Switch roles & conduct the experiment for each hand.

Which Hand? Dominant or Non-dominant?	Reaction Distance (centimeters)	Which Hand? Dominant or Non-dominant?	Reaction Distance (centimeters)

5. Next, calculate the following differences in your reaction distances:
    a. (**Non-Dominant, ND,** Reaction Dist.) – (**Dominant, D,** Reaction Dist.) = _________ cm
    b. (**Dominant, D,** Reaction Dist.) – (**Non-Dominant, ND,** Reaction Dist.) = _________ cm
    c. Compare your results in parts a. and b. with several classmates. What do you notice?

    d. If we switch the order in which we subtract our reaction distances, from (**ND – D**) to (**D – ND**), how will the two differences *always* be related?

6. Add your data to the class data sheet and copy it below.

ID	**Reaction Difference, ND – D (cm)**	**ID**	**Reaction Difference, ND – D (cm)**	**ID**	**Reaction Difference, ND – D (cm)**	**ID**	**Reaction Difference, ND – D (cm)**
1		9		17		25	
2		10		18		26	
3		11		19		27	
4		12		20		28	
5		13		21		29	
6		14		22		30	
7		15		23		31	
8		16		24		32	

7. Use the differences in reaction distances in the table above to answer the following questions.
    a. Create a well-labeled dot plot of the data.

b. Calculate the mean of the data. Illustrate the mean with a vertical line on the dot plot above.

$$\bar{x}_{\text{difference}} =$$

c. Interpret the information from the previous parts in the context of this problem. Does the data we collected suggest that people have a shorter reaction distance when they use their dominant hand? Explain your answer.

~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ **CHECKPOINT** ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~

Part II: Could this have happened by Chance?

Is the difference *suggested* by the calculations and dot plot above enough to conclude that people really are faster with their dominant hand than with their non-dominant hand? Could the differences between reaction distances have been obtained regardless of which hand we called "dominant"?

In order to investigate the claim that people are faster with their dominant hand, we will use a *matched pairs randomization test*.

8. If there really is no difference in the reaction distance for the two hands, then should we expect the differences in reaction distances to be close to zero, positive or negative? Explain your answer.

9. If it is true that people are faster with their dominant hand than with their non-dominant hand, then should we expect the differences in reaction distances to be close to zero, positive or negative? Explain your answer.

The above two questions are the basis of determining whether the differences we obtained provide strong evidence that people are faster with their dominant hand. If there really is no difference between reaction distances, then it really does *not* matter which hand we call "dominant". In this case we should obtain differences that are close to zero. (Check: how did you answer #8?)

If there really is a significant difference between reaction distances, then it *does* matter which hand we call "dominant", and we should obtain differences, **ND – D**, that are positive. (Check: how did you answer #9?) In fact, the further the differences are from zero, the stronger the evidence that one's dominant hand is faster.

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~ **CHECKPOINT** ~

If the hand used has no effect on reaction distance, then the difference "**ND – D**" is just as likely to be larger than zero as it is to be smaller than zero. If there is no difference in which hand we use, calling one hand "dominant" within any pair of reaction distances is just as likely as calling the other hand "dominant". This idea is the heart of a matched pairs randomization test.

To run a matched pairs randomization test, first imagine all possible rearrangements of which hand we call dominant in the class data set, keeping pairs of reaction distances for each person together. We could create all possible rearrangements of the class data, calculate the mean difference for each rearrangement, create a dot plot of all such means, and then compare our original mean difference to the distribution of mean values from the rearrangements. For a class data set of 20 reaction distances, we would be making 2^{20}, or over 1 million, such arrangements!

In the next part of the activity, each pair of students is going to find several rearrangements as described above and calculate the mean of each one.

To find the first randomization, we will flip a coin to determine which hand we call "dominant". If we obtain a tail, we will keep the original assignment of "**D**" and "**ND**" from our collection of data. If we obtain a head, we will switch the assignment of "**D**" and "**ND**".

10. If we switch the assignment of "**D**" and "**ND**", what happens to the sign of our original difference **ND – D**? Hint: consider your answers to #5.

11. Perform the randomization described above: For each ID, flip a coin, and record the new difference in reaction distances in the table below. Remember, keep the original assignment of hands if you flip a "tail" and change the assignment if you flip a "head".

ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)
1			9			17			25		
2			10			18			26		
3			11			19			27		
4			12			20			28		
5			13			21			29		
6			14			22			30		
7			15			23			31		
8			16			24			32		

12. Use the randomized differences in reaction distances in the table above to calculate the mean of the data.

$$\bar{x}_{\text{rearrangement\_1}} =$$

Recall that we said we needed to find <u>all</u> such rearrangements. We are going to find only three per group. Then we will pool all mean differences and count how many times we obtained means as extreme (farther from zero) as that of our original mean reaction distance.

13. Run a re-randomization on your original differences two more times. Calculate and record the mean difference for each re-randomization.

ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)
1			9			17			25		
2			10			18			26		
3			11			19			27		
4			12			20			28		
5			13			21			29		
6			14			22			30		
7			15			23			31		
8			16			24			32		

$$\bar{x}_{\text{rearrangement\_2}} =$$

ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)
1			9			17			25		
2			10			18			26		
3			11			19			27		
4			12			20			28		
5			13			21			29		
6			14			22			30		
7			15			23			31		
8			16			24			32		

$\bar{x}_{\text{rearrangement\_3}} =$

14. Add your three mean differences ($\bar{x}_{\text{rearrangement\_1}}$, $\bar{x}_{\text{rearrangement\_2}}$, $\bar{x}_{\text{rearrangement\_3}}$) to the class dot plot. Sketch a well-labeled dot plot below.

~ **CHECKPOINT** ~

15. What does each dot on the dot plot represent?

16. Describe the shape, mean, and standard deviation of the resulting dot plot. Use correct statistical language!

We plotted all of our randomized mean differences on the dot plot above. (Check: how did you answer #15?). Why did we do this? We are trying to see whether the results from the data collected by our class was due to chance alone or whether there is evidence there actually was a difference in reaction times. If a dominant hand is faster than a non-dominant hand, then we should expect to see few randomized differences like the ones in our experiment. If our results are not that unusual (and the dominant hand is not faster than the dominant hand), then we should expect to see many randomized differences like the ones in our experiment.

17. Recall that we calculated the mean of the original data in #7b, $\bar{x}_{\text{difference}}$. Draw and label a vertical line on the dot plot in #14 representing $\bar{x}_{\text{difference}}$.

18. How many $\bar{x}_{\text{rearrangement}}$'s from the randomization process are at least as extreme (farther from zero) as the vertical line representing the original mean difference? These dots represent mean differences where the dominant hand was faster than the non-dominant hand by more than our original mean difference.

The value p, the p-value, of a randomization test is **the proportion of observations from the process that are at least as extreme (farther from zero) as our original mean difference**. In general, if this proportion is less than 5%, the test provides strong evidence that reaction distances for dominant hands are less than those of non-dominant hands.

19. Estimate p using the pooled data from the class dot plot.

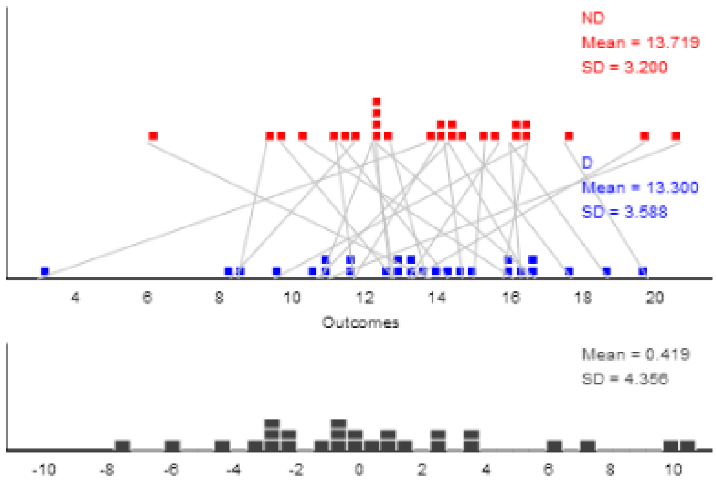
20. Based on the estimated p, what can you conclude about reaction distances between dominant and non-dominant hands?

