You Will Soon Analyze Categorical Data (Classifying Fortune Cookie Fortunes)

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Overview of Lesson Plan
In this activity students will have the opportunity to collect and explore real data using two different brands of fortune cookies. Students will open each brand of fortune cookie and classify their fortunes into one of four categories. Students will then construct a two-way frequency table to display their data and they will investigate their results using joint relative frequencies and marginal and conditional distributions. In an extension students will use a chi-square test of homogeneity to determine if the proportions of fortunes within the categories differ for the two brands.

GAISE Components
This activity follows all four components of statistical problem solving put forth in the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report. The four components are: formulate a question, design and implement a plan to collect data, analyze the data by measures and graphs, and interpret the results in the context of the original question. The main activity is a GAISE Level B Activity. The extension of the activity is a GAISE Level C Activity.

Common Core State Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them.  
2. Reason abstractly and quantitatively.  
4. Model with mathematics.  
5. Use appropriate tools strategically.  
6. Attend to precision.

Common Core State Standard Grade Level Content (High School)
S-ID. 5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.  
S-IC. 1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
NCTM Principles and Standards for School Mathematics

Data Analysis and Probability Standards for Grades 9-12

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them:
- understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable.

Select and use appropriate statistical methods to analyze data:
- display and discuss bivariate data where at least one variable is categorical.

Prerequisites
For the activity students must know how to calculate relative frequencies. For the extension, some exposure to hypothesis testing would be helpful.

Learning Targets
After completing the activity, students will be able to create a two-way frequency table from raw data and proceed to examine marginal and conditional distributions in order to help answer a question of interest.

If the extension is completed students will learn how to perform the chi-square test of homogeneity and will be able to distinguish between the chi-square test of homogeneity and the chi-square test of independence.

Time Required
The time required for the activity is roughly 1 class period.

Materials Required
Students will need a copy of the Activity Sheet (see the end of the lesson); to complete the lesson interactively, each student will need two or three of each of two brands of fortune cookies.

Note:
(1) A case of fortune cookies, containing 100 cookies, can be purchased for roughly $15.
(2) With monetary constraints in mind, a collection of fortune cookie sayings for two different brands of fortune cookies appears at the end of this lesson. The teacher could potentially provide each student with a single fortune cookie and use the sayings that are included with this lesson as part of the data collection process.
(3) Some top selling fortune cookie brands are: Golden Bowl (made by Wonton Foods, Inc.), Shang Pin, and Peking Noodle.
Instructional Lesson Plan

The GAISE Statistical Problem-Solving Procedure

I. Formulate Question(s)
Begin the activity by discussing some history on fortune cookies. Some historical background is provided on the activity worksheet. The worksheet also provides an introduction of and definitions and examples of four categories of fortunes that will be used in the activity: Prophecy, Compliment, Advice, and Wisdom.

Explain to students that there are two brands of fortune cookies available and that we would like to determine if the percentage of fortunes falling into the four categories differs for the two brands.

II. Design and Implement a Plan to Collect the Data
Have students open their fortune cookies, read the fortunes, and tally them into the categories: Prophecy, Advice, Wisdom, and Misc. Note that the Misc. category was created to incorporate Compliments and ‘Other’ types of fortunes. Create regions on the white board where the students can put their tallies.

The following table contains example data that might be collected when completing this activity. To replicate this data, each student will need to be given 3 or 4 of each brand of fortune cookie. Text of the individual fortunes extracted from these cookies is provided at the end of the activity worksheet.

<table>
<thead>
<tr>
<th>Type of Fortune</th>
<th>Shang Pin</th>
<th>Golden Bowl</th>
<th>Column Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand of Cookie</td>
<td>Prophecy</td>
<td>Advice</td>
<td>Wisdom</td>
</tr>
<tr>
<td>Shang Pin</td>
<td>16</td>
<td>34</td>
<td>49</td>
</tr>
<tr>
<td>Golden Bowl</td>
<td>15</td>
<td>21</td>
<td>52</td>
</tr>
<tr>
<td>Column Totals</td>
<td>31</td>
<td>55</td>
<td>101</td>
</tr>
</tbody>
</table>

*The Misc. category includes Compliments and Other (such as this fortune from a Golden Bowl cookie: “Great! You’re ready for a party.”).

III./IV. Analyze the Data/Interpret the Results
In order to help determine if the two brands of fortune cookies have similar fortunes students are lead through a series of questions.

Students begin by calculating the marginal distribution of the Type of Fortune. Students determine that the percentage of all of the fortune cookie sayings that are Prophecy is 16%. The corresponding percentages for Advice, Wisdom, and Misc. are: 28%, 52%, and 4%.
Discuss with students that these percentages collectively make up what is called the marginal distribution of the Type of Fortune and ask students to explain why it makes sense to call these percentages a marginal distribution. The term marginal seems appropriate since the percentages were calculated using the table column totals divided by the overall total number of fortunes. The column totals appear in the margin of the table.

Next, students are asked to calculate selected joint percentages. For example, the percentage of all of the fortunes that came from a Golden Bowl cookie and contained a Prophecy is 8%. The percentage of all of the fortunes that came from a Shang Pin cookie and contained Wisdom is 25%.

