A NEW STOCHASTIC PROGRAMMING METHOD FOR VEHICLE STRUCTURES

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ABSTRACT

This paper presents an application of a new stochastic programming method for vehicle side impact design. Nonlinear response surface models are employed as the ‘real’ models for the side impact related performance functions to conduct this study. The main goal is to enhance vehicle side impact crash performance while minimizing vehicle weight under various uncertainties. The new algorithm alleviates the computational burden of excessive model evaluations by estimating the objective and constraint functions during the optimization process through a reweighting method. The efficiency and accuracy of this algorithm are demonstrated by solving a real-world vehicle safety design problem.

Keywords: Stochastic Programming, Optimization Under Uncertainty, Nonlinear Programming, Side Impact

INTRODUCTION

The computational analysis of crashworthiness for vehicle impact has become a powerful and efficient tool in reducing the cost and development time for a new product. Today, nonlinear finite element-based crash analysis codes are commonly used to simulate many laboratory vehicle crash events, e.g. frontal impact, side impact, interior head impact, and rear impact.

Numerical optimization is a useful and systematic tool for automatically selecting appropriate design parameters. It has been widely used and achieved significant results in the automotive industry. Sobieski et al. (2001) conducted the multidisciplinary design optimization for a car body structure under constraints of Noise, Vibration, and Harshness (NVH) and roof crush. Kodiyalam et al. (2001) reported a multidisciplinary optimization of a vehicle system in a scalable high-performance computing environment. For vehicle safety optimization, Yang et al. (1994) conducted a feasibility study of vehicle safety CAE optimization. Stander (1999) investigated the crashworthiness problem using both the response surface method and massively parallel programming. Yang et al. (2000) compared approximate methods for safety optimization of large systems in terms of accuracy and effectiveness.


This paper employs a new stochastic programming method called better optimization of nonlinear uncertain systems, BONUS (Sahin and Diwekar 2002), for vehicle side impact design. BONUS uses sampling to estimate objective and constraint functions with uncertainties. It reduces the computational burden of excessive model simulations during the optimization process through a reweighting method. Since crash analysis is computationally intensive, global response surface models, which are generated using stepwise regression coupled with the optimal Latin hypercube sampling for design of experiment, are treated as the ‘real’ models for this study (Gu et al. 2001).

The following sections are organized as follows: First, the details of BONUS is introduced, and then a vehicle side impact design problem aiming at minimizing vehicle weight while maintaining or enhancing vehicle side impact performance is presented. It follows by the robustness assessment of the baseline and deterministic optimal designs as well as obtaining stochastic optimal designs with the consideration of various uncertainties using both the traditional approach and BONUS. Results of both methods are then compared. Finally, the conclusion is summarized at the end.

BEETTER OPTIMIZATION OF NONLINEAR UNCERTAIN SYSTEMS (BONUS)

A general stochastic optimization approach involves two computationally intensive recursive loops: (1) the inner sampling loop, and (2) the outer optimization loop (Figure 1). The commonly used method for the inner sampling loop is Monte Carlo sampling (MCS) technique, which uses random samples selected from assumed input distribution to obtain estimates of the output distributions and associated statistical characteristics, such as mean, variance, or percentiles. A numerical method for solving the outer optimization loop of nonlinear programming problems (NLPs) is sequential quadratic programming (SQP). As optimization progresses and new decision variables are determined, shifts in uncertain...
variables are observed, resulting in a new input distribution. The model is re-evaluated for another sample set which is generated from the new distribution. During the optimization iterations, even for small sample sizes, the repeated evaluation of the model is a significant bottleneck.

A new stochastic programming method called BONUS developed by Sahin and Diwekar (2002) is employed for stochastic optimization. The main idea of BONUS shown in Figure 2 can be summarized as an efficient approximation for calculating the objective and constraint functions. The traditional approach, shown in the center, relies on developing a sampling loop and evaluating this loop for every sample that is generated using the input distribution. The new approach follows the big arrows in Figure 2. Instead of running the model for every sample point, the output distributions are estimated based on the base case input and output distributions.