Sample Solutions for Student Handouts and Assessments

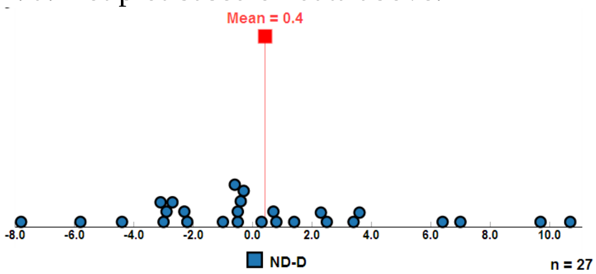
4. Answers will vary.
5. a. & b. Answers will vary.
5. c. Students should begin to see that the answers to a. and b. are opposite in sign but have the same magnitude.
5. d. They will always be opposite in sign; for example, if **ND – D** > 0, then **D – ND** < 0. Students may need to be reminded that if **ND – D** = 0, then **D – ND** = 0.
6. Answers will vary. Sample data is included below.

ID	Which Hand First?	Reaction Distance (cm)	Reaction Distance (cm)	ND-D	ID	Which Hand First?	Reaction Distance (cm)	Reaction Distance (cm)	ND-D
1	D	3.0	13.7	10.7	15	ND	12.2	12.8	-0.6
2	D	13.3	19.7	6.4	16	ND	20.7	11.0	9.7
3	D	16.3	16.0	-0.3	17	ND	12.3	12.7	-0.4
4	ND	11.2	11.7	-0.5	18	ND	16.0	18.7	-2.7
5	ND	14.8	17.7	-2.9	19	ND	9.7	12.8	-3.1
6	D	13.7	12.7	-1.0	20	D	9.5	16.5	7.0
7	ND	14.3	10.7	3.6	21	ND	11.3	14.3	-3.0
8	D	11.7	14.0	2.3	22	ND	17.5	19.8	-2.3
9	D	8.3	11.7	3.4	23	D	8.5	9.3	0.8
10	ND	14.3	16.5	-2.2	24	D	16.1	10.3	-5.8
11	ND	6.0	13.8	-7.8	25	D	10.8	12.2	1.4
12	D	16.7	12.3	-4.4	26	D	13.2	15.7	2.5
13	ND	14.2	14.7	-0.5	27	D	15.8	16.5	0.7
14	ND	15.3	15.0	0.3					

ID	ND	D	ND - D	ID	ND	D	ND - D	ID	ND	D	ND - D
1	13.7	3.0	10.7	10	14.3	16.5	-2.2	19	9.7	12.8	-3.1
2	19.7	13.3	6.4	11	6.0	13.8	-7.8	20	16.5	9.5	7.0
3	16.0	16.3	-0.3	12	12.3	16.7	-4.4	21	11.3	14.3	-3.0
4	11.2	11.7	-0.5	13	14.2	14.7	-0.5	22	17.5	19.8	-2.3
5	14.8	17.7	-2.9	14	15.3	15.0	0.3	23	9.3	8.5	0.8
6	12.7	13.7	-1.0	15	12.2	12.8	-0.6	24	10.3	16.1	-5.8
7	14.3	10.7	3.6	16	20.7	11.0	9.7	25	12.2	10.8	1.4
8	14.0	11.7	2.3	17	12.3	12.7	-0.4	26	15.7	13.2	2.5
9	11.7	8.3	3.4	18	16.0	18.7	-2.7	27	16.5	15.8	0.7



7. a. Dot plot based on data above:



7. b. $\bar{x}_{\text{difference}} \approx .419$

c. The mean reaction distance is greater than zero, so it seems likely that people are slightly faster with their dominant hand.

8. They should be close to zero because the reaction distances should be about the same.

9. If the dominant hand is faster, then the reaction distance should be less for the dominant hand than for the non-dominant hand. Thus, the difference **ND – D** should be positive.

10. The sign will change from positive to negative or from negative to positive. If the reaction distance difference was zero, it will remain zero.