Discuss with students that percentages such as these are referred to as joint percentages (relative frequencies) and ask them to explain why it makes sense to call these percentages joint. The percentages describe two characteristics: Brand of Cookie and Type of Fortune, so it seems reasonable to refer to them as joint.

Next, students will calculate the conditional distribution of the Type of Fortune given the Brand of fortune cookie. That is, for each brand, the percentages of the Types of Fortunes will be calculated. Note that when the conditional distribution is calculated the Row Totals should be approximately 100%. Table 2 contains the conditional distribution for the data appearing in Table 1.

Table 2. Conditional distribution of Type of Fortune given Brand of fortune cookie.

<table>
<thead>
<tr>
<th>Brand of Cookie</th>
<th>Type of Fortune</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shang Pin</td>
<td>Prophecy 15%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>Advice 33%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wisdom 48%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Misc 4%</td>
<td></td>
</tr>
<tr>
<td>Golden Bowl</td>
<td>Prophecy 16%</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>Advice 23%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wisdom 57%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Misc 4%</td>
<td></td>
</tr>
</tbody>
</table>

Based upon the conditional distribution ask students if they think that the two brands Shang Pin and Golden Bowl have the same Type of Fortunes. Of course, if the fortunes for Shang Pin and Golden Bowl were exactly the same, then all of the conditional percentages shown in the table above would be equal. In this case, we can see that Shang Pin and Golden Bowl tend to have the same percentage of fortunes that are Prophetic and that fall into the Misc. category. However, the Shang Pin cookie fortunes have a higher percentage of Advice, by 10% and a lower percentage of Wisdom, by 9%. So, the two brands may not have the same types of fortunes.

Finally, students are referred to the results obtained by Yin and Miike when they analyzed the text of fortune cookie sayings in the article A Textual Analysis of Fortune Cookie Sayings: How Chinese Are They? For their data collection, Yin and Miike categorized 595 fortune cookies from a variety of Chinese restaurants. The results of their analysis appear in the table below:
Table 3. The results obtained by Yin and Miike.

Categories and Themes of Fortune Cookie Sayings (p. 22)

<table>
<thead>
<tr>
<th>Categories</th>
<th>Numbers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prophecy</td>
<td>367 (61.7)</td>
</tr>
<tr>
<td>Compliments</td>
<td>66 (11.1)</td>
</tr>
<tr>
<td>Advice</td>
<td>72 (12.1)</td>
</tr>
<tr>
<td>Wisdom</td>
<td>90 (15.1)</td>
</tr>
<tr>
<td>Total</td>
<td>595 (100)</td>
</tr>
</tbody>
</table>

Tell students that we want to see if our data collection produced results comparable to Yin and Miike.

In order to make this comparison first have students combine their results for the Shang Pin and Golden Bowl fortune cookies. Have them fill in the 15 cells in the following table.

Table 4. Two way frequency table of class results and Yin and Miike’s results.

<table>
<thead>
<tr>
<th>Type of Fortune</th>
<th>Shang Pin/Golden Bowl</th>
<th>Yin and Miike’s Brands</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prophecy</td>
<td>31</td>
<td>367</td>
<td>195</td>
</tr>
<tr>
<td>Advice</td>
<td>55</td>
<td>72</td>
<td>127</td>
</tr>
<tr>
<td>Wisdom</td>
<td>101</td>
<td>90</td>
<td>191</td>
</tr>
<tr>
<td>Misc.</td>
<td>8</td>
<td>66</td>
<td>74</td>
</tr>
<tr>
<td>Column Totals</td>
<td>398</td>
<td>595</td>
<td>790</td>
</tr>
</tbody>
</table>

Ask students to explain what types of percentages should be used to compare the class results for Shang Pin and Golden Bowl cookies to the results of Yin and Miike: marginal, joint, or conditional.

They should respond that the appropriate percentages to use to make this comparison are conditional percentages. After a brief discussion, have them calculate the conditional distribution of Type of Fortune given Brand of cookie. The conditional distribution is shown in Table 5.
Table 5. Conditional distribution of Type of Fortune given Brand of cookie.

<table>
<thead>
<tr>
<th>Brand of Cookie</th>
<th>Type of Fortune</th>
<th></th>
<th></th>
<th></th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prophecy</td>
<td>Advice</td>
<td>Wisdom</td>
<td>Misc.</td>
<td></td>
</tr>
<tr>
<td>Shang Pin/Golden Bowl</td>
<td>16%</td>
<td>28%</td>
<td>52%</td>
<td>4%</td>
<td>100%</td>
</tr>
<tr>
<td>Yin and Miike’s Brands</td>
<td>62%</td>
<td>12%</td>
<td>15%</td>
<td>11%</td>
<td>100%</td>
</tr>
</tbody>
</table>

After they calculate the conditional distribution students should discuss if they think that the class data collection produced results that are comparable to the results of Yin and Miike.

Obviously, the class results are not comparable. Yin and Miike’s cookies overwhelming produced Prophetic fortunes whereas the Shang Pin/Golden Bowl cookies’ fortunes were predominantly fortunes that contained Wisdom.