An initial set of samples $X^*_i, i = 1, \cdots, N$ following uniformly distributed base case distribution is generated and the model is run to determine the base case output distributions. Let's assume $X$ is multivariate with $K$-dimension ($K = 1$ if $X$ is univariate) and its density can be calculated for one variable at a time according to the kernel density approach as:

$$g(X_{i,k}^*) = \frac{1}{N \cdot h} \sum_{j=1}^{N} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{X_{i,k}^* - X_{j,k}^*}{h} \right)^2}$$  \hspace{1cm} (1)$$

where $h$ is the width for the Gaussian kernel, which depends on the sample size and variance of the samples.

For independent variables, the joint probability density for each point is calculated as:

$$g(X^*_k) = \prod_{k=1}^{K} g(X_{i,k}^*)$$  \hspace{1cm} (2)$$

After determining the model output $Q(X^*_i), i = 1, \cdots, N$ for each input sample $X^*_i, i = 1, \cdots, N$, the optimization algorithm generates new design variables. This is followed by the generation of a new set of samples $(X_j, j = 1, \cdots, N)$ for the new input distribution. Let's assume $X$ is multivariate with $K$-dimension ($K = 1$ if $X$ is univariate), and the probability density of each $X^*_i, i = 1, \cdots, N$ in the new input distribution is determined for one variable at a time through the kernel density approximation as:

$$f(X_{i,k}^*) = \frac{1}{N \cdot h} \sum_{j=1}^{N} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{X_{i,k}^* - X_{j,k}^*}{h} \right)^2}$$  \hspace{1cm} (3)$$

For independent variables, the joint probability density for each point $X^*_i, i = 1, \cdots, N$ in the new input distribution is calculated as:

$$f(X^*_k) = \prod_{k=1}^{K} f(X_{i,k}^*)$$  \hspace{1cm} (4)$$
At this stage, the model is not re-run; instead, a reweighting approach is applied to approximate the probabilistic behaviors of the new output distributions. The mean (\( \hat{\mu}_{\text{ratio}} \)) and variance (\( \hat{\sigma}^2_{\text{ratio}} \)) of the output distributions are calculated by Equation 5 and 6, and then the objective and constraint functions can be obtained.

\[
\hat{\mu}_{\text{ratio}} = \sum_{i=1}^{N} \frac{f(X_i^*)}{g(X_i^*)} \cdot Q(X_i^*) - \sum_{i=1}^{N} \frac{f(X_i^*)}{g(X_i^*)} \cdot \omega_i \cdot Q(X_i^*)
\]

\[
\hat{\sigma}^2_{\text{ratio}} = \sum_{i=1}^{N} \frac{\omega_i}{N} \cdot \left[ Q(X_i^*) - \left( \hat{\mu}_{\text{ratio}} \right)^2 \right]
\]

where \( \omega_i = \frac{f(X_i^*)}{g(X_i^*)} \) is a weight

\[
\hat{\sigma}^2_{\text{ratio}} = E[Q(X^*)] - (E[Q(X)])^2
\]

The detailed explanation of BONUS can be found in Sahin and Diwekar (2002). The advantage of this technique is bypassing of the model evaluations for the new sample set during each iteration, which is computationally intensive for stochastic optimization. This is particularly critical for optimization algorithms that rely on evaluating the gradients numerically, by perturbing every design variable by a small increment and calculating the change in the objective and constraint functions. Evaluating the model for an entire sample set (due to uncertainties) over and over again during the optimization process is inefficient for complex models.

VEHICLE SIDE IMPACT DESIGN PROBLEM

A large-scale application of the proposed BONUS algorithm in the design of vehicle side impact is illustrated in Figure 3. The system model includes a full-vehicle finite element (FE) structural model, a FE side impact dummy model, and a FE deformable side impact barrier model. The system model consists of 85,941 shell elements and 96,122 nodes. In the FE simulation of the side impact event, the barrier has an initial velocity of 49.89 kph (31 mph) and impacts the vehicle structure. The CPU time for one nonlinear FE simulation using the RADIOSS software is approximately 20 hours on a SGI Origin 2000 machine. The design goal is to maintain or enhance side impact test performance while minimizing the vehicle weight.