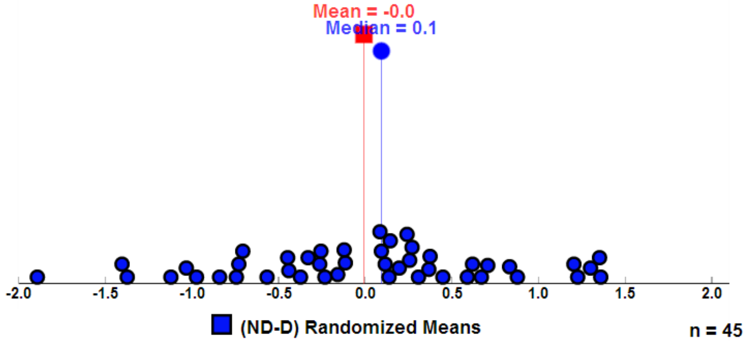
11. Possible answer:

ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)	ID	Heads or Tails?	ND – D (cm)
1	T	10.7	9	H	-3.4	17	T	-0.4	25	T	1.4
2	H	-6.4	10	H	2.2	18	H	2.7	26	T	2.5
3	T	-0.3	11	H	7.8	19	H	3.1	27	T	0.7
4	T	-0.5	12	T	-4.4	20	H	-7			
5	T	-2.9	13	T	-0.5	21	T	-3			
6	H	1	14	H	-0.3	22	H	2.3			
7	H	-3.6	15	H	0.6	23	T	0.8			
8	H	-2.3	16	H	-9.7	24	H	5.8			

12. $\bar{x}_{\text{rearrangement\_1}} = -0.115$ for the table above

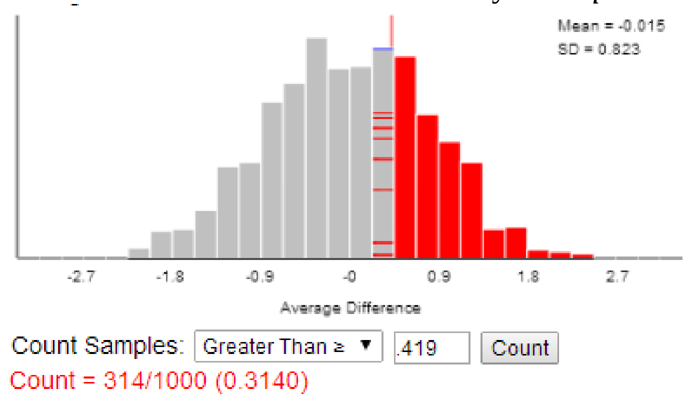
13. $\bar{x}_{\text{rearrangement\_2}} =$ answers will vary $\bar{x}_{\text{rearrangement\_3}} =$ answers will vary

14. Possible dot plot from re-randomization process:



Randomized Mean Differences (ND-D)				
-1.374	0.27	1.3	0.115	0.833
-0.448	-1.122	0.144	0.448	-0.233
0.307	0.196	1.226	-0.707	0.619
-0.159	0.589	1.204	-0.974	0.67
-0.374	-0.256	1.352	0.374	0.085
-0.567	0.093	-0.33	-0.841	-1.404
0.241	0.256	0.137	-0.744	-0.122
-0.73	-1.893	-0.441	1.359	0.878
-0.115	0.707	-1.033	-0.263	0.367

Randomization of the data 1000 times yielded $p \approx .3140$:



15. randomized mean differences

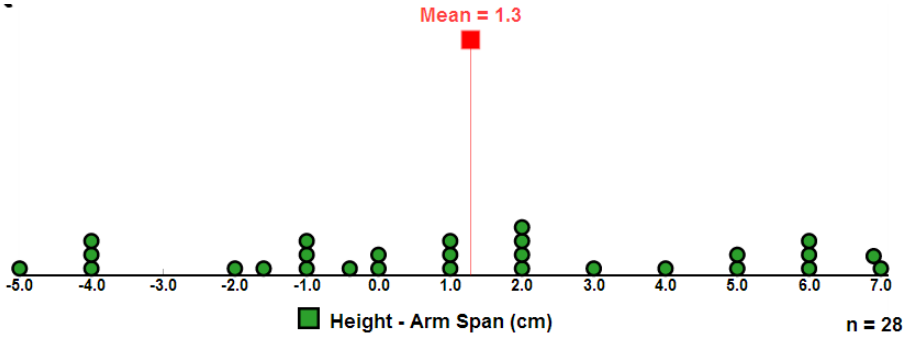
16. For the data in the dot plot, the mean is ≈-.008. Means are between roughly -1.9 and 1.4. The standard deviation is roughly 0.8. The plot is roughly symmetric and unimodal with median approximately the same as the mean.

17. Students should have a vertical line on the dot plot near $\bar{x}_{\text{difference}} \approx 0.419$ (the answer from #7b).