Ask students to provide a possible explanation for the discrepancies in the Types of Fortunes. One thing that comes to mind is that we are not certain of the brands of cookies that Yin and Miike extracted fortunes from. It does not seem as though they were Shang Pin or Golden Bowl cookies.
Assessment
In the General Social Survey, respondents were asked, “Do you agree with the following statement? “In spite of what some people say, the lot (situation/condition) of the average man is getting worse, not better.” The results, for 990 respondents by gender, are shown below.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Agree</th>
<th>Disagree</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>357</td>
<td>200</td>
<td>557</td>
</tr>
<tr>
<td>Male</td>
<td>234</td>
<td>199</td>
<td>433</td>
</tr>
<tr>
<td>Total</td>
<td>591</td>
<td>399</td>
<td>990</td>
</tr>
</tbody>
</table>

1. What percentage of the respondents were female and believed that the lot of the average man is getting worse, not better?

2. Calculate the marginal distribution of gender.

3. Calculate the conditional distribution of opinion of the lot of the average man, given gender.
Answers
1. $357/990 = .3606$ so 36.06%
2. Female: $557/990 = .5626$ or 56.26% and Male: $433/990 = .4374$ or 43.74%
3. 

<table>
<thead>
<tr>
<th>Gender</th>
<th>Agree</th>
<th>Disagree</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>357/557 = .6409 or 64%</td>
<td>200/557 = .3591 or 36%</td>
<td>100%</td>
</tr>
<tr>
<td>Male</td>
<td>234/433 = .5404 or 54%</td>
<td>199/433 = .4596 or 46%</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Extension of Introductory Activity**

Typically a two-way frequency table analysis will be extended to a chi-square hypothesis test. When analyzing data from a frequency table, there are two types of chi-square tests that might be utilized.

A test of independence answers the question, “Are the two categorical variables independent for a population under study?” It assesses whether there is a relationship between two variables for a single population. The null hypothesis for the test of independence is that the two categorical variables are not related (independent) for the population of interest.

A test of homogeneity answers the question, “Do two or more populations have the same distribution for one categorical variable?” It assesses whether a single categorical variable is distributed the same in two (or more) different populations. The null hypothesis for the test of homogeneity is that the distribution of the categorical variable is the same for the two (or more) populations.

The mechanics of tests of independence and tests of homogeneity are the same. The distinction is the way in which the data was collected. If two categorical variables are collected for each subject, then a test of independence should be performed. If a single categorical variable is collected for each of two (or more) groups, then a test of homogeneity should be performed.

Students first determine the null and alternative hypotheses to be tested in order to answer our question: Do Shang Pin and Golden Bowl fortune cookies have the same distribution of Type of Fortune? The null hypothesis is that the percentages of the fortunes that are Prophecy, Advice, Wisdom, and Misc. are the same for Shang Pin and Golden Bowl fortune cookies. And the alternative hypothesis is that the percentages of the fortunes that are Prophecy, Advice, Wisdom, and Misc. are not the same for Shang Pin and Golden Bowl fortune cookies.
Then, students are introduced to the necessary data conditions along with the formula for calculating the chi-square test statistic.

The necessary data conditions for the chi-square test of homogeneity are that: (1) all expected counts are greater than 1 and (2) at least 80% of the table cells have an expected count greater than 5. To compute the expected count for each table cell the following formula is applied:

$$\text{Expected count} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Total } n}.$$  

Once the expected counts have been calculated, they are used to calculate the chi-square test statistic:

$$\text{Chi-Square} = \chi^2 = \sum_{\text{all cells}} \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}.$$  

Explain to students that the chi-square test statistic measures the difference between the observed counts and the counts that would be expected if the null hypothesis were true. So, a large difference between the counts is evidence against the null hypothesis (or in other words a large test statistic value is evidence against the null hypothesis).

Students are asked to calculate the expected counts for the class two-way frequency table. The expected counts are shown in Table 6.

Table 6. Expected cell counts for the example class data.

<table>
<thead>
<tr>
<th>Type of Fortune</th>
<th>Brand of Cookie</th>
<th>Prophecy</th>
<th>Compliment</th>
<th>Advice</th>
<th>Wisdom</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shang Pin</td>
<td>103×31/195 = 16.37</td>
<td>103×55/195 = 29.05</td>
<td>103×101/195 = 53.35</td>
<td>103×8/195 = 4.23</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>Column Totals</td>
<td>31</td>
<td>55</td>
<td>101</td>
<td>8</td>
<td>195</td>
<td></td>
</tr>
</tbody>
</table>

Students see that none of the expected counts are less than 1. However, only 75% of the expected counts are greater than 5. After noting that for the class data the necessary conditions have not been met for the chi-square test of homogeneity, explain to students that the test will be performed anyway, for purposes of illustration.
Applying the formula for the chi-square test statistic to the example class data:

\[ \chi^2 = \frac{(16-16.37)^2}{16.37} + \frac{(34-29.05)^2}{29.05} + \frac{(49-53.35)^2}{53.35} + \frac{(4-4.23)^2}{4.23} \]

\[ + \frac{(15-14.63)^2}{14.63} + \frac{(21-25.95)^2}{25.95} + \frac{(52-47.65)^2}{47.65} + \frac{(4-3.77)^2}{3.77} \approx 2.58. \]

In order to have students calculate the p-value ask them to recall that a large test statistic is evidence against the null hypothesis. Thus the p-value will be the probability that the chi-square test statistic could have been as large or larger if the null hypothesis were true.