Figure 3: Vehicle Side Impact Model

For side impact protection, the vehicle design must meet or exceed regulated side impact requirements specified by the vehicle market. Two primary side impact protection regulations are Federal Motor Vehicle Safety Standard No. 214 in the United States and ECE United Nations Economic Commission For Europe Regulation No. 95 in Europe. In this study, the ECE side impact test configuration is used. The dummy's responses are the main metric in side impact studies. The crash dummy criteria specified in the ECE side impact regulation include abdomen load, viscous criteria (upper, middle, and lower), rib deflections (upper, middle, and lower), and pubic symphysis force. The dummy's responses must meet or exceed ECE criteria. Other concerns in side impact design are the velocity of the B-pillar at the middle point \( V_{B\text{-Pillar}} \) and the velocity of the front door at the B-pillar \( V_{Door} \).

<table>
<thead>
<tr>
<th>Table 1. Regulations and Design Targets</th>
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</thead>
<tbody>
<tr>
<td>Performance</td>
</tr>
<tr>
<td>Abdomen Load (kN)</td>
</tr>
<tr>
<td>Viscous Criteria (m/s)</td>
</tr>
<tr>
<td>Middle</td>
</tr>
<tr>
<td>Lower</td>
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<tr>
<td>Rib Deflection</td>
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<tr>
<td>Upper</td>
</tr>
<tr>
<td>Middle</td>
</tr>
<tr>
<td>Lower</td>
</tr>
<tr>
<td>Pubic Symphysis Force (kN)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Properties of Design and Uncertain Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name of Variable</td>
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<tr>
<td></td>
</tr>
<tr>
<td>1 Thickness of B-Pillar inner</td>
</tr>
<tr>
<td>2 Thickness of B-Pillar reinforcement</td>
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<tr>
<td>3 Thickness of floor side inner</td>
</tr>
<tr>
<td>&amp; #2 Thickness of cross member #1</td>
</tr>
<tr>
<td>4 Thickness of door beam</td>
</tr>
<tr>
<td>6 Thickness of door belt line reinforcement</td>
</tr>
<tr>
<td>7 Thickness of roof rail</td>
</tr>
<tr>
<td>8 Material of B-Pillar inner</td>
</tr>
<tr>
<td>9 Material of floor side inner</td>
</tr>
<tr>
<td>10 Barrier height</td>
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<tr>
<td>11 Barrier hitting position</td>
</tr>
</tbody>
</table>

Note: \( x_i^0 \) is the baseline value for design variable \( x_i \).

There are 9 design variables used in the design optimization of vehicle side impact. The design variables are the thickness (\( x_i - x_i^0 \)) and material property of critical parts \( (xs, x_s) \), as shown in Table 2. All thickness design variables (unit, mm) are continuous which are allowed to vary over the range from \( 0.5x_i^0 \) to \( 1.5x_i^0 \), where \( x_i^0 \) is the baseline value for design variable \( x_i \). The two material design variables (unit, GPa) are discrete, and can be either mild steel (MST) or high strength steel (HSST). In this side impact CAE model, it is assumed that there are normally distributed uncertainties around their nominal value for all the 9 design variables.
A response surface method (RSM) developed by Gu et al. (2001) is employed to demonstrate BONUS for vehicle crashworthiness as numerical computation is out of scope of this paper. The RSM used the quadratic backward-stepwise regression method coupled with the optimal Latin hypercube sampling to generate a global response surface model for the dummy test performance in vehicle side impact. The models can be summarized as Equation 7.