18. For the given dot plot, there are 11 values that are greater than $\bar{x}_{\text{difference}} \approx 0.419$.

19. The estimated p-value for the pooled data is $p \approx 11/45 \approx 0.24$.
20. For $p \approx 0.24$, this means that about 24% of the means we obtained are as extreme (farther from zero) as our original mean reaction distance. If there is no association between hand dominance and reaction distance, we would expect results like ours (an observed mean $\bar{x}_{\text{difference}} \approx 0.419$ or larger) to occur roughly 24% of the time (or about 1 out of 4 times). It is plausible that there is no difference in the reaction distances between our two hands.

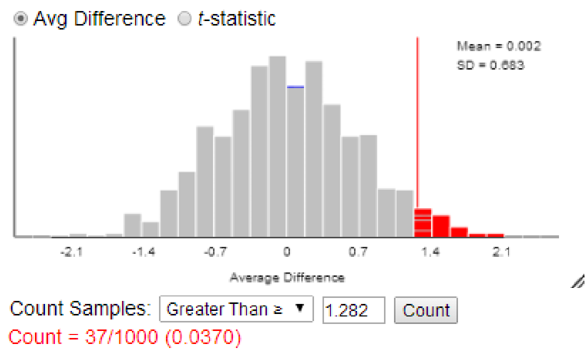
Assessment Answers

1. 

2. $\bar{x}_{\text{difference}} = 1.282 \approx 1.3$
3. No. It appears that with a mean of about 1.3, Da Vinci's claim doesn't hold for our population. We have $\frac{17}{28} \approx 60.7\%$ of our data points above the value of 0 which would be the difference if Height and Arm Span were the same.
4. In a matched pairs randomization we look for evidence that Height is larger than Arm Span. For each pair of data, we will flip a coin. If tails, then we will leave the paired data as is and use the difference in the table. If heads, we will switch which measurement we call Height; we will use the opposite/negative of the value in the table. Once we do this for each paired set of data points, we will find the mean of the differences. If we do this many times, we can count the number of our simulated mean differences that are as extreme (farther from zero) as the mean from our collection of paired data points. The proportion of values that are in the tail above our observed mean difference is the p-value. If the p-value is less than 0.05, then we have significant evidence that allows us to conclude that height is greater than arm span for 11th grade females in the U.S.
5. Answers will vary. Examples of possible mean differences can be found in the table below.
6. The plot was created with the following 99 simulated randomized means:

-0.689	-0.525	0.239	0.689	-0.546	1.004	-0.896	-0.468	0.139	0.218
-0.504	-1.068	-0.354	-0.711	-1.525	-0.425	1.711	0.311	0.396	-0.339
-0.004	-0.389	-0.139	-0.175	-1.425	-0.896	-0.025	-1.182	-1.211	0.454
-1.46	0.382	0.354	0.282	-0.425	-1.168	0.189	0.246	0.382	0.068
-0.832	0.075	0.475	0.218	0.075	-0.146	-0.854	0.318	-0.568	-0.962
-1.068	0.425	0.182	-0.546	1.075	-0.882	0.668	-0.282	0.425	-0.639
0.004	0.425	0.496	-0.968	-0.046	-0.025	-0.246	-0.461	1.396	0.675
0.311	-0.932	0.289	-0.032	-0.068	0.289	0.254	0.711	1.039	-0.668
0.825	0.568	0.539	0.068	0.2821	0.782	0.218	-1.611	0.311	0.954
-0.925	-0.289	-0.325	-0.575	0.118	-0.261	0.925	-0.854	1.246	

7. For the data given, using a rounded mean of $\bar{x}_{\text{difference}} \approx 1.3$, the dot plot given has 2 means above 1.3; thus the percentage that students should obtain is $p \approx \frac{2}{99} \approx 0.02$. Running 1000 simulations returned a simulated p-value of 0.037. (http://www.rossmanchance.com/applets/MatchedPairs/MatchedPairs.htm)



8. If we use a cut-off of $p = 0.05$, then we have strong evidence that the true mean difference is greater than 0. Thus, we can say that we have strong evidence that height is greater than arm span for females in our population.
9. Since the data was self-reported in a class census, we need to be careful not to generalize these results beyond what was collected. Therefore, our population is U.S. females from grade 11 that self-reported their arm span and height.