On the TI-84 PLUS calculator students can use the test statistic value to find the corresponding p-value. Select 2nd \( \Rightarrow \) DISTR \( \Rightarrow \) \( \chi^2 \text{cdf}( \Rightarrow \) ENTER. Within the parentheses, the students need to enter the lower bound, upper bound, degrees of freedom. The lower bound will always be the test statistic due to the shape of the chi-square distribution. For the upper bound, students can enter any very large number such as 10000000. The degrees of freedom are found using the formula \( df = (r - 1)(c - 1), \) where \( r \) is the number of rows in the table and \( c \) is the number of columns. Note that in our two-by-four table, the degrees of freedom are equal to 3. So for our example class data the p-value is .4610.

Based upon the p-value, students decide whether or not to reject the null hypothesis and provide a conclusion in this problem’s context. Since the p-value is rather large, at any reasonable level of significance, the null hypothesis will not be rejected. The data do not provide significant evidence to indicate that the percentages of the fortunes that are Prophecy, Advice, Wisdom, and Misc. differ for Shang Pin and Golden Bowl fortune cookies.

Finally, discuss with students that the assumptions, test statistic calculation, and p-value calculation are the same for the chi-square test of homogeneity and the chi-square test of independence. The distinction lies in how the data were collected and in the formulation of the hypotheses.
Assessment

1. Gender (female or male) and handedness (right-handed or left-handed) are recorded for a randomly selected sample of adults. Of the 100 women in the sample, 92 women are right-handed. Of the 80 men in the sample, 70 men are right-handed.
   (a) Write a two-way table of observed counts.

   (b) Determine expected counts for all combinations of gender and handedness.

   (c) Calculate the value of the chi-square test statistic.

2. State the most appropriate chi-square test to use to address each of the following research questions.
   (a) A researcher wants to determine if scoring high or low on an artistic ability test depends on being right or left-handed.

   (b) A national organization wants to compare the distribution of level of highest education completed (high school, college, masters, doctoral) for Republicans versus Democrats.
Answers

1. (a) 

<table>
<thead>
<tr>
<th></th>
<th>Right-handed</th>
<th>Left-handed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td>92</td>
<td>8</td>
<td>100</td>
</tr>
<tr>
<td>Men</td>
<td>70</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>Total</td>
<td>162</td>
<td>18</td>
<td>180</td>
</tr>
</tbody>
</table>

(b) The following solution uses the fact that expected counts have the same row and column totals as the observed counts do.

<table>
<thead>
<tr>
<th></th>
<th>Right-handed</th>
<th>Left-handed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td>100 \times \frac{162}{180} = 90</td>
<td>100 - 90 = 10</td>
<td>100</td>
</tr>
<tr>
<td>Men</td>
<td>162 - 90 = 72</td>
<td>80 - 72 = 8</td>
<td>80</td>
</tr>
<tr>
<td>Total</td>
<td>162</td>
<td>18</td>
<td>180</td>
</tr>
</tbody>
</table>

(c) \[ \chi^2 = \frac{(92 - 90)^2}{90} + \frac{(8 - 10)^2}{10} + \frac{(70 - 72)^2}{72} + \frac{(10 - 8)^2}{8} = 1.0 \]

2. (a) The chi-square test of independence.
(b) The chi-square test of homogeneity.

References


General Background

Fortune cookies are an American invention. They originated in California, but who the actual inventor was, and which city in California is the true home of the fortune cookie, has continued to be a matter of debate. Unequivocally not Chinese, the fortune cookie may in fact not even be Chinese American. Fortune cookies became common in Chinese restaurants after World War II. Desserts were not traditionally part of Chinese cuisine, and the cookies thus offered Americans something familiar with an exotic flair. Although there have been a few cases reported of individuals actually liking the texture and flavor of fortune cookies, most consider the fortune to be the essence of the cookie. Early fortunes featured Biblical sayings, or aphorisms from Confucius, Aesop, or Ben Franklin. Later, fortunes included recommended lottery numbers, smiley faces, jokes, and sage, if hackneyed, advice. Today’s messages are variously cryptic, nonsensical, feel-good, hectoring, bland, or mystifying.

People often take fortune cookie messages to heart. They crack open the yellow crescent moon cookies that conclude their Chinese restaurant meal, and eagerly hunt for predictions, revelations, and deeper meaning. Many save their favorites, carrying them around in a wallet. According to Hendrick (2004), “Research indicates that about 96 percent of people who eat Chinese food open their cookies and read the fortunes, and that 67 percent read them aloud so that everyone dining with them will hear.”  (p. 6NW)

Specific Background
Background adapted from: A Textual Analysis of Fortune Cookie Sayings: How Chinese Are They? by Jing Yin and Yoshitaka Miike.