\[
\begin{align*}
\text{Weight} & = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7, \\
V_{B\text{-Pillar}} & = 10.58 - 0.674x_1x_2 - 1.95x_3x_8 + 0.02054x_7x_{10}, \\
V_{Door} & = 16.45 - 0.489x_1x_3 - 0.843x_7x_8 + 0.0432x_7x_{10}, \\
Ab\text{-Load} & = 1.16 - 0.371x_1x_3 - 0.00931x_7x_{10} - 0.484x_8x_9 + 0.01343x_8x_{10}, \\
V_{C\text{ }\text{ }\text{ }}_{\text{Upper}} & = 0.261 - 0.0159x_1x_3 - 0.188x_8x_9 - 0.019x_2x_7, \\
& + 0.0144x_9x_3 + 0.0008757x_8x_{10} + 0.08045x_8x_9, \\
& + 0.00139x_8x_{10} - 0.0001575x_{10}x_{11}, \\
V_{C\text{ }\text{ }\text{ }}_{\text{Middle}} & = 0.214 + 0.00517x_3 - 0.131x_5x_8 - 0.0704x_7x_9, \\
& + 0.08045x_8x_9 + 0.03099x_5x_8 - 0.018x_7x_7, \\
& + 0.0208x_5x_8 + 0.121x_5x_9 - 0.00364x_5x_6, \\
& + 0.0007715x_8x_{10} - 0.0005354x_5x_{10}, \\
& - 0.00121x_5x_{11} - 0.00184x_8x_{10} - 0.02x_7, \\
V_{C\text{ }\text{ }\text{ }}_{\text{Lower}} & = 0.74 - 0.61x_1x_3 - 0.163x_8x_9 + 0.00123x_3x_{10}, \\
& - 0.166x_7x_9 + 0.227x_7^2, \\
RibDefl.\text{ }\text{ }_{\text{Upper}} & = 28.98 + 3.818x_3 - 4.2x_1x_3 + 0.0207x_7x_{10}, \\
& + 6.63x_8x_3 + 7.7x_8x_9 + 0.32x_7x_{10}, \\
RibDefl.\text{ }\text{ }_{\text{Middle}} & = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2, \\
& - 11.0x_3x_9 - 0.0215x_3x_{10} - 9.8x_8x_9 - 22x_8x_9, \\
RibDefl.\text{ }\text{ }_{\text{Lower}} & = 46.36 - 9.9x_3 - 12x_9x_8 + 0.1107x_7x_{10}, \\
PSF & = 4.72 - 0.5x_4 - 0.19x_3 - 0.0122x_4x_{10}, \\
& + 0.009325x_8x_{10} + 0.000191x_{11}^2.
\end{align*}
\]

The stochastic optimization problem for crashworthiness of vehicle side impact is formulated as Equations 8. It is noted that if a response (output) distribution follows normal distribution, \( \mu + 1.2816\sigma \) with respect to design specification limit is the equivalent of designing for a reliability level of 90% (or probability of failure of 10%). Even for a non-normal response distribution, \( \mu + 1.2816\sigma \) still provides a good indicator for a reliable design under uncertainty. It is also noted that the proposed method can be easily implemented for robust design by changing the objective to minimize the variance of a response, such as \( \sigma^2\text{[Weight]} \).

\[
\begin{align*}
\text{Baseline Design} & \\
1 \text{ Thickness of B-Pillar inner} & = x_1^0 = 0.50x_1^0, \\
2 \text{ Thickness of B-Pillar reinforcement} & = x_2^0 = 1.36x_2^0, \\
3 \text{ Thickness of floor side inner} & = x_3^0 = 0.50x_3^0, \\
4 \text{ Thickness of cross member} & = x_4^0 = 1.15x_4^0, \\
5 \text{ Thickness of door beam} & = x_5^0 = 0.50x_5^0, \\
6 \text{ Thickness of door belt line reinforcement} & = x_6^0 = 1.14x_6^0, \\
7 \text{ Thickness of roof rail} & = x_7^0 = 0.50x_7^0, \\
8 \text{ Material of B-Pillar inner} & = \text{MST}, \\
9 \text{ Material of floor side inner} & = \text{MST}, \\
10 \text{ Material of B-Pillar inner} & = \text{MST}, \\
11 \text{ Material of B-Pillar inner} & = \text{MST}.
\end{align*}
\]

\[
\begin{align*}
\text{Deterministic Optimal Design} & \\
1 \text{ Thickness of B-Pillar inner} & = x_1^0, \\
2 \text{ Thickness of B-Pillar reinforcement} & = x_2^0, \\
3 \text{ Thickness of floor side inner} & = x_3^0, \\
4 \text{ Thickness of cross member} & = x_4^0, \\
5 \text{ Thickness of door beam} & = x_5^0, \\
6 \text{ Thickness of door belt line reinforcement} & = x_6^0, \\
7 \text{ Thickness of roof rail} & = x_7^0, \\
8 \text{ Material of B-Pillar inner} & = \text{MST}, \\
9 \text{ Material of floor side inner} & = \text{MST}, \\
10 \text{ Material of B-Pillar inner} & = \text{MST}.
\end{align*}
\]
assessment using MCS with a very large number of samples (50,000) are conducted for both the baseline and deterministic optimal designs. The results are shown in Table 3. The deterministic optimal design can improve the test performance in vehicle side impact as well as reducing vehicle weight without considering uncertainties. However, the results also show that by the consideration of the uncertainties, some of the constraints are significantly violated by the baseline design (e.g. 13.9% violation on the lower rib deflection) and by the deterministic optimal design (e.g. 3.3% on the pubic symphysis force and total of three violations).