The Longman Dictionary of Contemporary English (1995) defined a fortune cookie as “a Chinese biscuit that contains a piece of paper that says what will happen to you in the future” (p. 718). This definition probably coincides with people’s common expectations about fortune cookies. However, an analysis performed by Yin and Miike revealed that they do more than telling about the future. According to Yin and Miike, fortune cookies have four primary functions. In addition to prophecy, they also offer compliments and provide advice and wisdom. (p. 21)

1Image generated from: http://roadrunner.pacprod.com/cgi-bin/GRCard.exe?ACTION=CREATE&CONFIG=fortunecookie
**Fortune Cookies as Prophecies**
One category of fortune cookie sayings is prophecy. The prophecy category is characterized by the use of the future tense. The typical prophetic fortune cookie starts with “you will” or “something (some-one) will….” (p. 22)

However, unlike fortunetellers or other types of prophecy that hopefully give signs of the future as accurately as possible, whether positive or negative, fortune cookie sayings make only positive predictions. (p. 22)

**Examples of prophecies:** (pp. 23, 26)
You will be richer day by day after this moment.
An exciting opportunity lies ahead if you are not timid.

**Fortune Cookies as Compliments**
People generally may not associate fortune cookies with compliments. However, some fortune cookie sayings can be lumped under the category of compliments. Fortune cookies pay people compliments by praising their good character. (p. 28)

**Examples of compliments:** (pp. 28, 30)
Your presence livens up any conversation.
You have the making of a leader, not a follower.

**Fortune Cookies as Advice**
The third category of fortune cookie sayings is advice. This type of fortune cookie saying is written in the directive form, telling someone to do something. Unlike the prophetic and complimentary fortune cookies that use the second person to address customers, advisory fortune cookies often omit direct address. And yet, they imply a dialogue with their targeted consumers. (p. 32)

**Examples of advice:** (pp. 32, 33)
Be innovative, take charge of new ideas.
Remember to share good fortune as well as bad with your friends.

**Fortune Cookies as Wisdom**
The last category of fortune cookie sayings is wisdom. Unlike the advice category, this category is written in the assertive mode, stating a fact or proposition. (p. 35)

**Examples of wisdom:** (p. 35)
A merry heart does good like a medicine.
No man is rich enough to buy back his past.
Introductory Activity

Question of Interest: Are all fortune cookie fortunes the same? That is, do different brands of fortune cookies have different types of fortunes?

1. You will be given 3 fortune cookies of each of two brands: Shang Pin and Golden Bowl. Open your cookies, read their fortunes, and classify the Type of Fortune as: Prophecy, Advice, Wisdom, or Miscellaneous (Miscellaneous will include Compliments and Other). Once you have classified your fortunes, put your tallies on the white board in the appropriate location. Once the class has completed all of their tallies, fill in all of the cells in the frequency table below.

<table>
<thead>
<tr>
<th>Type of Fortune</th>
<th>Prophecy</th>
<th>Advice</th>
<th>Wisdom</th>
<th>Misc.</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand of Cookie</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shang Pin</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Golden Bowl</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. (a) In the context of this problem, what is a sampling unit?

(b) In the context of this problem, what are the two variables of interest?

3. In order to help answer the question of interest, we will calculate some percentages from our two-way frequency table.

(a) i. What percentage of all of the fortune cookie sayings were Prophecy? 

ii. What percentage of all of the fortune cookie sayings were Advice? 

iii. What percentage of all of the fortune cookie sayings were Wisdom? 

iv. What percentage of all of the fortune cookie sayings were Misc.? 

You have just calculated what we call the marginal distribution of the Type of Fortune. Explain why it makes sense to call this a marginal distribution:
(b)  

i. What percentage of all of the fortunes came from a Golden Bowl cookie and were a Prophecy? ________

ii. What percentage of all of the fortunes came from a Shang Pin cookie and contained Wisdom? ________

You have just calculated two examples of what we call a joint percentage (relative frequency). Explain why it makes sense to call these joint relative frequencies:

(c) Now, you will calculate the conditional distribution of the Type of Fortune given the Brand of fortune cookie. That is, for each brand, what are the percentages of the Types of Fortunes. When you calculate this conditional distribution, your Row Totals should be approximately 100%.

<table>
<thead>
<tr>
<th>Brand of Cookie</th>
<th>Prophecy</th>
<th>Advice</th>
<th>Wisdom</th>
<th>Misc.</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shang Pin</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Golden Bowl</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based upon the conditional distribution that you calculated, would you say that the two brands Shang Pin and Golden Bowl have the same Type of Fortunes? Explain.
(d) For their data collection, Yin and Miike categorized 595 fortune cookies from a variety of Chinese restaurants. The results of their analysis appear in the table below:

**Categories and Themes of Fortune Cookie Sayings (p. 22)**

<table>
<thead>
<tr>
<th>Categories</th>
<th>Numbers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prophecy</td>
<td>367 (61.7)</td>
</tr>
<tr>
<td>Compliments</td>
<td>66 (11.1)</td>
</tr>
<tr>
<td>Advice</td>
<td>72 (12.1)</td>
</tr>
<tr>
<td>Wisdom</td>
<td>90 (15.1)</td>
</tr>
<tr>
<td>Total</td>
<td>595 (100)</td>
</tr>
</tbody>
</table>

i. We want to see if our data collection produced results comparable to Yin and Miike. Combine the results for the Shang Pin and Golden Bowl fortune cookies and fill in all of the cells in the following frequency table.

<table>
<thead>
<tr>
<th>Brand of Cookie</th>
<th>Shang Pin/Golden Bowl</th>
<th>Yin and Miike’s Brands</th>
<th>Column Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Fortune</td>
<td>Prophecy</td>
<td>Advice</td>
<td>Wisdom</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ii. If we want to compare our class results to the results of Yin and Miike; should we use marginal, joint, or conditional percentages? Why?
iii. Calculate the Conditional Distribution of Type of Fortune given Brand.

<table>
<thead>
<tr>
<th>Brand of Cookie</th>
<th>Type of Fortune</th>
<th>Prophecy</th>
<th>Advice</th>
<th>Wisdom</th>
<th>Misc.</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shang Pin/Golden Bowl</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yin and Miike’s Brands</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based upon the conditional distribution, does it appear that our data collection has produced results that are comparable to the results of Yin and Miike? If not, what are the differences? And what might account for the differences?
Activity Extension

Earlier in Question 3, part (c) you calculated the conditional distribution of Type of Fortune given brand of cookie (Shang Pin or Golden Bowl). You determined that the conditional percentages seemed to differ. But, were the differences significant?

In order to answer the question: Do Shang Pin and Golden Bowl fortune cookies have the same distribution of Type of Fortune? We need to use a chi-square test of homogeneity.

In general, the test of homogeneity answers the question, “Do two or more populations have the same distribution for one categorical variable?” It assesses whether one categorical variable is distributed the same in two (or more) different populations. The null hypothesis for the test of homogeneity is that the distribution of the categorical variable is the same for the two (or more) populations. (Gunderson, 2012.)

Recall that we can use five steps for assessing statistical significance.

1. Determine the null and alternative hypotheses to be tested in order to answer our question.

   \[ H_0: \]

   \[ H_a: \]

2. Check necessary data conditions and calculate the test statistic.

   The necessary data conditions for the chi-square test of homogeneity:
   1. All expected counts should be greater than 1.
   2. At least 80% of the table cells should have an expected count greater than 5.

   To compute the expected count for each cell:
   
   \[ \text{Expected count} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Total } n} \]

   And, to compute the test statistic:  
   \[ \text{Chi-Square} = \sum_{\text{all cells}} \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}} \]

   Note: The chi-square test statistic measures the difference between the observed counts and the counts that would be expected if the null hypothesis were true. So, a large difference between the counts is evidence against the null hypothesis (a large test statistic value is evidence against the null hypothesis).
3. (a) Calculate the expected counts for our two-way frequency table. Fill your answers in to the table below.

<table>
<thead>
<tr>
<th>Type of Fortune</th>
<th>Row Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand of Cookie</td>
<td></td>
</tr>
<tr>
<td>Shang Pin</td>
<td></td>
</tr>
<tr>
<td>Golden Bowl</td>
<td></td>
</tr>
<tr>
<td>Column Totals</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Prophecy</th>
<th>Compliment</th>
<th>Advice</th>
<th>Wisdom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shang Pin</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Golden Bowl</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Are any of the expected counts less than 1?

(c) ________% of the table cells have an expected count greater than 5?

(d) Are the necessary data conditions met for performing a chi-square test of homogeneity? Why or why not?

(e) Even if you answered ‘no’ to part (d), calculate the chi-square test statistic:

Chi-Square Test Statistic:
4. Calculate the $p$-value. 
Recall that a large chi-square test statistic is evidence against the null hypothesis. So how large is large enough to declare significance? The $p$-value will be the probability that the chi-square test statistic could have been as large or larger if the null hypothesis were true. To calculate the $p$-value we use the chi-square probability distribution with degrees of freedom, 
$$ df = (\text{Rows} - 1)(\text{Columns} - 1). $$

$p$-value =

5. Based upon the $p$-value decide whether or not to reject the null hypothesis and provide a conclusion in this problem’s context.

Note: The mechanics of tests of independence and tests of homogeneity are the same. The distinction is the way in which the data was collected. If two categorical variables are collected for each subject, then a test of independence should be performed. If a single categorical variable is collected for each of two (or more) groups, then a test of homogeneity should be performed.
Text of the Fortunes Extracted from the Cookies

Brand: Shang Pin – Distributed by Asian Foods, Inc. – St. Paul, MN  
Purchase Date: May 15, 2014  