The results obtained by both the traditional approach and BONUS with different numbers of HSS samples are shown in Table 4. It is shown that BONUS with minimum simulations yields a similar stochastic optimal design compared to that found by the traditional approach.

Table 4 also shows that the first four stochastic optimal designs found by BONUS (with the number of HSS points less than 1,000) save slightly more vehicle weight than that found by the traditional approach, however, they slightly violate (by less than 0.5%) two of the ten constraints. The stochastic optimal design with 1,000 HSS points satisfies the constraint of the lower rib deflection better than the first four, but with slight violation (by 1.15%) on the constraint of the pubic symphysis force and provides less vehicle weight saving. The stochastic optimal design with 2,000 HSS points satisfies the constraints better than the others, but results in slightly higher vehicle weight. It is noted that all stochastic optimal designs found by BONUS have less weight and better test performance in vehicle side impact than those of the baseline and deterministic optimal designs (shown in Table 3) under the uncertainties.

Table 5 presents the relative errors of BONUS estimations compared to 50,000 Monte Carlo simulations for all the responses of the stochastic optimal designs (Table 5) found by BONUS. The relative error is calculated as follows:

$$e_j = \frac{(\mu_j + 1.2816\sigma_j)_{MCS-50,000} - (\mu_j + 1.2816\sigma_j)_{BONUS}}{(\mu_j + 1.2816\sigma_j)_{MCS-50,000}} \times 100\% \quad (9)$$

where $j$ represents response $j$.

In general, the estimation technique can provide good predictions of the actual response values under the uncertainties. The maximum relative error reduces as the number of HSS samples increases. There are a few relative errors that are larger than 10%. However, they do not affect the results in negative manners, as the constraints with relatively larger errors are mostly inactive. The accuracies of the most critical constraints, such as lower rib deflection and pubic symphysis force, have greater impacts on the results than the others.

### Table 4. Comparison of Stochastic Optimal Designs

<table>
<thead>
<tr>
<th>Methods</th>
<th>SQP + MCS (50,000)</th>
<th>BONUS (HSS-100)</th>
<th>BONUS (HSS-150)</th>
<th>BONUS (HSS-300)</th>
<th>BONUS (HSS-500)</th>
<th>BONUS (HSS-1000)</th>
<th>BONUS (HSS-2000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean values of variables</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Thickness of B-Pillar inner (mm)</td>
<td>0.500x^1</td>
<td>0.500x^2</td>
<td>0.500x^3</td>
<td>0.540x^4</td>
<td>0.506x^5</td>
<td>0.625x^6</td>
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<tr>
<td>2</td>
<td>Thickness of B-Pillar reinforcement (mm)</td>
<td>1.454x^1</td>
<td>1.444x^2</td>
<td>1.444x^3</td>
<td>1.427x^4</td>
<td>1.444x^5</td>
<td>1.470x^6</td>
</tr>
<tr>
<td>3</td>
<td>Thickness of floor side inner (mm)</td>
<td>0.500x^1</td>
<td>0.500x^2</td>
<td>0.500x^3</td>
<td>0.500x^4</td>
<td>0.502x^5</td>
<td>0.577x^6</td>
</tr>
<tr>
<td>4</td>
<td>Thickness of cross member #1 &amp; #2 (mm)</td>
<td>1.404x^1</td>
<td>1.400x^2</td>
<td>1.400x^3</td>
<td>1.352x^4</td>
<td>1.397x^5</td>
<td>1.233x^6</td>
</tr>
<tr>
<td>5</td>
<td>Thickness of door beam (mm)</td>
<td>0.500x^1</td>
<td>0.500x^2</td>
<td>0.500x^3</td>
<td>0.500x^4</td>
<td>0.500x^5</td>
<td>0.500x^6</td>
</tr>
<tr>
<td>6</td>
<td>Thickness of door belt line reinforcement (mm)</td>
<td>1.500x^1</td>
<td>1.500x^2</td>
<td>1.500x^3</td>
<td>1.500x^4</td>
<td>1.500x^5</td>
<td>1.500x^6</td>
</tr>
<tr>
<td>7</td>
<td>Thickness of roof rail (mm)</td>
<td>0.500x^1</td>
<td>0.500x^2</td>
<td>0.500x^3</td>
<td>0.500x^4</td>
<td>0.500x^5</td>
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<tr>
<td>8</td>
<td>Material of B-Pillar inner</td>
<td>HSST</td>
<td>HSST</td>
<td>HSST</td>
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<td>9</td>
<td>Material of floor side inner</td>
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<td>MST</td>
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<td>10</td>
<td>Barrier height (mm)</td>
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<td>11</td>
<td>Barrier hitting position (mm)</td>
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<tr>
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<td>[Vb]</td>
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<td>15.622</td>
<td>15.622</td>
<td>15.622</td>
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<tr>
<td>3</td>
<td>[Path]</td>
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<td>0.496</td>
<td>0.496</td>
<td>0.528</td>
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</tr>
<tr>
<td>4</td>
<td>[Vc]</td>
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<td>0.216</td>
<td>0.216</td>
<td>0.214</td>
<td>0.216</td>
<td>0.210</td>
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<td>5</td>
<td>[VC, VMD]</td>
<td>0.298</td>
<td>0.298</td>
<td>0.298</td>
<td>0.299</td>
<td>0.298</td>
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<td>32.088</td>
<td>32.069</td>
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<td>32.074</td>
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<tr>
<td>9</td>
<td>[RibDefl, M]</td>
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<td>4.001</td>
<td>4.001</td>
<td>4.020</td>
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<td>4.046</td>
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<tr>
<td>10</td>
<td>[RibDefl, L]</td>
<td>3.999</td>
<td>4.001</td>
<td>4.001</td>
<td>4.020</td>
<td>4.001</td>
<td>4.046</td>
</tr>
<tr>
<td>11</td>
<td>[PSF]</td>
<td>4.001</td>
<td>4.001</td>
<td>4.001</td>
<td>4.020</td>
<td>4.001</td>
<td>4.046</td>
</tr>
</tbody>
</table>