1. A new friend helps you break out of an old routine.  
2. Bring something up from the back burner.  
3. The life of every woman or man – the heart of it – is pure and holy joy.  
4. Ambition knows no obstacles.  
5. Cooking is easy. Good taste is hard. That’s why you call take out.  
6. All happiness is in the mind.  
7. An ounce of care is worth a pound of cure.  
8. Accept something that you cannot change, and you will feel better.  
9. You will be fortunate in the opportunities presented to you.  
10. You can only live once, but if you do it right, once is enough.  
11. When you have no choice, mobilize the spirit of courage.  
12. Do not seek so much to find the answer as much as to understand the question better.  
13. A good beginning is half the task.  
14. A man cannot be comfortable without his own approval.  
15. The smallest deed is better than the biggest intention.  
16. An hour with friends is worth more than ten with strangers.  
17. Courage is the mastery of fear – not the absence of fear.  
18. What is temporary has to be temporary. Don’t let it last longer than a year.  
19. Action is the proper fruit of knowledge.  
20. This is a time for love and affection.  
21. You can’t have everything…where would you put it all?  
22. A couple of extra bucks could be floating in your direction.  
23. When you awaken tomorrow, solutions to your problems will become clear.  
24. Be prepared for a sudden, needed, and happy change in plans.  
25. You are kind and broad-minded.  
26. The man who rows the boat doesn’t have time to rock it.  
27. Today it’s okay to make a few choices by going with your instincts.  
28. Analyze only when necessary.  
29. A person of words and not of deeds is like a garden full of weeds.  
30. You have a friendly heart and are well admired.  
31. All things come to him who goes after them.  
32. You need to talk to someone about what’s on your mind.  
33. Do what is right, not what you should.  
34. Be calm and collected, peace is a virtue.  
35. A merry heart does good like a medicine.  
36. You must be willing to act today in order to succeed.  
37. A distant relative will phone you soon.  
38. Today’s profits are yesterday’s good well ripened.  
39. A calm sea does not make a skilled sailor.  
40. Awaken your divine nature within.  
41. A rolling stone gathers no moss…but it obtains a certain polish!
42. To forgive others one more time is to create one more blessing for yourself.  
Wisdom
43. For insight on quandary, turn to people with firsthand experience.  
Advice
44. The reward for having feelings is great joy.  
Wisdom
45. To truly find yourself you should play hide and seek alone.  
Advice
46. Accept yourself.  
Advice
47. Discriminating mind leads you in the proper direction.  
Advice
48. A great pleasure in life is doing what others say you can’t.  
Wisdom
49. While you’re carrying a grudge, others are out dancing.  
Prophecy
50. Advancement will come with hard work.  
Wisdom
51. A smooth sea never made a skillful mariner!  
Wisdom
52. A window of opportunity won’t open itself.  
Wisdom
53. There are no limitations to the mind except those we acknowledge.  
Wisdom
54. A good memory is one trained to forget the trivial.  
Wisdom
55. A person with a determined heart frightens problems away.  
Advice
56. A refreshing change is in your future.  
Prophecy
57. You have a good start.  Work harder!  
Advice
58. What you will discover will be yourself.  
Prophecy
59. You can’t learn less.  
Wisdom
60. Be on the alert for new opportunities.  
Advice
61. Be concerned, not obsessed, with your health.  
Advice
62. Your ability to trust fuels your ability to love.  
Wisdom
63. True love is only found in the heart.  
Wisdom
64. Be broke or be wealthy, but never accept mediocrity.  
Advice
65. A smile increases your face value.  
Wisdom
66. A ship in harbor is safe, but that’s not why ships are built.  
Wisdom
67. Creative energy is up – capitalize on it.  
Advice
68. Don’t worry about the world coming to an end.  It’s already tomorrow in Australia.  
Advice
69. A new voyage will fill your life with untold memories.  
Prophecy
70. Tough times never last but tough people do.  
Wisdom
71. Your greatest asset is not the quantity of your friends, rather the quality of your friends.  
Wisdom
72. Time is not measured by a watch but by moments.  
Wisdom
73. Absence sharpens love, but presence strengthens it.  
Wisdom
74. You value freedom – grant it to others.  
Advice
75. You identified the issue.  Bring the attention to it.  
Advice
76. You only treasure what you cannot possess.  
Wisdom
77. The path of life shall lead upwards for you.  
Prophecy
78. A dose of adversity is often as needful as a dose of medicine.  
Wisdom
79. A bold attempt is half of success.  
Wisdom
80. You will be sought out for your diplomatic skills.  
Prophecy
81. You will receive a fortune.  (cookie)  
Prophecy
82. You have a pair of shining eyes.  
Compliment
83. Your reputation is your wealth.  
Wisdom
84. Vary your friendships.  
Advice
85. You will be graced by the presence of a loved one soon.  
Prophecy
86. A new outlook brightens your image and brings new friends.  
Wisdom
87. Advice comes in all forms; some help you and some hurt you.  
88. You will become more passionate and determined about your convictions.  
89. Three things cannot be long hidden: the sun, the moon, and the truth.  
90. A person is not wise simply because one talks a lot.  
91. Answer just what your heart prompts you.  
92. Allow your mind to absorb new knowledge.  
93. Watch your character, for it becomes your destiny.  
94. A leader is powerful to the degree he empowers others.  
95. All generalities are false.  
96. Today you should spend some time to search in yourself.  
97. Accept the challenges, so that you may feel the exhilaration of victory.  
98. At the end of each day, think “what has this day brought me, and what have I given it?”  
99. Act like a role model. The younger is watching.  
100. You should pay for this check. Be generous.  
101. A part of us remains wherever we have been.  
102. Value your present moments.  
103. About time I got out of that cookie.
1. Good luck bestows upon you. You will get what your heart desires.  
2. Happiness isn’t something you remember, it’s something you experience.  
3. Faith is the substance of things hoped for, the evidence of things unseen.  
4. Great ambition makes great men.  
5. Happiness lies in the joy of achievement and the thrill of creative effort.  
6. Good things are being said about you. (3 total)  
7. Grand adventures await those who are willing to turn the corner. (2 total)  
8. Don’t accept that others know better than you. (2 total)  
9. Good news will come to you from far away. (2 total)  
10. Do the thing you fear and the death of fear is certain.  
11. Happy news is on its way to you.  
12. Golden investment opportunities are arising.  
13. Failure is not defeat until you stop trying.  
14. Do not worry about holding a high position; worry rather about playing your proper role.  
15. Grasp opportunities to create the future.  
16. Great thoughts come from the heart.  
17. Gratitude is not only the greatest of virtues, but the parent of all others.  
18. Good things will come to you in due course of time. (2 total)  
19. Hard work pays off in the future, laziness pays off now. (2 total)  
20. Do not seek so much to find the answer as much as to understand the question better.  
21. Happiness is around the next corner, wealth down the street.  
22. Happiness measures a person’s real worth.  
23. Happiness isn’t something you remember, it’s something you experience. (2 total)  
24. Great things are made of little things. (2 total)  
25. Go shopping.  
26. He who bravely dares must sometimes risk a fall.  
27. Fate will find a way.  
28. Good bakers always make plenty of dough.  
29. Have a vision. Be demanding.  
30. Don’t be afraid of fear.  
31. Grant yourself a wish this year; only you can do it.  
32. Fate loves the fearless.  
33. Give to the world the best you have and the best will come back to you.  
34. Do not fear failure. (2 total)  
35. Do not underestimate yourself. Human beings have unlimited potentials.  
36. He who is shipwrecked the second time cannot lay the blame on Neptune. (2 total)  
37. Good character is more to be praised than outstanding talent.  
38. He who is afraid of asking is ashamed of learning.  
39. Hard work without talent is a shame, but talent without hard work
is a tragedy.