Note: $x^j$ is the baseline value for design variable $x_j$. All response values are confirmed by MCS with 50,000 samples.
Table 5. Relative Errors of BONUS Estimations Compared to 50,000 MCS

<table>
<thead>
<tr>
<th>Methods</th>
<th>Relative Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BONUS (HSS-100)</td>
</tr>
<tr>
<td>1</td>
<td>µ [Weight] + 1.2816σ [Weight] (Kg)</td>
</tr>
<tr>
<td>2</td>
<td>µ [Vb-1300] + 1.2816σ [Vb-1300] (m/s)</td>
</tr>
<tr>
<td>3</td>
<td>µ [Vd-1300] + 1.2816σ [Vd-1300] (m/s)</td>
</tr>
<tr>
<td>5</td>
<td>µ [VCmiddle] + 1.2816σ [VCmiddle] (m/s)</td>
</tr>
<tr>
<td>6</td>
<td>µ [VCmiddle] + 1.2816σ [VCmiddle] (m/s)</td>
</tr>
<tr>
<td>7</td>
<td>µ [VCmiddle] + 1.2816σ [VCmiddle] (m/s)</td>
</tr>
<tr>
<td>8</td>
<td>µ [RibDefl_L] + 1.2816σ [RibDefl_L] (mm)</td>
</tr>
<tr>
<td>9</td>
<td>µ [RibDefl_M] + 1.2816σ [RibDefl_M] (mm)</td>
</tr>
<tr>
<td>10</td>
<td>µ [RibDefl_U] + 1.2816σ [RibDefl_U] (mm)</td>
</tr>
<tr>
<td>11</td>
<td>µ [PSF] + 1.2816σ [PSF] (KN)</td>
</tr>
</tbody>
</table>

Maximum Relative Error % | 35.358 | 35.237 | 14.505 | 15.259 | 11.730 | 9.773

CONCLUSION

A new stochastic programming method (BONUS) is successfully applied to a vehicle side impact problem for obtaining stochastic optimal designs. The results demonstrate that BONUS can significantly reduce the computational resources by using an approximation technique for approximating performance functions instead of running model simulations during the optimization loops.

The results of this study are primarily based on the assumptions that both of the global response surface and finite element models are valid, and the distributions of random (uncertain) variables are also accurate. However, the validations of these models and the statistical analysis of these random variables are beyond the scope of this paper.

REFERENCES