40. Greatest fool of all is the man who fools himself.  
Wisdom

41. Domestic conditions demand your attention.  
Advice

42. Do what you wish, as long as it does not harm anyone.  
Advice

43. Good people are good because they’ve come to wisdom through failure.  
Wisdom

44. Great acts of kindness will befall you in the coming months.  
Prophecy

45. Happier days are definitely ahead for you. Struggle has ended.  
Prophecy

46. He can who thinks he can. And he can’t who thinks he can’t.  
This is an indisputable law.  
Wisdom

47. Good news is on the way.  
Prophecy

48. Doing the best at this moment puts you in the best place for the next moment.  
Wisdom

49. Goodness is its own reward. (3 total)  
Wisdom

50. Grant yourself a wish this year; only you can do it.  
Advice

51. Hardly anyone knows how much is gained by ignoring the future.  
Wisdom

52. Don’t ask, don’t say. Everything lies in silence.  
Advice

53. Face facts with dignity.  
Advice

54. Don’t be hasty; prosperity will knock on your door soon.  
Prophecy

55. Good luck is a hop, skip, and jump away. Hop to it!  
Prophecy

56. He who can take advice is sometimes superior to those who give it.  
Wisdom

57. He who is afraid of doing too much always does too little.  
Wisdom

58. Do not put off till tomorrow what can be enjoyed today.  
Advice

59. Failure is opportunity in disguise.  
Wisdom

60. Fear is interest paid on a debt you may not owe.  
Wisdom

61. Half of being smart is knowing what you are dumb about.  
Wisdom

62. Good to begin well, better to end well.  
Wisdom

63. Great minds must be ready not only to take opportunities, but to make them. (2 total)  
Wisdom

64. He who enjoys doing and enjoys what he has done is happy.  
Wisdom

65. Good sense is the master of human life.  
Wisdom

66. Good news of a long-awaited event will arrive soon.  
Prophecy

67. He climbs highest who helps another up.  
Wisdom

68. Good work, good life, good love, good-bye oppression. (2 total)  
Wisdom

69. Do not follow where the path may lead. Go where there is no path...and leave a trail.  
Advice

70. Good advice jars the ear.  
Wisdom

71. Happy news is on its way to you.  
Prophecy

72. Do not let what you cannot do interfere with what you can do.  
Advice

73. Gratitude is not only the greatest of virtues, but the parent of all others.  
Wisdom

74. Hard work is always appreciated.  
Wisdom

75. Good judgment comes from experience.  
Wisdom

76. Dogs have owners, cats have staff.  
Wisdom

77. Fearless courage is the foundation of victory.  
Wisdom

78. Do what you love and the necessary resources will follow.  
Prophecy

79. Guard yourself against evil temptations.  
Advice

80. Give yourself some peace and quiet for at least a few hours.  
Advice

81. Happiness is often a rebound from hard work.  
Wisdom
82. Good instincts usually tell you what to do before your head has figured it out.  
Wisdom
83. He who hesitates is last.  
Wisdom
84. Happy event will take place shortly in your home.  
Prophecy
85. Fear is the darkroom where negatives are developed.  
Wisdom
86. Do not let what you do not have, prevent you from using what you do have.  
Advice
87. Greed leads to poverty.  
Wisdom
88. Do you see difficulty behind every opportunity, or opportunity behind every difficulty.  
Wisdom
89. Faith answered. No one was there.  
Wisdom
90. Either way you are right.  
Misc
91. Hallelujah!  
Misc
92. Great! You're ready for a party. (2 total)  
Misc

*The Golden Bowl fortunes had some repeats as indicated above. The repeats were not included as part of the analysis.